A.P. 3302

STANDARD TECHNICAL TRAINING NOTES

FOR THE

RADIO ENGINEERING TRADE GROUP

(FITTERS)

PART 1A

ELECTRICAL AND RADIO FUNDAMENTALS

(BOOK 1—SECTIONS 1 to 7)

Promulgated by Command of The Air Council

M. J. Dean

AIR MINISTRY
November, 1962

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RECORD OF AMENDMENTS

The amendments promulgated in the undermentioned Amendment Lists have been incorporated in this Publication.

Amendment List No.	Dated	Amended By	Date Incorporated
Old A.L. Nos. 1 to 20 incorporated	November 1962	—	—
1			
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4			
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STANDARD TECHNICAL TRAINING NOTES
FOR THE
RADIO ENGINEERING TRADE GROUP (FITTERS)

FOREWORD

1. These Notes are issued to assist airmen and apprentices under training as Fitters in the Radio Engineering Trade Group. Fitters in this trade group "require a thorough knowledge of the electrical and radio principles, and the elementary mathematics, appropriate to the theory of the specified equipment" (see A.P. 3282A, Vol. 2). It is with the intention of helping to attain this standard that these Notes are written. They are not intended to form a complete text-book, but are to be used as required in conjunction with lessons and demonstrations given at the radio schools. They may also be used to assist airmen on continuation training at other R.A.F. stations.

2. The Notes, which are based on the syllabuses of training for aircraft apprentices, are sub-divided as follows:—

Part 1A: Electrical and Radio Fundamentals.

This deals with the principles of electricity, electronics and radio at a level suitable for the upper technician ranks and for the advanced stream of apprentices.

Because of its bulk Part 1A has been split into three separate books: Book 1 covers basic electricity; Book 2, basic electronics; and Book 3, basic radio.

Part 1B: Basic Electricity and Radio.

This deals with the principles of electricity, electronics and radio at a level suitable for the lower technician ranks and for the normal stream of apprentices.

Part 2: Communications.

This deals with the applications of the principles covered in Parts 1A and 1B to communication systems and is intended to be used as required by all fitters in the Radio Engineering Trade Group.

Part 3: Radar.

This deals with the applications of the principles covered in Parts 1A and 1B to radar and is intended to be used as required by all fitters in the Radio Engineering Trade Group.

3. In general, fitters employed on communications equipment (i.e. air and ground wireless fitters and telegraph fitters) will be interested mainly in Part 1A or 1B and Part 2 of these Notes. Similarly, radar fitters will be concerned mainly with Part 1A or 1B and Part 3. However it is difficult to draw a firm dividing line between the knowledge required by fitters engaged in communications and that required by radar fitters. There is considerable overlapping; much of what was once regarded as being exclusively in the province of the radar fitter is now a requirement for the communications fitter also, and *vice versa.* Therefore those under training in the radar trades may find much that is useful in Part 2, whilst those under training in the communications field may find much of interest in Part 3.

4. The Notes deal with the basic theory and the applied principles of electricity, electronics and radio in a general way. They do not cover specific details of equipment in use in the Service. Such details are to be found in the official Air Publication for the equipment and this should always be consulted during the servicing of the equipment.

5. No alteration to these Notes may be made without the authority of official Amendment Lists.

AIR MINISTRY
November, 1962

LIST OF AIR PUBLICATIONS ASSOCIATED WITH THE TRADE

Principles and Techniques

A.P. 1093	R.A.F. Signal Manual, Part 2 (Radio Communication)
A.P. 1093E	Interservices Radar Manual—Radar Techniques
A.P. 1093F	Radar Circuit Principles, with Aerials and Centimetric Techniques
A.P. 1093G	Radio Circuitry Supplement
A.P. 1093H	Suppressed Aerials
A.P. 1186V	C.V. Register of Electronic Valves
A.P. 2521A	V.H.F. Ground Station Aerial Systems
A.P. 2867	Interservices Standard Graphical Symbols
A.P. 2867A	Interservice Glossary of Terms used in Telecommunications
A.P. 2867B	Interservice Glossary of Terms used in Telecommunications (Radar)
A.P. 2878C	H.F. and M.F. Aerials for Ground Stations
A.P. 2900C	Handbook of Electronic Test Methods and Practices
A.P. 3158C	R.A.F. Technical Services Manual
A.P. 3214 (Series)	The Services Textbook of Radio.

Equipment

Air Publications applicable to specific radio equipment are listed in:—

A.P. 2463	Index to Radio Publications

INSTRUCTIONAL FILMS

Title	Reference
Current of Electricity	14L/52
Nuts and Bolts	14L/178
Micrometer Calipers	14L/273
Vernier Scale	14L/413
Hammers, Chisels, Punches and Drifts	14L/1605
Files and Filing	14L/1606
Spanners, Screwdrivers and Pliers	14L/1636
Taps, Dies and Reamers	14L/1727
Hacksaws, Shears, and Vice Clamps	14L/1728
Locking Devices	14L/1729
Measuring and Marking—Precision Instruments	14L/1730
Transmission Lines—Maintenance of Coaxial Cables	14L/3280
Transmission Lines and Waveguides	14L/3288

This leaf issued with A.L. 13

Title	Reference
Vacuum Tubes—Electronic Diode	14L/3953
Cathode Ray Tube	14L/4268
Electricity and Magnetism	14L/4708
Magnetism	14L/5557
Electrical Terms	14L/5607
What is Electricity?	14L/5609
Electricity and Heat	14L/5610
Electricity and Movement	14L/5611
Electrochemistry	14L/5612
Putting Free Electrons to Work	14L/5614
A.C. and D.C.	14L/5615
The Generation of Electricity	14L/5616
The Transmission of Electricity	14L/5617
Aircraft First Line Servicing	14L/5656
Audio Oscillator	14L/5666
Volts—Ohm Meter Operation	14L/5667
Radio Shop Technician	14L/5668
First Line Servicing, Fighter Aircraft	14L/5768
Radio Antennae Fundamentals, Parts 1 and 2	14L/5780–1
R.D.F. to Radar	14L/5826
Waveguides, Parts 1 to 5	14L/5958–5962
Tuned Circuits	14L/6037
Ground Handling of Aircraft	14L/6338
The Doppler Principle in Airborne Navigation Aids	14L/6388
Centimetric Oscillators, Parts 1 to 3	14L/6397
Servomechanisms	14L/6435
Radar Techniques, Part 1—Waveform Response of C.R. Circuits	14L/6500
Radar Techniques, Part 2—Multivibrator	14L/6502
Radar Techniques, Part 3—Miller Timebase	14L/6504
Radar Techniques, Part 4—Pulse Forming by Delay Lines	14L/6506
Radar Techniques, Part 5—Flip Flop	14L/6508
Problems of Radio and Electronic Fault Finding	14L/6594
Principles of the Transistor	14L/6620

INSTRUCTIONAL FILM STRIPS

Title	Reference
Primary Cells	14J/154
Time Constant	14J/155
Distribution of Electricity	14J/194
Electricity—its Production	14J/195
Uses of Electricity	14J/196
Radiation	14J/197
Thermionic Valve	14J/198
Electrical Measuring Instruments	14J/203
The D.C. Motor	14J/204
Basic Radio Trouble-shooting, Parts 1 to 5	14J/239–243
The Internal Combustion Engine	14J/369
Elementary Principles of Cathode Ray Oscillograph	14J/370
The Cathode Ray Tube	14J/404
Magnetism and Electricity	14J/407
Waveguide Theory	14J/495–511
Waveguide Theory	14J/512–517
Introduction to Control Engineering Theory	14J/578
Introduction to Electronics	14J/586
Electronic Devices—Electron Tubes	14J/587
Basic Valve Circuits, Parts 1 to 4	14J/588–9
The Meaning of Valve Characteristics	14J/590
Telecommunication Principles	14J/606

This leaf issued with A.L. 13

LIST OF SYMBOLS AND ABBREVIATIONS

TABLE 1

Greek Letters Used in the Text

Letter			Letter		
Small	**Capital**	**Name**	**Small**	**Capital**	**Name**
α	—	Alpha	λ	—	Lambda
β	—	Beta	μ	—	Mu
γ	—	Gamma	π	—	Pi
δ	Δ	Delta	ρ	—	Rho
ε	—	Epsilon	σ	—	Sigma
η	—	Eta	φ	Φ	Phi
θ	—	Theta	ω	Ω	Omega
κ	—	Kappa			

TABLE 2

Meaning of Symbols Used in the Text

Letter	Meaning	Letter	Meaning
A	Ampere, Amplification	j	Vector operator $=\sqrt{-1}$
B	Magnetic flux density, Susceptance, Bandwidth	k	Coupling coefficient, Kilo—(prefix), constant
C	Capacitance	l	Length
D	Electric flux density, Distance	m	Modulation factor, Metre, Mass, Milli—(prefix)
E	Electromotive force, Electric field strength	n	Number
F	Farad, Factor, Force	p	Pico—(prefix)
G	Conductance, Giga—(prefix)	q	Instantaneous charge
H	Magnetic field strength, Henry	r	Length (in polar co-ordinates)
I	Electric Current	r_a	Anode slope resistance

(*continued overleaf*)

J	Joule
L	Inductance
M	Mutual inductance, Mega—(prefix)
N	Number, Noise factor, Revs. per minute
P	Power
Q	Quantity or charge of electricity, Coil amplification factor
R	Resistance
S	Magnetic reluctance
T	Temperature (Abolute), Period, Transit Time, Transformation ratio
V	Potential difference, Volt, Volume
W	Energy or work, Watt
X	Reactance
Y	Admittance
Z	Impedance
a	Area
c	Velocity of light, Cycle
d	Distance
e	Instantaneous e.m.f., Electron charge
f	Frequency
g_c	Valve conversion conductance
g_m	Valve mutual conductance
i	Instantaneous current

(*continued overleaf*)

s	Second
t	Time, Temperature
u	Velocity
v	Instantaneous potential difference
x	Distance, Length
y	Length
α	Angle, Number
β	Number, Feedback factor
γ	Propagation constant
δ	Small increment, Loss angle
ε	Base of natural logs = 2·71828
η	Efficiency
θ	Angle
κ_r	Dielectric constant
κ_o	Permittivity of free space
λ	Wavelength
μ	Permeability, Valve amplification factor
μ_o	Permeability of free space
μ_r	Relative permeability
π	Ratio of circumference to diameter of a circle = 3·14159
ρ	Specific resistance
ϕ	Angle
Φ	Magnetic flux
ω	Angular velocity = $2\pi f$
Ω	Ohm
σ	Specific conductance

TABLE 3

Prefixes for Multiples and Sub-multiples

Multiple or sub-multiple	Name	Prefix	Multiple or sub-multiple	Name	Prefix
$1,000,000,000 = 10^9$	Giga-	G	$\frac{1}{1,000,000} = \frac{1}{10^6} = 10^{-6}$	Micro-	μ
$1,000,000 = 10^6$	Mega-	M			
$1,000 = 10^3$	Kilo-	k	$\frac{1}{10^{12}} = 10^{-12}$	Micro-micro- or Pico	μμ or *p*
$\frac{1}{1,000} = \frac{1}{10^3} = 10^{-3}$	Milli-	m			

TABLE 4

Abbreviations of Units

Unit	Abbreviation	Unit	Abbreviation
Ampere	A	Gramme	g
Ampere-hour	Ah	Henry	H
Ampere-turn	AT	Joule	J
Cycles per second	c/s	Metre	m
Decibel	db	Ohm	Ω
Degree	° Centigrade = C. Fahrenheit = F.	Second	s or sec.
		Volt	V
Electron-volt	eV	Watt	W
Farad	F	Weber	Wb

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SECTION 1

CHAPTER 1

ELECTRIC CURRENT, E.M.F. AND P.D.

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PART 1, SECTION 1, CHAPTER 1

ELECTRIC CURRENT, E.M.F. AND P.D.

FUNDAMENTAL UNITS

Introduction

1. Both electrical and radio engineering are based upon the same fundamental principles, and for a sound knowledge of the techniques of radio it is essential to have an understanding of these principles. Electricity may be used to make things move, to generate heat and light, to set the diaphragm of a telephone vibrating as a source of sound, and in many chemical manufacturing processes. The study of electricity is thus closely linked with that of a number of other branches of engineering and, as a result, many electrical units are defined in terms of mechanical units. It is, therefore, necessary to know something of fundamental mechanical terms like "force", "power", and "energy" before their use in electrical and radio engineering can be appreciated.

Definitions

2. **Force** is any push or pull which alters or tends to alter the state of a body, whether of rest or of uniform motion in a straight line. The practical interpretation of this definition is that the application of force to a body will change or tend to change its **velocity** (or speed) in some way. If the body is at rest and free to move it will do so ; if moving in a straight line at a constant speed it will be deflected or alter its speed. When the velocity of a body is increasing the body is said to undergo **acceleration** and this is measured by the rate at which the velocity is changing.

3. When a force moves the body to which it is applied, **work** is done, the amount of work being equal to the product (force X distance moved in the direction of the force). Thus, in lifting a pencil work is done by the person on the pencil. This can be accomplished quickly or otherwise, and the *rate* of doing work is termed **power.**

4. Any body which has the ability to do work possesses **energy.** When work is done by one body on another, energy is transferred from the one to the other. In lifting a pencil energy is transferred from the person to the pencil. While held above the table, the pencil has *potential* energy which can be regarded as work stored for future use. When the pencil is allowed to fall the potential energy is converted to *kinetic* energy or energy of motion. Energy exists in various forms, and although it can be converted from one form to another, it cannot be created or destroyed.

Units

5. The units in which the above quantities are measured depend on the fundamental units used for length, mass, and time. In accordance with modern practice in radio the unit of length is the *metre* (m) ; that of mass, the *kilogramme* (kg) ; and that of time, the *second* (sec.) Measurement of quantities derived from these are said to be made in *metre-kilogramme-second* (*m.k.s.*) units.

6. (*a*) Velocity or speed is measured in *metres per second* (m/sec.)

(*b*) If the velocity of a body is increasing at the rate of one metre per second every second, the acceleration is *one metre per second per second* (m/sec^2).

(*c*) The unit of force is the *newton.* One newton is that force which gives one kilogramme mass an acceleration of one metre per second per second.

(*d*) The unit of work is the *joule* (J). One joule is the work done when a force of one newton acts through a distance of one metre.

(*e*) The unit of energy is the same as that of work—the *joule* (J). When one joule of work is done by one body on another, one joule of energy is transferred from the one to another.

(*f*) The unit of power is the *watt* (W). One watt is the power developed when work is done at the rate of one joule per second.

ELECTRIC CURRENT

Structure of Matter

7. The study of electricity has grown up during the past two hundred years, keeping pace with the growth of knowledge of

chemistry and of the nature of the substances (called matter) found in the world. It should be realized that chemical actions and electrical effects both happen because matter is made up in a certain way, and to understand what is meant by " an electric current" it is necessary to have an elementary knowledge of the atomic structure of matter.

8. (*a*) **Matter** is defined as anything which occupies space and is acted upon by gravitational forces ; it can exist in three states—solid, liquid, or gaseous.

(*b*) All matter is constructed from **molecules**—the smallest particles into which a substance may be divided while still retaining the chemical properties of the substance.

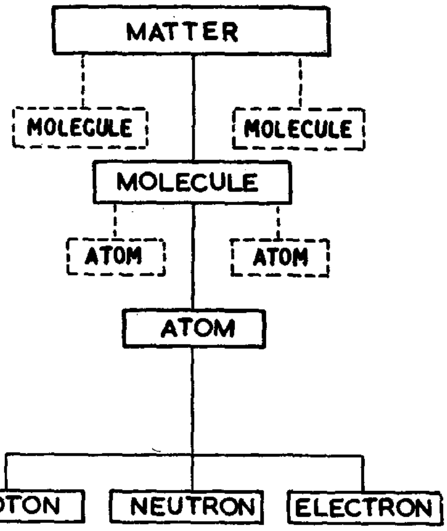


FIG. 1 — THE CONSTRUCTION OF MATTER

(*c*) Molecules are made up of **atoms**—the smallest particles of matter which can enter into chemical combination or which are obtainable by chemical separation, e.g., two hydrogen atoms and one oxygen combine to form a molecule of water.

(*d*) An **element** is a substance whose molecules are made up of the same kinds of atoms, e.g., iron, copper, and nickel.

(*e*) A **compound** is a substance whose molecules are made up of different kinds of atoms, e.g., water, salt, and lime.

Atomic Structure

9. An atom in any material contains three fundamental particles in close association:—

(*a*) **Proton.** The elementary particle of *positive* electricity or charge.

(*b*) **Electron.** The elementary particle of *negative* electricity ; its charge is equal and opposite to that on a proton.

(*c*) **Neutron.** A particle having mass approximately equal to that of a proton but having zero electric charge.

10. The protons and neutrons, comprising almost all the mass of the atom, are confined to the central core or nucleus of the atom; the electrons rotate at high speed in orbits of various sizes around the nucleus, like the planets round the sun.

11. The number of planetary electrons in an atom varies with the element and gives the *atomic number* of the element. The atomic number of hydrogen is 1, indicating that it has one electron ; that of uranium is 92. At the present day the number of known elements (and hence, of different kinds of atoms) is 98, although research indicates that this may well be extended. Table 1 gives a short list of some of the better-known elements.

Atomic Number	Element	Symbol	Atomic Number	Element	Symbol
1	Hydrogen	H	33	Arsenic	As
6	Carbon	C	47	Silver	Ag
8	Oxygen	O	50	Tin	Sn
13	Aluminium	Al	74	Tungsten	W
26	Iron	Fe	79	Gold	Au
28	Nickel	Ni	82	Lead	Pb
29	Copper	Cu	92	Uranium	U

TABLE 1. SOME WELL-KNOWN ELEMENTS

12. An atom, under normal conditions, is electrically neutral. The numbers of electrons and protons are *equal*, and since the negative charge on an electron is neutralized by the equal positive charge on a proton, the atom as a whole has zero charge. The neutron increases the mass of the atom without contributing to the charge. Fig. 2 gives a simple idea of the construction of a hydrogen atom and of a lithium atom (atomic number 3).

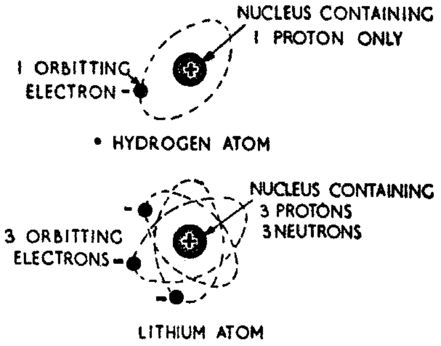

NUCLEUS CONTAINING 1 PROTON ONLY
1 ORBITTING ELECTRON
• HYDROGEN ATOM
NUCLEUS CONTAINING 3 PROTONS 3 NEUTRONS
3 ORBITTING ELECTRONS
LITHIUM ATOM

Fig. 2.—THE ATOMIC STRUCTURE

Ionisation

13. Normally, the electron orbits are maintained by attractive forces between the negative charge on the electron and the positive charge in the nucleus. However, under certain conditions, one or more of the planetary electrons in the outer orbits can be detached from the atom. Such an atom, having " lost " a negative charge, is termed a **positive ion.** The electron so stripped off may attach itself to a neighbouring atom which then becomes a **negative ion.** The process whereby atoms are caused to lose or gain electrons is termed **ionisation.** During the time that an electron remains unattached to any atomic system it is termed a **free electron.**

Conductors and Insulators

14. A **conductor** is a substance in which there is a constant random movement of free electrons between atoms. Pure metals are good conductors, silver being the best and copper ranking second.

15. An **insulator** is a substance in which there is practically no random movement of free electrons. In this case, the outer orbital electrons are tightly bound to their parent nuclei and will not normally break away. Examples of good insulators are dry air, rubber, mica, ebonite, and porcelain.

16. No rigid line can be drawn between conductors and insulators. Copper is a very good conductor ; mica is a very good insulator. Between these extremes lies a group of materials which are neither good conductors nor good insulators. Some of these materials are termed **semi-conductors** and have important special properties which will be dealt with in Section 8.

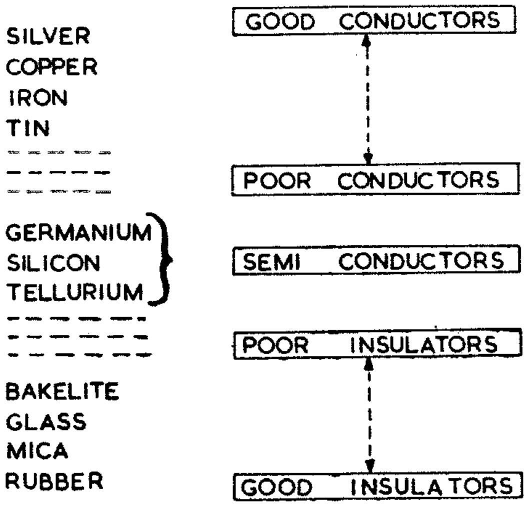

SILVER
COPPER
IRON
TIN
GOOD CONDUCTORS
POOR CONDUCTORS
GERMANIUM
SILICON
TELLURIUM
SEMI CONDUCTORS
POOR INSULATORS
BAKELITE
GLASS
MICA
RUBBER
GOOD INSULATORS

Fig. 3.—CONDUCTORS AND INSULATORS

Electric Current

17. An electric current is an average movement of electric charges through a material in one general direction. The electric charges can be electrons, ions, or both. For instance, in a conductor under normal conditions there is a continual random movement of free

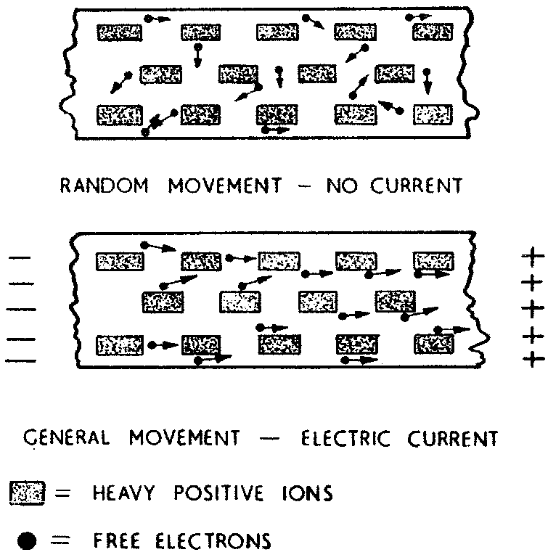

RANDOM MOVEMENT – NO CURRENT
GENERAL MOVEMENT — ELECTRIC CURRENT
▨ = HEAVY POSITIVE IONS
● = FREE ELECTRONS

Fig. 4.—CONDUCTION CURRENT

electrons between the static atoms. This does not constitute a current. Under certain conditions, however, (such as that shown in Fig. 4) the free electrons can be caused to move through the conductor in one general direction towards a point which is positive and, therefore, attracts them. This constitutes a **conduction current,** which occurs in conductive circuits and in a vacuum, e.g., in a thermionic valve.

18. By convention, the *direction* of an electric current is from positive to negative. This was assigned as the direction of current before the discovery of the electron and the result is that a large number of electrical laws are defined in terms of this **conventional current,** notwithstanding the fact that electrons move in the reverse direction in a conductive circuit.

Effects of an Electric Current

19. The more important effects observed when an electric current is flowing are :—

(*a*) **The heating effect.** A movement of electrons through a conductor always causes the conductor to become hot. This effect is used in devices such as electric fires, electric irons, electric lamps, and fuses. Consideration must always be given to the heat produced in radio instruments by electric currents.

(*b*) **The chemical effect.** An electric current through an electrolyte causes a change in the chemical composition of the electrolyte and of any electrodes immersed in the solution. This effect is used in battery charging and in electroplating.

(*c*) **The magnetic effect.** An electric current through any medium always produces a magnetic field (see Section 2). This cause and effect are inseparable and are put to use in devices such as electric bells, relays, electric motors, and transformers.

Unit of Charge

20. The charge on an electron (or a proton) is extremely small and is inconvenient for practical measurements. The practical unit of charge or quantity of electricity (symbol Q) is the **coulomb.** A charge of one coulomb is equal to the charge on $6{\cdot}29 \times 10^{18}$ electrons. This rather awkward figure arose from the fact that the coulomb was assigned as the unit of charge before the discovery of the electron.

Unit of Current

21. The practical unit of current (symbol I) is the **ampere.** If, in any circuit, the quantity of electricity passing a given point in one second is one coulomb, the current will be one ampere.

22. Since one ampere equals a rate of flow of one coulomb per second :—

$$\text{Current, I amperes} = \frac{\text{Quantity or charge, Q coulombs}}{\text{Time, t seconds}}$$

$$\text{i.e. } I = \frac{Q}{t}\,(\text{Amperes}) \text{ — — } (1)$$

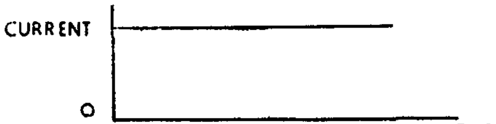
Constant and Varying Currents

23. The rate of flow of electrons in a circuit (i.e., the current) can be in one of the following forms :—

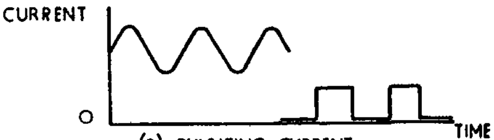
(*a*) **Direct current, d.c.** An electric current flowing continuously in one direction at a steady rate.

(*b*) **Pulsating current.** An electric current which flows in one direction but which undergoes regular, recurring variations in magnitude.

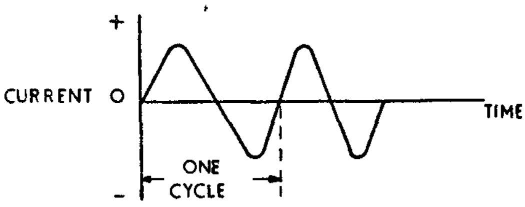
(*c*) **Alternating current, a.c.** An electric current which alternately reverses its direction in a circuit in a regular manner. One *cycle* is a complete variation as shown in Fig. 5. The number of such cycles occurring in one second is termed the *frequency* and is given in cycles per second or c/s (see Sections 3 and 5).



(A) D.C.



(B) PULSATING CURRENT



(C) AC

Fig. 5.—TYPES OF CURRENT

E.M.F. AND P.D.

Electrical Energy

24. When an electric current flows in a conductor, the conductor becomes hot, i.e., energy is dissipated in the conductor in the form of heat. Electrons during their passage through the conductor collide with the molecules and give up some of their kinetic energy to them. The molecules, already in a state of agitation, move faster and further from one another. This results in both heat and expansion of the conductor.

25. As energy can neither be created nor destroyed, this heat energy must be derived from some other form of energy—in this case, **electrical energy.**

26. By conversion, electrical energy may be obtained from other forms of energy. The more practical methods are :—

(*a*) **Chemical.** The electric cell.

(*b*) **Thermal.** Thermionic emission.

(*c*) **Electromagnetic.** The electric generator.

(*d*) **Thermo-electric.** The thermo-junction.

(*e*) **Photo-electric.** The photo-electric cell.

Electromotive Force (E.M.F.) and Potential Difference (P.D.)

27. Consider a simple closed electric circuit, such as a conductor connected between the terminals of a battery. There are two energy transformations going on concurrently. Chemical energy is being converted to electrical energy by the battery, and electrical energy is being converted to heat energy in the conductor. These two processes provide the basis of two important ideas in the description of electrical phenomena :—

(*a*) Whenever there is introduced in any part of an electric circuit any form of energy capable of being converted into electrical energy, an **electromotive force (e.m.f.)** is said to be acting in that circuit. Thus, in the simple circuit described above, the battery supplies an e.m.f., as chemical energy is there being converted to electrical energy.

(*b*) If between any two points in an electric circuit it is possible to convert electrical energy into any other form, a **potential difference (p.d.)** is established between the two points. In the simple circuit described above, between any two points on the conductor electrical energy is being converted to heat energy ; there is, therefore, a p.d. between any two points on the conductor.

Unit of E.m.f. and P.d.

28. The unit of e.m.f. is the **volt.** The e.m.f. of a supply is one volt if the amount of energy converted into electrical energy is one joule for each coulomb of electricity passing.

$$\text{E.m.f., E volts} = \frac{\text{Energy or work, W joules}}{\text{Quantity or charge, Q coulombs}}$$

$$\text{i.e., } E = \frac{W}{Q} \text{ (volts)} \quad — \quad — \quad — \quad (2)$$

29. The unit of p.d. is the **volt.** The p.d. between two points in a circuit is one volt if the amount of electrical energy converted into some other form is one joule for each coulomb which passes between the two points.

$$\text{P.d., V volts} = \frac{\text{Energy or work, W joules}}{\text{Quantity or charge, Q coulombs}}$$

$$\text{i.e., } V = \frac{W}{Q} \text{ (volts)} \quad — \quad — \quad — \quad (3)$$

Potential

30. For practical purposes the earth is regarded as being electrically neutral (i.e. zero charge). Any point having a deficiency of electrons has then a positive **potential** with respect to the earth ; between the two a p.d. will exist. Conversely, a point having a surplus of electrons has a negative potential with respect to earth.

31. Any point in an electric circuit can be taken as a reference point ; any other point will then have a potential (either positive or negative) *with respect to* the reference point and a p.d. will exist between the two. In Fig. 6 :—

(*a*) "A" is at a potential of one volt positive with respect to "B", and *two* volts positive with respect to "C". A p.d. of one volt exists between "A" and "B", and two volts between "A" and "C".

(*b*) "C" is at a potential of one volt negative with respect to "B", and *two* volts negative with respect to "A". A p.d.

of one volt exists between "C" and "B", and two volts between "C" and "A".

(*c*) "B" is at a potential of one volt *negative* with respect to "A", and one volt *positive* with respect to "C". Between "A" and "B", and between "B" and "C" p.d.s of one volt exist.

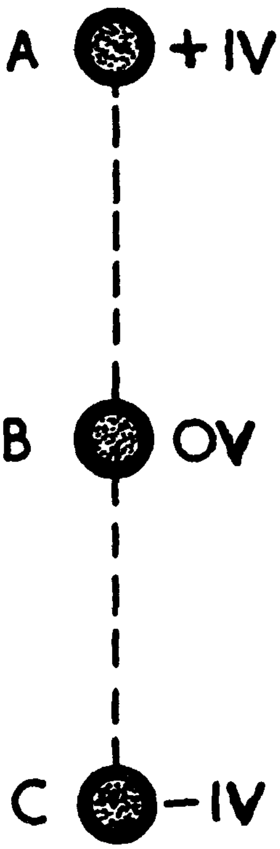


Fig. 6.—POTENTIAL AND POTENTIAL DIFFERENCES

Expressions for Electrical Energy and Power

32. Electrical energy is the ability or the capacity of an electrical system for doing work. The practical unit is the **joule** (see Para. 6(*e*)). The work done when a charge of Q coulombs moves through a p.d. of V volts is given by equation (3) :—

$$W = V.Q \text{ (joules)}$$

From equation (1) :—

$$Q = I.t \text{ (coulombs)}$$

By substitution :—

$$W = V.I.t \text{ (joules)} \quad — \quad — \quad (4)$$

33. Electrical power is the *rate* at which work is done. The practical unit is the **watt**—the rate of working when one joule of energy is transformed per second. Thus:—

$$\text{Power, P watts} = \frac{\text{Energy or work, W joules}}{\text{Time, t seconds}}$$

$$\text{i.e., } P = \frac{W}{t} = \frac{V.I.t.}{t} = V.I. \text{ (watts)} \quad (5)$$

34. From Paras. 32 and 33 it is seen that if a current of one ampere is established in a circuit where the p.d. is one volt, a power of one watt is developed. If this condition is maintained for one second, the energy expended is one joule.

SECTION 1

CHAPTER 2

RESISTANCE AND OHM'S LAW

RESISTANCE AND OHM'S LAW *Paragraph*

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PART 1, SECTION 1, CHAPTER 2

RESISTANCE AND OHM'S LAW

Introduction

1. Chapter 1 has shown that when a source of e.m.f. is acting in a circuit such as to cause a p.d. to be developed across a conductor, a current is established in that conductor. It is now necessary to find the relationship between the current in the conductor and the p.d. across it.

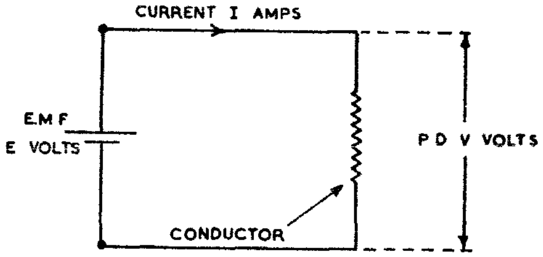


Fig. 1.—SIMPLE ELECTRIC CIRCUIT

Ohm's Law

2. This states :—

> " In any given conductor, provided the temperature remains constant, the ratio of the p.d. across the conductor to the current established in the conductor is a constant."

$$\text{Thus :—}\frac{\text{P.d., V volts}}{\text{Current, I amperes}} = \text{Constant}$$

$$\text{i.e., } \frac{V}{I} = \text{Constant} \quad — \quad — \quad (1)$$

3. Ohm's law is a **linear** law,—i.e., plotting a graph of current in the conductor for various values of p.d. across it results in a straight line through the origin. Fig. 2 shows such a graph for two different conductors.

In graph (*a*) :—

$$\frac{V}{I} = \text{Constant} = 1$$

In graph (*b*) :—

$$\frac{V}{I} = \text{Constant} = 2$$

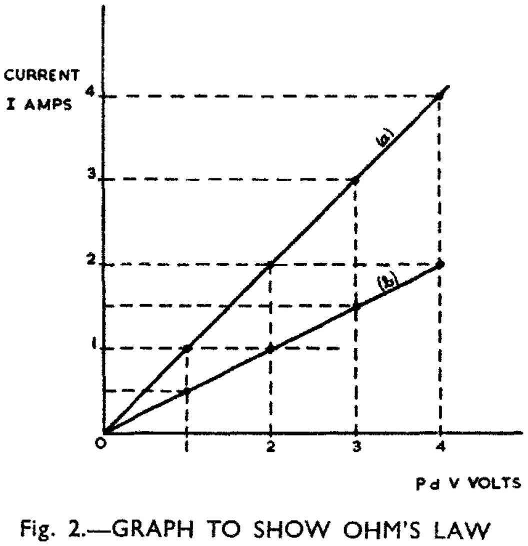


Fig. 2.—GRAPH TO SHOW OHM'S LAW

4. Any material which gives a linear graph through the origin as in Fig. 2 is said to obey Ohm's law. Any device which does not obey Ohm's law is termed a **non-linear device**,—e.g., a thermionic valve.

Resistance

5. The constant of Paras. 2 and 3 is termed the **resistance** of the conductor (symbol R). The unit of resistance is the ohm (symbol Ω). A conductor has a resistance of one ohm if a p.d. of one volt across the conductor establishes a current of one ampere. Equation (1) can now be re-written :—

$$\frac{\text{P.d., V volts}}{\text{Current, I amps}} = \text{Resistance, R ohms}$$

$$\text{i.e., } \frac{V}{I} = R \text{ (ohms)} \quad — \quad — \quad — \quad (2)$$

Thus, if a p.d. of 5 volts across a conductor establishes a current of 50 milli-amperes (mA), the resistance of the conductor is :—

$$R = \frac{V}{I} = \frac{5}{50 \times 10^{-3}} = 100 \text{ ohms.}$$

6. The resistance of a material varies from a very low value for a good conductor to a very high value for a good insulator.

Factors Affecting Resistance

7. The resistance of a material is directly proportional to its length (l) and inversely proportional to its cross-sectional area (*a*) :—

$$R = \rho \frac{l}{a} \quad — \quad — \quad — \quad — \quad (3)$$

8. In this expression, ρ is called the **resistivity** or the **specific resistance** of the material and is defined as the resistance between the opposite faces of a specimen of that material of unit length and unit cross-sectional area ;

that is, of unit cube. The value of ρ depends on the units chosen and is generally given in ohms per cm. cube.

Temperature Coefficient of Resistance.

9. The resistivity (and hence the resistance) of a material varies with temperature. The resistance of pure metals increases with temperature rise, while the resistance of insulators decreases. Pure metals change more than alloys, some of which actually decrease in resistance as the temperature rises. The resistances of carbon and electrolytes decrease with a rise in temperature. A material whose resistance increases with a rise in temperature has a *positive temperature coefficient* ; those whose resistances decrease with a rise in temperature have *negative temperature coefficients.*

10. For the purposes of calculation, the formula

$$R_t = R_o(1 + \alpha t) \text{ ohms} \quad — \quad — \quad (4)$$

is used, where R_t stands for the resistance at t°C, R_o for the resistance at O°C and α for the temperature coefficient taking O°C as standard.

Conductance

11. In direct current circuits, the **conductance** of a material (symbol G) is the reciprocal of its resistance. The unit of conductance is the **mho.** Thus :—

$$\text{Conductance, } G = \frac{1}{R} \text{ (mhos)} \quad — \quad — \quad (5)$$

Resistors in Series

12. When a conductor is specially constructed to provide resistance it is termed a **resistor,** and as such is used extensively in radio.

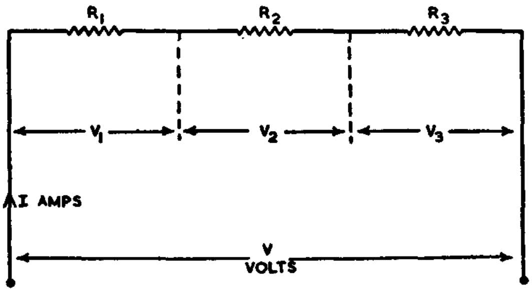


Fig. 3.—RESISTORS IN SERIES

Resistors are " in series " when they are connected end to end in such a manner that only one path is provided for the current, the same current flowing through each resistor. This is shown in Fig. 3.

13. The p.d. across each resistor is obtained by applying the general formula for Ohm's law, i.e., V = I.R (volts). From this it is seen that the p.d. across individual resistors connected in series is the same only if their resistance values are the same. The total resistance of several resistors connected in series is the sum of the individual resistances:-

$$R_T = R_1 + R_2 + R_3 + — \text{ (ohms)} —(6)$$

Resistors in Parallel

14. Resistors are " in parallel " when they are connected in such a manner that they provide alternative paths for the current, the p.d. across each resistor being the same. The current through each resistor will be the same only if the resistors are equal in value. This is shown in Fig. 4.

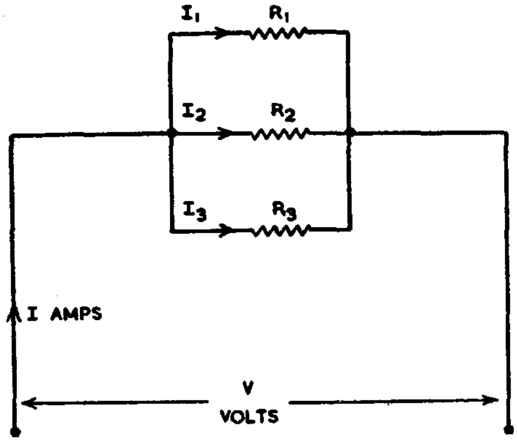


Fig. 4.—RESISTORS IN PARALLEL

15. By applying Ohm's law to the circuit it is found that :—

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \quad — \quad — \quad (7)$$

i.e., the reciprocal of the total resistance is the sum of the reciprocals of the individual resistances ; the total resistance is always less than the smallest individual resistance.

16. Since conductance is the reciprocal of resistance in d.c. circuits, equation (7) may be written :—

$$G_T = G_1 + G_2 + G_3 + \text{ (mhos)} \quad — (8)$$

Resistors in Series-Parallel

17. The procedure for solving a circuit consisting of combinations of resistors in

series and parallel is to reduce any parallel combination to a single equivalent resistance, and add to any resistance which may be in series in that arm, repeating the procedure as necessary.

Internal Resistance

18. When a current of I amperes flows through a resistance of R ohms, a p.d. or "volts drop", given by $V = I.R$ (volts) is developed across the resistance. Any source of supply (e.g., a battery) must have some "internal resistance" and when a current flows in the circuit a volts drop is developed across this internal resistance.

19. In the circuit shown in Fig. 5, the internal resistance of the supply is 0·5 ohms. The total resistance is :—

$$R_T = R_1 + R_2 = 0{\cdot}5 + 5{\cdot}5 = 6 \text{ ohms.}$$

$$\therefore\ I = \frac{E}{R_T} = \frac{12}{6} = 2 \text{ amps.}$$

This current flows through the two resistances in series developing a volts drop across each. The volts drop across the internal resistance is :—

$$V = I.R = 2 \times 0{\cdot}5 = 1 \text{ volt.}$$

The polarity is as shown and is such as to *subtract* from the e.m.f. of the supply. The terminal p.d. of the supply across AB is, thus:—

$$12 - 1 = 11 \text{ volts.}$$

The conclusion is that when a current is taken from a supply, the terminal p.d. is less than the e.m.f. by the volts drop developed across the internal resistance of the supply.

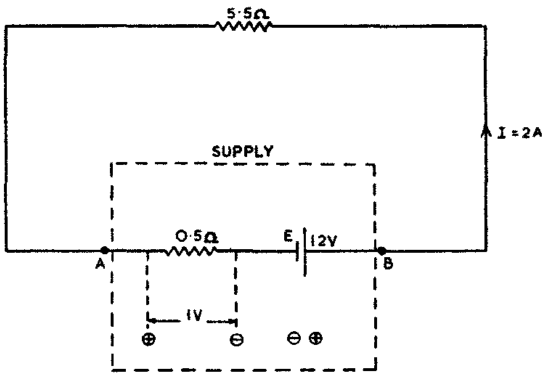


Fig. 5.—INTERNAL VOLTS DROP

Expressions for Power and Energy

20. It has been shown in Chap. 1 that :—

Energy, $W = V.I.t$ (joules)
Power, $P = V.I$ (watts).

By substitution for V or I from Ohm's law relationship $\frac{V}{I} = R$, other expressions for energy and power are obtained :—

$$\text{Energy, } W = V.I.t = I^2.R.t = \frac{V^2}{R}.t \quad \text{(joules)} \text{ — — — — (9)}$$

$$\text{Power, } P = V.I = I^2.R = \frac{V^2}{R} \quad \text{(watts)} \text{ — — — (10)}$$

Maximum Power Transfer Theorem

21. This states :—

> "Maximum power is transferred from a source of supply to an external circuit when the resistance of the external circuit is equal to the internal resistance of the supply".

22. In the circuit shown in Fig. 6, to find that value of load R for which maximum power will be transferred from the supply to the load :—

(*a*) Let $R = 2\Omega$; then $R_T = 3 + 2 = 5\Omega$

$$\therefore I = \frac{E}{R_T} = \frac{12}{5} = 2{\cdot}4\text{A}$$

The power developed across the load is :—

$$P = I^2R = (2{\cdot}4)^2 2 = 11{\cdot}52\text{W}$$

(*b*) Let $R = 3\Omega$; then $R_T = 3 + 3 = 6\Omega$

$$\therefore\ I = \frac{12}{6} = 2\text{A}$$

And $P = (2)^2\, 3 = 12\text{W}$.

(*c*) Let $R = 4\Omega$; then $R_T = 3 + 4 = 7\Omega$

$$\therefore\quad I = \frac{12}{7} = 1{\cdot}7\text{A}$$

And $P = (1{\cdot}7)^2\, 4 = 11{\cdot}56\text{W}$

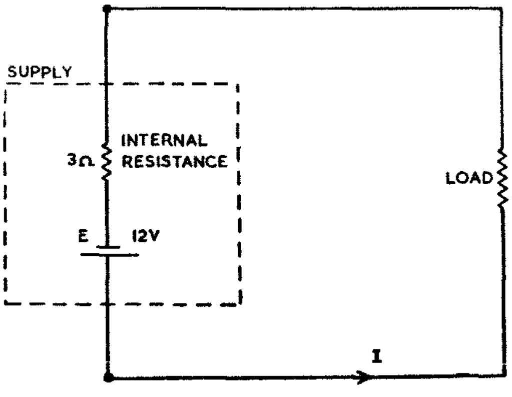


Fig. 6.—MAXIMUM POWER TRANSFER

(*d*) A graph of the power P developed across the load for several values of load R is shown in Fig, 7. This proves the theorem.

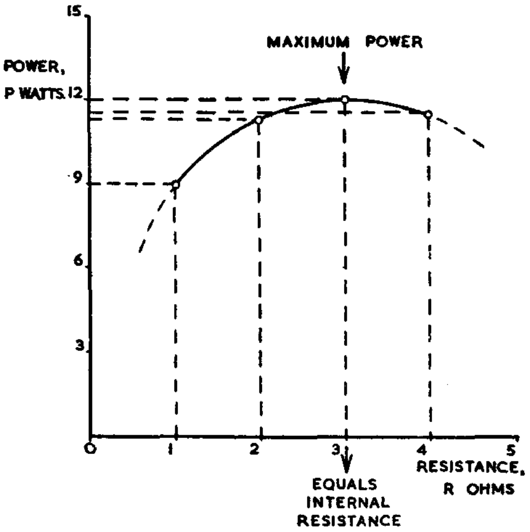


Fig. 7.—GRAPH TO SHOW MAXIMUM POWER TRANSFER CONDITIONS

The Potential Divider

23. The fact that a volts drop is developed across a resistance when a current flows through it can be used to give a division of potential in a circuit.

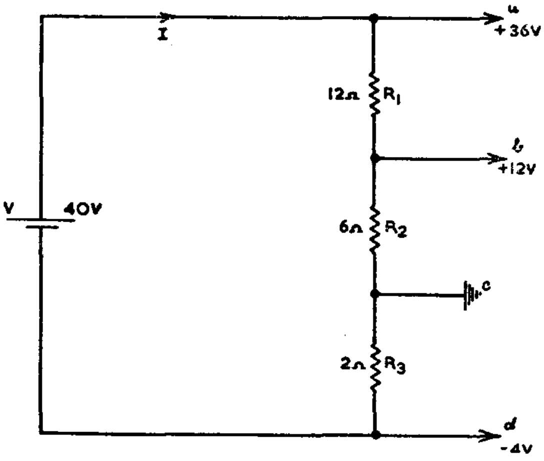


Fig. 8.—A POTENTIAL DIVIDER

24. Consider Fig. 8. The p.d. across each resistor is found by applying Ohm's law to the circuit. Point "c" is at earth potential. The p.d. across R_3 is 4 volts, so that point "d" is at a potential of 4 volts *negative* with respect to earth. Similarly point "b" is at a potential of 12 volts *positive* with respect to earth. Point "a" is at a potential of 24 + 12 = 36 volts *positive* with respect to earth. These potentials can be applied to various parts of a system. In any potential divider of this type, the p.d. across any resistor R is :—

$$V_R = \frac{\text{Total p.d.} \times R}{\text{Total Resistance}} = \frac{V \times R}{R_T} \text{ (volts)} \quad — \quad — \quad — \quad — \quad (11)$$

KIRCHHOFF'S LAWS

Kirchhoff's Laws

25. Circuits which do not come within the category of simple series or parallel circuits can be solved by means of Kirchhoff's laws :—

(*a*) **First Law.** " The algebraic sum of currents meeting at any point in a circuit is zero ". Thus, at any junction in a circuit, the current entering that junction must equal the current leaving it.

(*b*) **Second law.** " In any closed circuit, the algebraic sum of the e.m.f.s is equal to the algebraic sum of the p.d.s." Thus, if all the p.d.s in a circuit are added (with due regard to their polarity) the sum will equal the e.m.f. acting in that closed circuit.

Application of Kirchhoff's Laws

26. To demonstrate the method by which Kirchhoff's laws are applied consider Fig. 9.

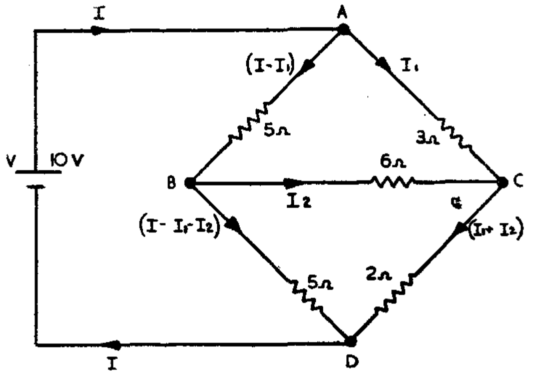


Fig. 9.—APPLICATION OF KIRCHHOFF'S LAWS

It may be necessary to find the current in the centre (6 ohms) arm. To obtain this, the sequence is as follows :—

(*a*) Apply Kirchhoff's first law to the circuit and annotate the currents at various parts of the circuit. For instance, the current entering A is I. This is also the current leaving A so that if I_1 is established between AC, the current established between AB is $(I - I_1)$, since $I_1 + (I - I_1) = I$. By similar reasoning the currents established at other parts of the circuit will be as shown.

(*b*) Three unknown quantities I, I_1, and I_2 are thereby produced, and to obtain the value of any one of them, three simultaneous equations are obtained and solved in the usual manner.

(*c*) To obtain such equations, Kirchhoff's second law is applied to three of the closed circuits (one containing the battery e.m.f.). For instance, mesh ACB is a closed circuit where the e.m.f. acting is zero. The algebraic sum of the p.d.s must, therefore, be zero. The correct polarity of the p.d. across each resistor must be observed, a minus sign indicating that the p.d. is negative when taken against the conventional flow of current. Thus :—

$$3I_1 - 6I_2 - 5(I - I_1) = 0$$

$$\therefore 5I - 8I_1 + 6I_2 = 0$$

(*d*) A further two simultaneous equations may be obtained in a similar manner by considering circuits BCD and ACD (and battery). The three equations are then solved to give I_2 — the current in the centre arm.

Wheatstone's Bridge

27. The bridge arrangement of resistors shown in Fig. 10 is used extensively in radio for a variety of applications, including the measurement of component values. The principle of Wheatstone's bridge is simple. If there is no reading in the galvanometer (an instrument used to indicate that current is flowing in a circuit) the bridge is said to be " balanced ". If no current is flowing between B and C these points must be at the same potential ; the current flowing in the other branches will then be as indicated in Fig. 10.

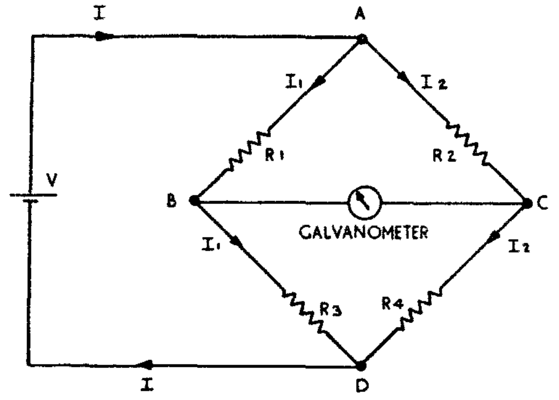


Fig. 10.—WHEATSTONE'S BRIDGE

28. (*a*) For the points B and C to be at the same potential, the p.d. across AB must equal that across AC.

i.e., $I_1R_1 = I_2R_2$

$$\therefore \frac{I_1}{I_2} = \frac{R_2}{R_1} \quad - - - - - - \quad \text{(i)}$$

(*b*) Similarly, the p.d. across BD must equal that across CD.

i.e., $I_1R_3 = I_2R_4$

$$\therefore \frac{I_1}{I_2} = \frac{R_4}{R_3} \quad - - - - - - \quad \text{(ii)}$$

(*c*) From equations (i) and (ii) :—

$$\frac{I_1}{I_2} = \frac{R_2}{R_1} = \frac{R_4}{R_3}$$

$$\therefore \frac{R_1}{R_2} = \frac{R_3}{R_4} \quad - - - - \quad (12)$$

This gives the condition under which the bridge is balanced.

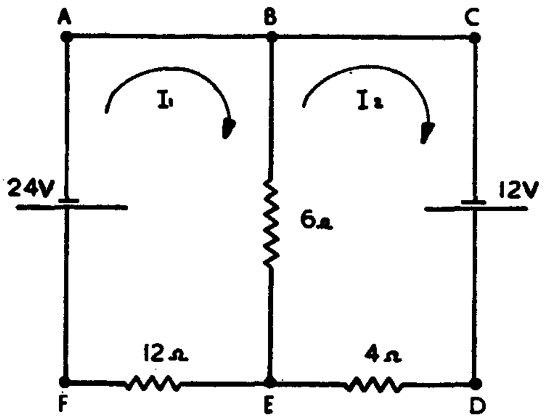
29. The normal arrangement in a Wheatstone's bridge used for resistance measurement is for two resistors (say R_1 and R_2) to be fixed and known in value ; R_3, say, to be variable and calibrated ; R_4 is then the unknown resistor whose resistance is to be measured. For adjustment, R_3 is varied until there is no deflection in the galvanometer. The bridge is then balanced and from equation (12) :—

$$R_4 = \frac{R_2 R_3}{R_1} \text{ (ohms).}$$

30. In a Wheatstone's bridge, at balance, there is no p.d. across BC for any voltage applied across AD. The converse is also true. In either case, however, a p.d. is developed across any single resistor in the bridge. These facts are used in practice where it is required to prevent two voltages, which feed into a common circuit, from affecting each other.

Maxwell's Circulating Currents

31. In the circuit shown in Fig. 11, two currents I_1 and I_2 are assumed to flow in a


A
B
C
I_1
I_2
24V
12V
6Ω
12Ω
4Ω
F
E
D

11.—MAXWELL'S CIRCULATING CURRENTS

clockwise direction round each loop. Kirchhoff's second law is applied to each loop and the simultaneous equations so obtained are solved to give the required result.

The Superposition Theorem

32. With this theorem, each battery is considered to be short-circuited in turn and two separate calculations are made by applying Ohm's law to each circuit. The separate answers are then superposed to give the final result. Fig. 11 can be redrawn in accordance with the superposition theorem to give Fig. 12.

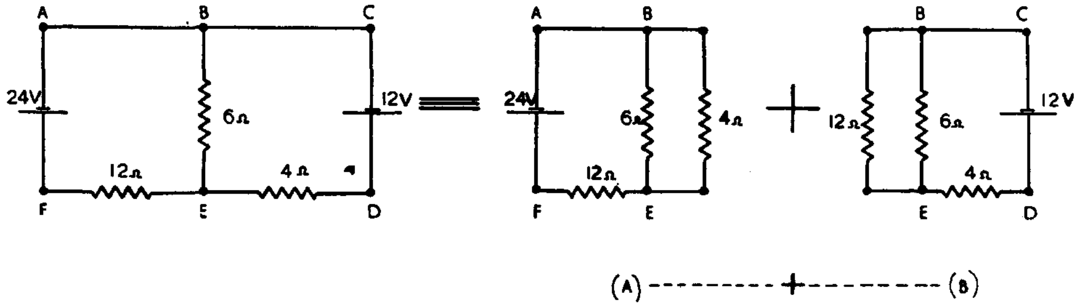

A
B
C
24V
6Ω
12V
12Ω
4Ω
F
E
D
A
B
24V
6Ω
4Ω
12Ω
F
E
B
C
12Ω
6Ω
12V
4Ω
E
D
(A)
(B)

Fig. 12.—THE SUPERPOSITION THEOREM

SECTION 1

CHAPTER 3

RESISTOR CONSTRUCTION

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Cracked Carbon Film Resistors 5
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PART 1, SECTION 1, CHAPTER 3

RESISTOR CONSTRUCTION

Introduction

1. A modern radar equipment may contain as many as 5,000 resistors, each used for a specific purpose such as to provide a p.d. from a current flowing through it, or to form part of a potential divider network.

2. Values of resistance from a fraction of an ohm to several million ohms are used. Methods of resistor construction include :—

   (*a*) Carbon composition type.
   (*b*) Cracked carbon film type.
   (*c*) Wire-wound type.
   (*d*) Metal film type.

Carbon Composition Resistors

3. The method of construction of this type varies considerably. The following method is typical :—

Powdered carbon dust with a filling material and a binder of resin, are mixed together, moulded to shape, and then fired at a high temperature. The resultant composition rod forms the resistor whose resistance varies according to the carbon content. Connection is made to the resistor either by metal caps clamped at each end of the rod, or by wires firmly wound round the ends of the rod. The resistor is then varnished, or placed inside a ceramic tube, and colour-coded.

4. This type of resistor has a very poor stability. The resistance value varies considerably with temperature, load, and even with shelf life. Consequently, it can only be used where a quite large change in resistance is of no importance to the operation of the equipment. In addition, the power that can be developed in this type of resistor is relatively small (of the order of 2 watts).

Cracked Carbon Film Resistors

5. In this type, a hard carbon film is deposited on to a ceramic rod and the required resistance obtained by cutting a spiral track through the film. This rod is then varnished, or placed inside a ceramic tube, and colour-coded. The stability in this type is much better than that in the carbon composition type.

Colour Code

6. (*a*) The colour code used to indicate the resistance value on carbon composition and cracked carbon film types of resistors is given in Table 1. Two of the more common methods of marking the resistor are shown in Fig. 1.

Colour	1st Band or Body, 1st Figure	2nd Band or Tip, 2nd Figure	3rd Band or Spot, Multiplying Factor	4th Band, Tolerance
Black	0	0	1	—
Brown	1	1	10	± 1%
Red	2	2	100	± 2%
Orange	3	3	1,000	± 3%
Yellow	4	4	10,000	± 4%
Green	5	5	100,000	—
Blue	6	6	1,000,000	—
Violet	7	7	10,000,000	—
Grey	8	8	—	—
White	9	9	—	—
Gold	—	—	0·1	± 5%
Silver	—	—	0·01	± 10%
No Colour	—	—	—	± 20%

TABLE 1—COLOUR CODE

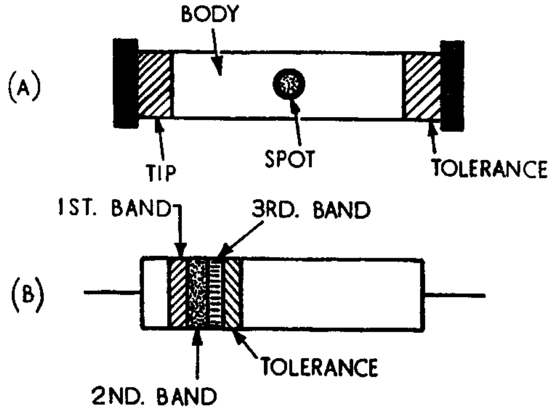

Fig. 1.—COLOUR CODING

Example. (i) Body = Yellow (4)
Tip = Violet (7)
Spot = Orange (3)
∴ Resistance = 47,000 ± 20% (ohms)

(ii) 1st Band = Blue (6)
2nd Band = Grey (8)
3rd Band = Red (2)
4th Band = Gold
∴ Resistance = 6,800 ± 5% (ohms).

(*b*) The accuracy of a resistance value is indicated by a tolerance marking as shown in Fig. 1. Because a tolerance of ± 20% is more common in carbon composition type resistors, manufacturers produce a limited range of resistors which, by the tolerance variation, can cover the whole range of resistance, e.g., 10, 15, 22, 33, 47, 68, 100 (all ± 20%). These are known as " preferred values ".

Wire-Wound Resistors

7. In this type, wire of nickel-chrome or copper-nickel is wound on a ceramic former. These wires have a high resistivity and the small amount of wire required for a particular resistance gives a resistor of reasonable size. In addition, the temperature coefficient of resistance is low, and therefore resistivity remains fairly constant with variations in temperature. This type has the best stability characteristics. The resistor is finished with a coating of vitreous enamel so that high temperatures will not oxidise the wire. The value of resistance in ohms is stencilled or stamped on the resistor.

8. Wire-wound resistors suffer from the disadvantage that the wire is wound in the form of a coil and, unless special precautions are taken, they may be unsuitable for many applications in radio (see Sect. 2, Chap. 4).

Metal Film Resistors

9. These combine the stable characteristics of the wire-wound type with the simplicity of the cracked carbon film type. The metal film type resistor is constructed as follows :—

(*a*) A thin film of platinum-gold compound is coated on to a glass plate or tube.

(*b*) This is fired to about 400°C.

(*c*) The resistance value is adjusted by etching a spiral track through the film.

(*d*) The resistor is then fired to about 700°C. to give in effect, a wire-wound resistor with each individual " wire " bonded to the glass base.

(*e*) The whole element is finally sealed in an outer tube which gives complete protection from the atmosphere and ensures the long term stability of the resistor.

Variable Resistors

10. These are termed rheostats and potentiometers.

(*a*) A **rheostat** is a variable resistor which is inserted in series with other devices in a circuit. Its value can be varied to alter the current in the circuit. It has only two connections, as shown in Fig. 2.

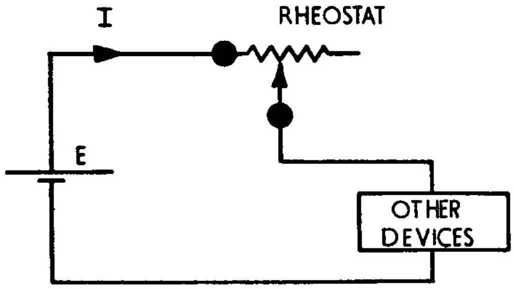

Fig. 2.—USE OF A RHEOSTAT.

(*b*) A **potentiometer** is a variable resistor arranged in such a manner that a certain proportion of the applied voltage can be " tapped off " for application to another part of the circuit. A typical use is as a volume control in a radio receiver. The potentiometer has three connections. It is seen from Fig. 3 that

V_1 is variable between 0% and 100% of E.

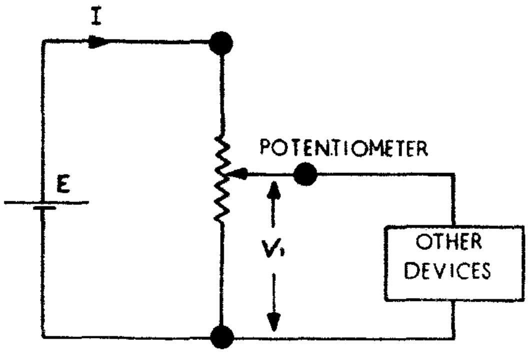

Fig. 3.—USE OF A POTENTIOMETER

(*c*) Variable resistors are constructed as carbon composition, cracked carbon film, or wire-wound types depending on the stability and power requirements. A potentiometer consists of an incomplete ring (either wire on a ceramic former, or a carbon track) the ends of which are taken out to two terminals. A wiper arm, operated by a shaft, bears on this ring and is connected to the centre terminal. As the wiper arm is moved, the resistance value between either of the end terminals and the centre terminal is varied. The whole is enclosed in a sealed bakelite or metal container.

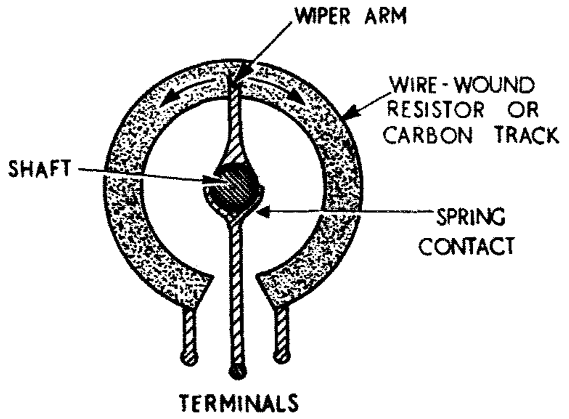

Fig. 4.—CONSTRUCTION OF A POTENTIOMETER

Summary of Resistors

11. The main characteristics of the resistors discussed in this Chapter are given in Table 2. Fig. 5 shows a selection of these resistors.

Type	Wattage Rating	Resistance	Remarks
Carbon Composition, fixed.	$\frac{1}{16}$W to 3W.	10Ω to 25 MΩ	Used in low power circuits, at all frequencies, where stability is not important.
Cracked Carbon Film, fixed.	$\frac{1}{8}$W to 2W	10Ω to 10MΩ	Used in low power circuits where greater stability is required.
Wire-wound, fixed.			
(*a*) Precision	$\frac{1}{4}$W to 3W	0·1Ω to 5MΩ	Used in low power circuits where consistent accuracy in critical applications is essential.
(*b*) Power	1W to 300W	0·5Ω to 400kΩ	Used in circuits where reliability under all conditions is necessary and where a high dissipation of power is to be achieved.
Metal Film, fixed.	$\frac{1}{8}$W to 2 W	1Ω to 1 MΩ	Used in low power circuits where greater stability is required than that obtainable with a cracked carbon resistor. This type is smaller than a wire-wound resistor of comparable ohmic value.

TABLE 2—COMPARISON OF RESISTORS

Type	**Wattage Rating**	**Resistance**	**Remarks**
Carbon Composition, variable.	⅕W to 2W	10Ω to 2MΩ	Used in low power circuits where stability is not important.
Wire-wound, variable.	1W to 100W	1Ω to 100kΩ	Used in circuits where a good stability is required or where a high dissipation of power is to be achieved.

TABLE 2—COMPARISON OF RESISTORS

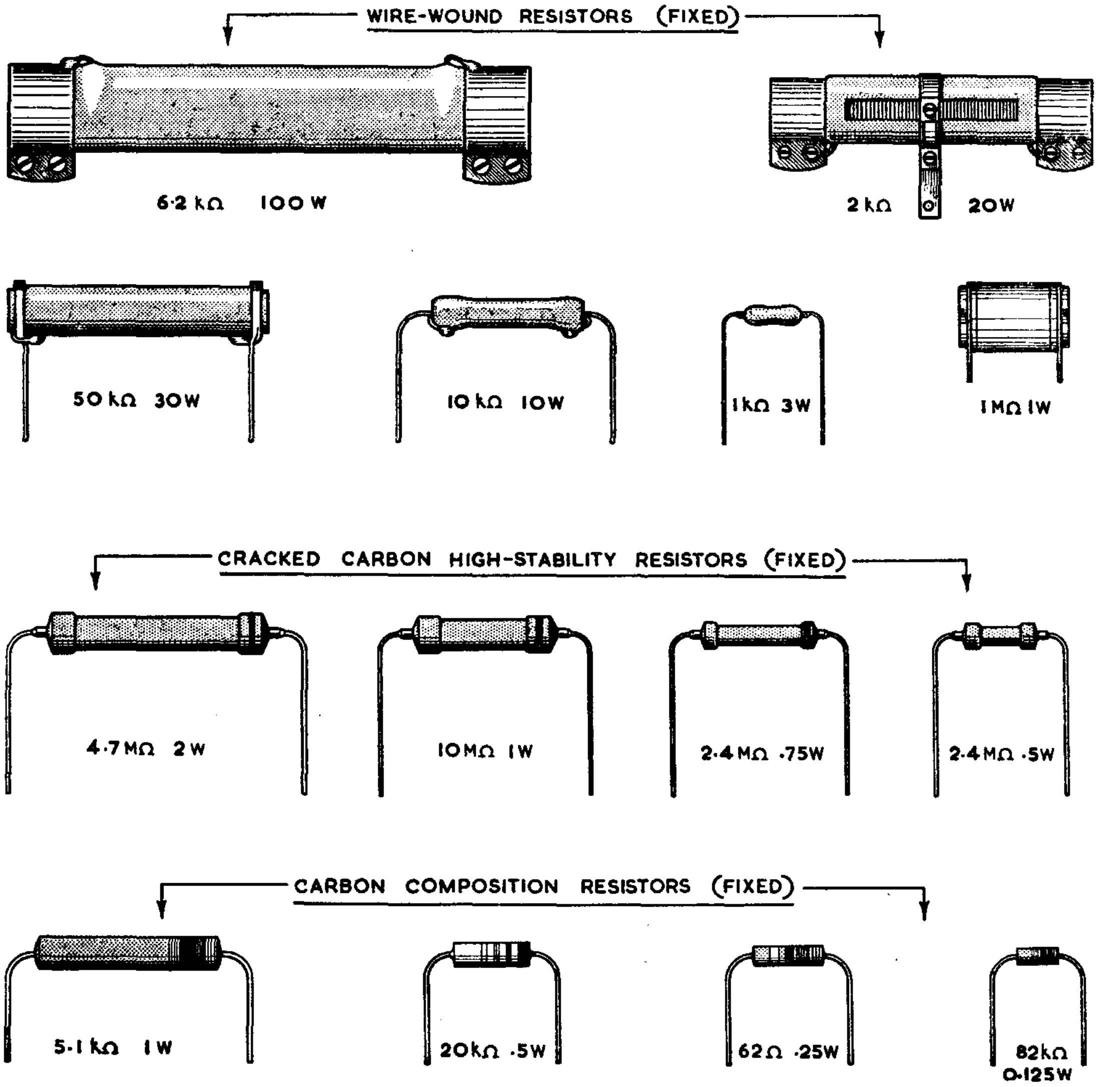
WIRE-WOUND RESISTORS (FIXED)
6·2 kΩ 100 W
2 kΩ 20W
50 kΩ 30W
10 kΩ 10W
1 kΩ 3W
1 MΩ 1W
CRACKED CARBON HIGH-STABILITY RESISTORS (FIXED)
4·7 MΩ 2 W
10 MΩ 1W
2·4 MΩ ·75W
2·4 MΩ ·5W
CARBON COMPOSITION RESISTORS (FIXED)
5·1 kΩ 1 W
20kΩ ·5W
62Ω ·25W
82kΩ
0·125W

Fig. 5—TYPICAL RESISTORS USED IN RADIO

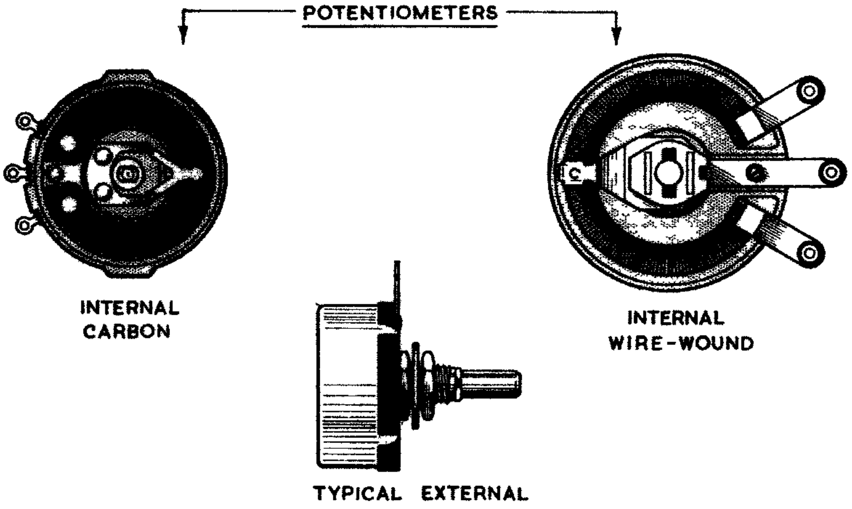
POTENTIOMETERS
INTERNAL CARBON
INTERNAL WIRE-WOUND
TYPICAL EXTERNAL

FIG 5—TYPICAL RESISTORS USED IN RADIO

SECTION 2

MAGNETISM AND ELECTROMAGNETIC INDUCTION

Chapter 1 .. .. **The Magnetic Circuit**
Chapter 2 .. .. **Electromagnetic Induction**
Chapter 3 .. .. **Inductive Circuits**
Chapter 4 .. .. **Construction of Inductors**

SECTION 2

CHAPTER 1

THE MAGNETIC CIRCUIT

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PART 1, SECTION 2, CHAPTER 1

THE MAGNETIC CIRCUIT

M[redacted]ISM

Introduction

1. Certain spe[redacted]of iron ore called natural magnets [redacted]estones, possess the following properties :—

(*a*) When freely suspended they come to rest pointing in a North-South direction.

(*b*) They attract small fragments of iron and steel.

2. These "magnetic" properties can be imparted artificially to iron and steel to produce permanent magnets and electromagnets.

(*a*) **Permanent magnets** are made of steel or alloys of steel, and once magnetised they retain their magnetic properties for a long period of time under normal conditions.

(*b*) **Electromagnets** are made with a core of soft iron or alloys of iron and have the property that, although easily magnetised, they lose their magnetic properties almost immediately once the magnetising influence is removed.

The Magnetic Field

3. That end of a permanent magnet which, when freely suspended in a horizontal plane, points to the earth's north magnetic pole is termed the north-seeking end, or the north (N) pole, of the magnet. The other end is the south (S) pole.

4. If the N-pole of another magnet is brought near the N-pole of the suspended magnet, the two poles repel each other. Attraction occurs between a N-pole and a S-pole. Thus :—

"*Like poles repel; unlike poles attract each other*".

5. The **magnetic field** of the bar magnet shown in Fig. 1 is the space around a magnet where magnetic forces are experienced. This field can be detected by the use of iron filings or by a compass needle, and is represented by **lines of magnetic flux**. Although some lines are shown incomplete they are always continuous.

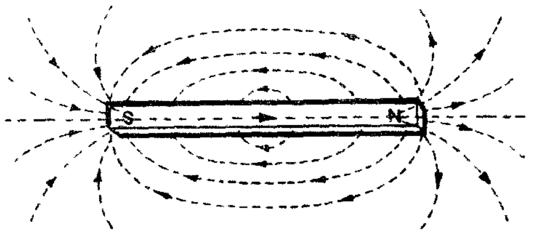

Fig. 1.—THE FIELD OF A PERMANENT MAGNET

6. The direction of a magnetic field at any point is that in which the North pole of a compass needle would point if placed in the field. The direction is indicated by arrowheads in Fig. 1, and is seen to be from N. to S. in the space around the magnet (and from S. to N. "inside" the magnet).

Flux and Flux Density

7. The total number of lines of magnetic flux leaving a magnetic pole is termed **the magnetic flux** (symbol Φ). The practical unit is the **Weber** (Wb), which corresponds to 10^8 lines.

8. The **flux density** (symbol B) is defined as the flux per unit area and is measured in **Webers per square metre** (Wb/m²). Thus :—

$$\text{Flux density} = \frac{\text{Total magnetic flux}}{\text{Area}}$$

$$\text{i.e.,}\quad B = \frac{\Phi}{a}\ (\text{Wb/m}^2)\ldots\quad\ldots\quad (1)$$

9. Fig. 2 (*a*) shows the magnetic field that exists between two unlike poles. Fig. 2(*b*) shows the effect of inserting an iron bar in the space between the two poles. The flux lines appear to be concentrated in the vicinity of the iron. The iron has in fact, become magnetised from the external field and produces flux lines of its own, which combine with the original flux to give an increase in the total flux in that area. Thus iron has the property of increasing the flux density of a magnetic field.

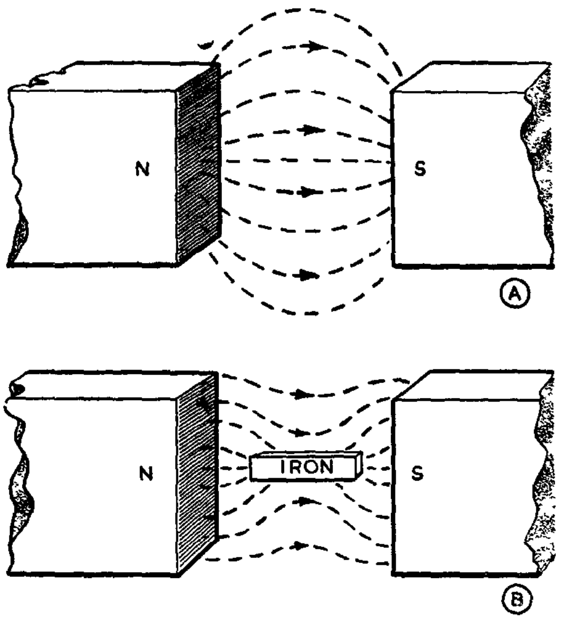

Fig. 2.—EFFECT OF INSERTING IRON IN A MAGNETIC FIELD.

The Magnetic Effect of a Current

10. Fig. 3 shows the cross-section of a conductor assumed to be carrying a current. The convential flow of current is denoted by " + ", which indicates that the current is in a direction " into the paper ". The resultant magnetic field (both inside and outside the conductor) takes the form of concentric circles. The direction of the field is seen to be clockwise in this case.

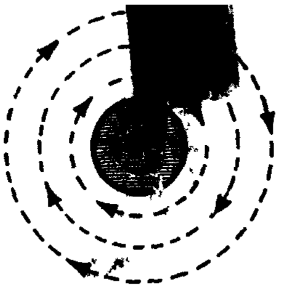

Fig. 3.—THE MAGNETIC FIELD ROUND A CURRENT-CARRYING CONDUCTOR.

The direction can be found from **the corkscrew rule**, which states :—

> " *If a corkscrew is rotated so that its direction of travel is the same as the conventional flow of current in the conductor, the direction of rotation indicates the direction of the magnetic field around the conductor* ".

This is shown in Fig. 4.

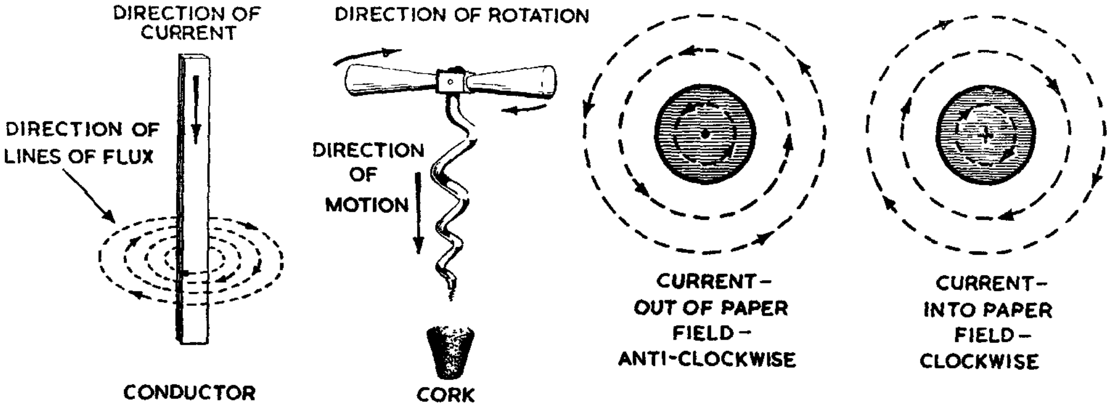
DIRECTION OF CURRENT
DIRECTION OF LINES OF FLUX
CONDUCTOR
DIRECTION OF ROTATION
DIRECTION OF MOTION
CORK
CURRENT - OUT OF PAPER FIELD - ANTI-CLOCKWISE
CURRENT - INTO PAPER FIELD - CLOCKWISE

Fig. 4.—THE CORKSCREW RULE

The Magnetic Field of a Solenoid

11. A solenoid consists of a number of turns of wire wound in the same direction as in a spiral spring, to form a coil. Fig. 5 shows a solenoid connected to a battery such that the conventional current is in the direction indicated.

12. The direction of the magnetic field around any elementary part of the solenoid can be obtained from the corkscrew rule. If this is done for a large number of elementary parts, and the fields combined, the

resultant field will be as indicated in Fig. 5. It is seen that the flux lines form a complete closed path in every case, the field being very similar to that for a permanent magnet.

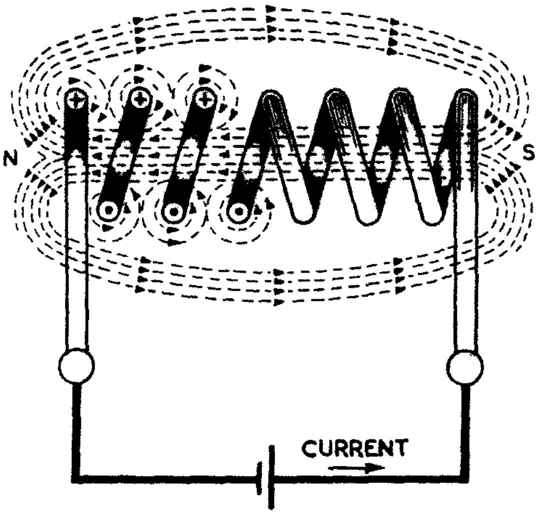

Fig. 5.—THE MAGNETIC FIELD OF A SOLENOID.

13. The polarity of the magnetic field of a solenoid can be obtained from either of two rules :—

(*a*) **The N—S rule.** Looking at one end of the coil, if the current flows in a *clockwise* direction, the end nearer the observer is a *South* pole. If the current is in an *anti-clockwise* direction, the nearer end is a *North* pole. This is illustrated in Fig. 6.

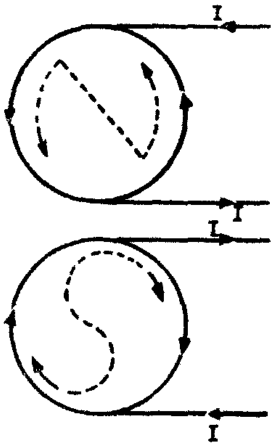

Fig. 6.—THE N-S END RULE

(*b*) **The gripping rule.** If the coil is gripped with the right hand and the fingers are wrapped round the coil in the direction of the current, the *thumb* will point to the *North* pole of the solenoid (Fig. 7).

14. Placing an iron core inside the solenoid will increase the flux density as noted in Para. 9. The iron also becomes magnetised in the polarity shown in Fig. 7. When the current is switched off, the magnetic field associated with that current collapses. If the core is made of soft iron or other similar material, the core also loses its magnetism almost immediately. This is the basis of electromagnets.

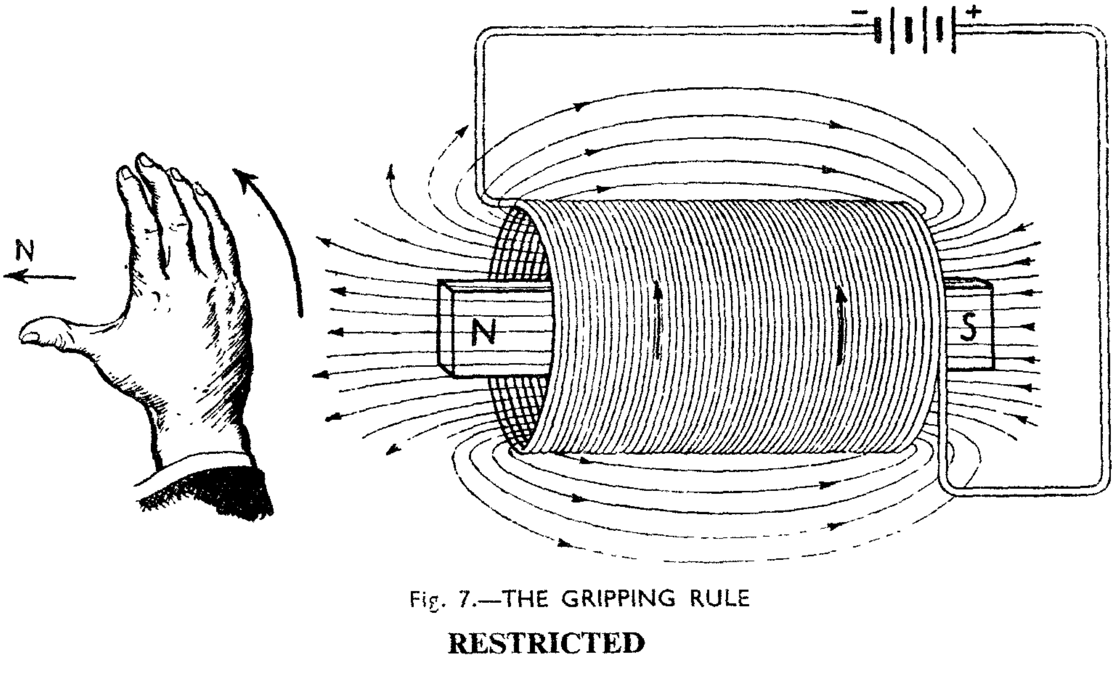

Fig. 7.—THE GRIPPING RULE

THE MAGNETIC CIRCUIT

The Magnetic Circuit

15. The closed path formed by a line (or lines) of flux is referred to as the **magnetic circuit.** One of the simplest forms of magnetic circuit is shown in Fig. 8, where part of the magnetic circuit is in the iron and part in the air gap.

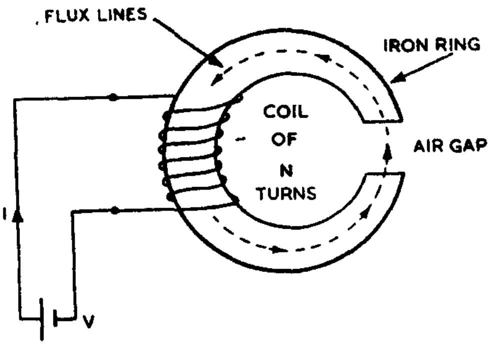

Fig. 8.—A SIMPLE MAGNETIC CIRCUIT.

Magnetomotive Force (M.m.f.)

16. In an electric circuit, a current is established due to the existence of an electromotive force (e.m.f.). In the same way, in a magnetic circuit, a magnetic flux is established due to the existence of a **magnetomotive force (m.m.f.)**; the m.m.f. is produced by the current flowing in the coil and its value is proportional to the current and to the number of turns in the coil. Appropriately, the unit of m.m.f. is the **ampere-turn** (AT). Thus, in Fig. 8 the m.m.f. is IN ampere-turns.

Magnetising Force or Field Strength

17. The **magnetising force** (symbol H) of a magnetic circuit is a measure of the intensity of the magnetic effects at any given point in the magnetic field. It is defined as the m.m.f. per unit length and is measured in **ampere—turns per metre (AT/m).**

$$\text{Magnetising force} = \frac{\text{m.m.f.}}{\text{length}}$$

$$\text{i.e.,} \quad H = \frac{\text{m.m.f.}}{l}\ (\text{AT/m}) \quad .. \ (2)$$

Permeability

18. If a coil of N turns has a current of I amps flowing through it the m.m.f. will be IN (AT). If this coil is in free space, a certain value of flux density B in Wb/m^2 may be observed. On inserting an iron core the value of B will increase. The ratio of "*flux density produced with the iron core*" to "*flux density produced in free space*" for the same applied m.m.f. is termed the **relative permeability** (symbol μ_r) of the iron.

$$\mu_r = \frac{\text{B in a medium}}{\text{B in free space}}$$

$$(\text{for the same m.m.f.}) \quad .. \quad .. \ (3)$$

For air, μ_r may be taken as unity; for certain other materials it may be as high as 100,000.

19. The ratio of "*flux density, B*" to "*magnetising force, H*" at any point in free space is termed the **magnetic space constant** (or the permeability of free space); it is represented by the symbol μ_o. The value of this constant is $4\pi \times 10^{-7}$ m.k.s. units. Thus :—

$$\mu_o = \frac{B}{H}(\text{in free space}) = 4\pi \times 10^{-7} \quad (4)$$

20. From equation (4) :—

$$\text{B in free space} = \mu_o H$$

By substitution in equation (3) :—

$$\mu_r = \frac{\text{B in a medium}}{\mu_o H}$$

Thus, in any medium :—

$$\frac{B}{H} = \mu_r\mu_o = 4\pi \times 10^{-7} \times \mu_r = \mu \quad (5)$$

The symbol $\mu = \mu_r\ \mu_o$ denotes the **absolute permeability** of a medium.

Reluctance

21. The reluctance of a magnetic circuit is a criterion of the opposition of the circuit to the establishment of magnetic flux and may be likened, by analogy, to the resistance of an electric circuit.

22. It was shown in Section 1 that :—

$$R = \rho\frac{l}{a}\ (\text{Ohms}).$$

Replacing resistivity (ρ) by specific conductance (σ) :—

$$R = \frac{l}{\sigma a}\ (\text{Ohms})$$

Similarly, reluctance S is given by :—

$$S = \frac{1}{\mu a} = \frac{1}{\mu_r \mu_0 a} \text{ (AT/Wb)} \quad .. \quad .. \quad (6)$$

Comparison of Electric and Magnetic Circuits

23. By analogy, flux Φ in a magnetic circuit is equivalent to current I in an electric circuit, m.m.f. is equivalent to e.m.f., and reluctance S to resistance R. Fig. 9 (*a*) shows a simple electric circuit ; Fig. 9 (*b*) a magnetic circuit.

From Ohm's law :—

$$I = \frac{E}{R} \text{ (amps)}$$

By analogy :—

$$\text{Flux} = \frac{\text{m.m.f.}}{\text{Reluctance}} \text{ (Wb)}$$

$$\text{i.e. } \Phi = \frac{\text{m.m.f.}}{S} \text{ (Wb)} \quad .. \quad .. \quad (7)$$

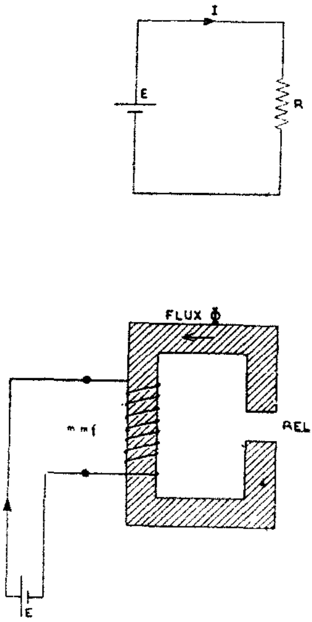

Fig. 9.—SIMPLE ELECTRIC AND MAGNETIC CIRCUITS

Relays

24. As shown in Fig 10, if an equipment is switched on from a remote control point, the heavy current taken by the equipment will develop a considerable volts drop in the connecting cable. The terminal p.d. V_1 will then be very much less than the supply voltage E.

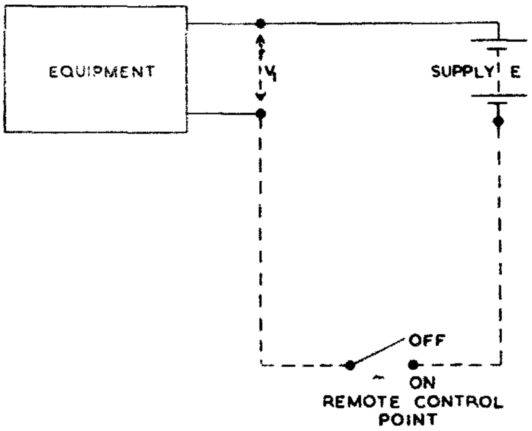

Fig. 10.—REMOTE SWITCHING WITHOUT A RELAY.

25. To prevent this excessive volts drop, a relay can be used (Fig. 11). When the equipment is switched on at the remote control point, the relay becomes " *energised* " and contact RL1 closes to connect the supply to the equipment. There will be very little volts drop in the connecting cable because :—

(*a*) There is only a short length of cable between the supply and the equipment.

(*b*) The current taken by the relay is small.

Note. In radio equipments, relays are annotated as, say $\frac{\text{RL2.}}{3}$ The numerator gives the number of the relay in the equipment and the denominator gives the number of contacts on the relay. The relay contacts are then given as RL2/1, RL2/2, and RL2/3.

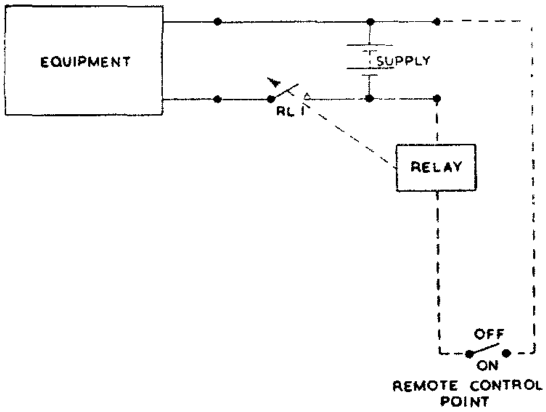

Fig. 11.—REMOTE SWITCHING BY RELAY.

26. Fig. 12 shows the magnetic circuit of a typical relay. It is seen that the flux path

is mainly in iron so that the reluctance of the circuit will be low. Thus, since m.m.f. = flux × reluctance, the ampere—turns required to produce the necessary value of flux will be small. When the circuit is switched on, the field around the solenoid rises and the resultant flux round the magnetic circuit *energises* the relay. The armature is attracted towards the core and closes the relay contact. The circuit to the equipment is then completed. The reverse applies when the on-off switch is opened.

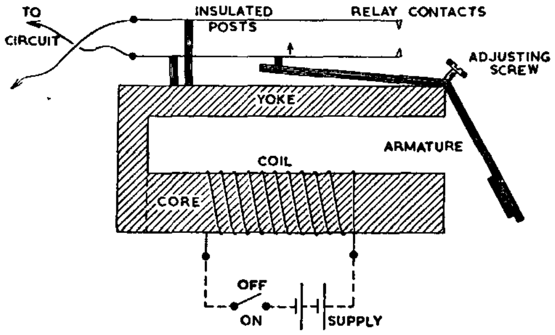

Fig. 12.—TYPICAL MAGNETIC RELAY

The Magneto-striction Effect

27. The theory of magnetism shows that each atom in a magnetic material acts as a tiny electromagnet, the magnetism being produced by the rotation of the electrons around the nucleus of the atom at high speed. The orbiting electrons may be regarded as forming current rings in the material. When the material is not magnetised, the current rings are arranged in small, self-contained, symmetrical groups, and any such group has no influence on any other. When the material is magnetised, the current rings become distorted such that their axes point, more or less, in the direction of the magnetising force. In some materials such as permanent magnets, the current rings continue to have their axes distorted even when the magnetising force is removed.

28. Some materials when magnetised, expand in a lengthwise direction due to the distorted lengthwise axes of the current rings. Conversely, if a bar of such material is caused to expand and contract lengthwise, a varying magnetic field is produced because of the forced distortion of the current rings. This is termed the **magneto-striction effect** of a material (most marked in nickel)—an effect which is used in radio in such devices as the underwater hydrophone.

MAGNETIC MATERIALS

Classification

29. Magnetic materials are used extensively in radio for such applications as the cores of transformers and coils, relay magnetic circuits, magnetic amplifiers, telephones, and loudspeakers. An elementary knowledge of the properties of such materials is required.

30. All materials are classified for magnetic purposes under one of three headings :—

(*a*) **Ferro-magnetic.** These materials (e.g. iron, nickel, and cobalt) have high values of relative permeability μ_r. Since $B = \mu_0 \mu_r H$, ferromagnetic materials have the property of greatly increasing the flux density B in a circuit for a given magnetising force H.

(*b*) **Para-magnetic.** These materials (e.g. platinum and aluminium) are virtually non-magnetic, having values of μ_r only slightly *greater* than unity. Platinum, for instance, has a relative permeability of 1·000017.

(*c*) **Dia-magnetic.** These materials (e.g. lead, copper, and bismuth) are virtually non-magnetic, having values of μ_r only slightly *less* than unity. Bismuth, for instance, has a relative permeability of 0·99996.

B—H Curves

31. In the simple circuit shown in Fig. 13, the current I amps through the coil of N turns can be varied by means of a rheostat. Thus, the m.m.f., given by IN (AT), can be varied, and so can the magnetising force H since :—

$$H = \frac{\text{m.m.f.}}{l} \quad \text{(AT/m)}.$$

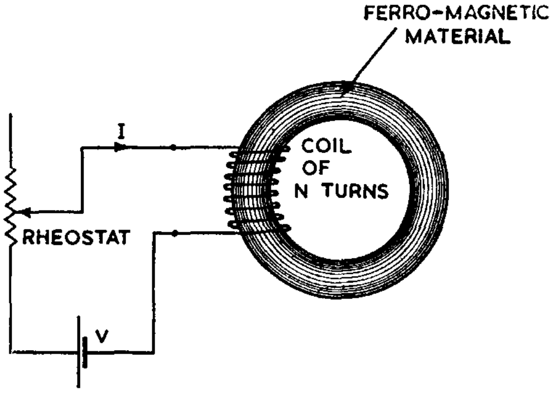

Fig. 13.—A SIMPLE MAGNETIC CIRCUIT

Further, the flux density B can be varied since :—

$$B = \mu H \ (\text{Wb/m}^2).$$

It is possible, therefore, with such a circuit to plot a graph of the flux density B against the magnetising force H ; such a graph is termed a **B—H curve** for the material used. For air, such a graph would be a straight line.

32. A B—H curve for a typical ferromagnetic material is shown in Fig. 14 :—

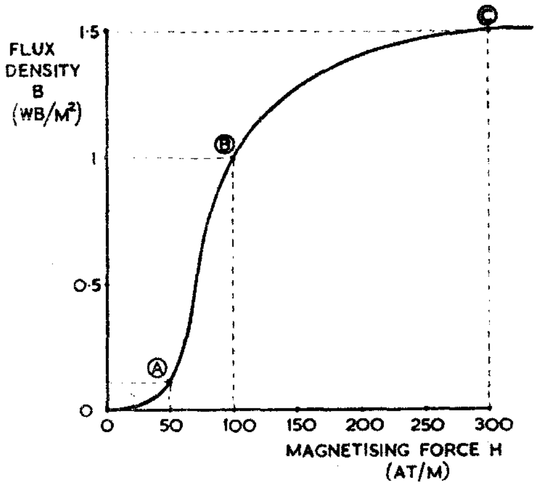

Fig. 14.—A TYPICAL B-H CURVE

(*a*) When the magnetising force H is small there is very little flux density B. According to the electron theory of magnetism quite a large force is necessary to cause the electron current rings to distort their axes in the direction of the applied force.

(*b*) When H reaches a certain value, the axes of the current rings start to change and very quickly align themselves according to the polarity of the applied force. Thus, B rises very quickly for small changes in H.

(*c*) When the current rings have their axes completely aligned with the applied force the material has reached **saturation ;** the only increase now in B is that due to H alone.

33. The ratio $\frac{B}{H}$ gives the absolute permeability of a material. From this, the relative permeability can be found from the relationship $\mu_r = \frac{\mu}{\mu_0}$ (see Para. 20).

By making the necessary calculations at three points A, B, and C in Fig. 14 it is seen that the relative permeability of a material is *not* a constant. It varies with the magnetising force H in the manner shown in Fig. 15.

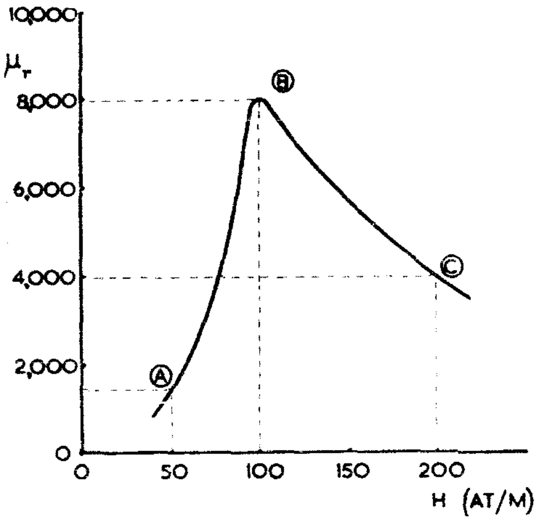

Fig. 15.—VARIATION OF RELATIVE PERMEABILITY

Hysteresis

34. Fig. 16 shows that if H is increased from zero in a positive direction, B rises as in a normal B—H curve along the points OPQ. On reducing H to zero again, however, B does not follow its original path to zero, but follows the path QR. Completing a full cycle for H gives the loop for B as shown by QRSTQ. Such a loop is known as a **hysteresis loop.** The flux density B always "*lags behind*" the magnetising force H because of the inertia of the electron current rings in the material.

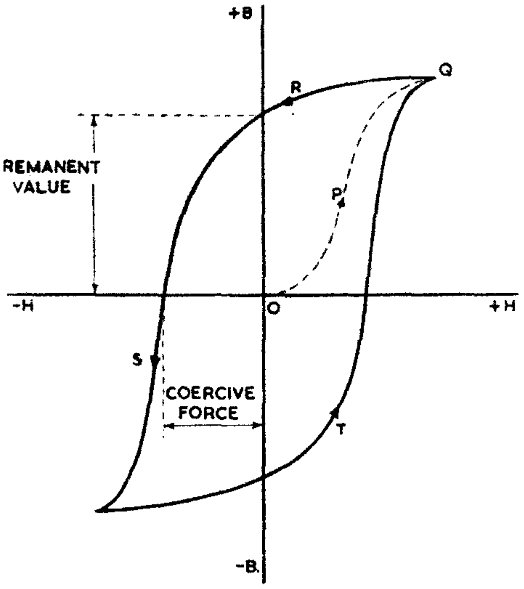

Fig. 16.—A TYPICAL HYSTERESIS LOOP.

35. A negative value of H is necessary to reduce B to zero. The actual value of H in AT/m necessary for this is termed the **coercive force.** If the material has first reached saturation this value is termed the **coercivity** of the material.

36. The value of B in Wb/m^2 remaining in the material when H has reached zero is termed the **remanent value.** If the material has first reached saturation this value is termed the **remanence** of the material.

37. **Retentivity** is a measure of the magnetism retained by a material over a long period of time. It is not to be confused with remanence since in some materials remanence is quickly reduced to zero under normal conditions. Retentivity is given by :—

$$\text{Retentivity} = \frac{\text{Coercivity}}{\text{Remanence}} \text{ of the material.}$$

This relationship shows, for instance, that for a permanent magnet a material with a high value of coercivity is required—i.e., the force required to " remove " the magnetism must be large.

A.C. Permeability

38. It was shown in Para. 33 that the relative permeability μ_r of a material was a function of the magnetising force H. Where H is alternating as in a.c. circuits, the hysteresis loop for the material must be used. The value of μ_r under these conditions is indicated by the *slope* of the hysteresis loop. If the line joining the tips of the loop has a steep slope, the a.c. relative permeability of the material will be high. Further, the slope will decrease as the magnitude of the a.c. input increases.

Hysteresis Loss

39. Energy is expended in a magnetic material as the flux density B follows the a.c. magnetisation in a hysteresis loop. The axes of the current rings in the material have been caused to alternate and in so doing, produce heat by friction in the atomic structure of the material. Thus, some of the original electrical energy supplied to the circuit has been converted into heat energy ; this loss of energy from the circuit is termed **hysteresis loss.**

40. The area enclosed by a hysteresis loop is a measure of the hysteresis loss of the material ; the greater this area, the larger is the loss. Thus, where hysteresis loss must be kept to a minimum, a material with a narrow hysteresis loop should be selected. In addition, the loss increases with frequency —i.e., the number of times the hysteresis loop is traced out per second.

Factors Affecting the Choice of Material

41. The choice of ferro-magnetic material for any application in radio is determined by several factors :—

(*a*) The relative permeability μ_r of the material.

(*b*) The values of B and H at which the material saturates.

(*c*) The hysteresis loss as determined by the hysteresis loop.

(*d*) The coercivity and remanence which determine the retentivity of the material.

Comparison of Typical Materials

42. The main characteristics of ferro-magnetic materials can be determined from their hysteresis loops. The details of a few materials are given below:—

(*a*) **35% Cobalt-iron**

- (i) Very low value of μ_r (100).
- (ii) Very high hysteresis loss.
- (iii) Very high retentivity : makes very good permanent magnet material.

(*b*) **Tungsten-steel**

- (i) Very low value of μ_r (200).
- (ii) Very high hysteresis loss.

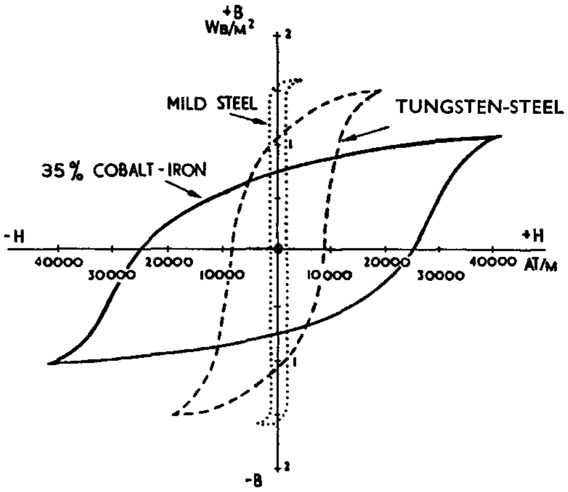

Fig. 17.—HYSTERESIS LOOPS FOR MATERIALS (*a*), (*b*), AND (*c*).

(iii) High retentivity : makes good permanent magnet material.

(*c*) **Mild steel**
- (i) Low value of μ_r (1,000).
- (ii) High hysteresis loss.
- (ii) Low retentivity : seldom used.

(*d*) **Soft iron**
- (i) High value of μ_r (10,000).
- (ii) High hysteresis loss.
- (iii) Low retentivity : used in d.c. circuits in relays.

(*e*) **Stalloy**
- (i) High value of μ_r (15,000).
- (ii) Low hysteresis loss.
- (iii) High saturation values of B and H : used considerably in a.c. circuits.

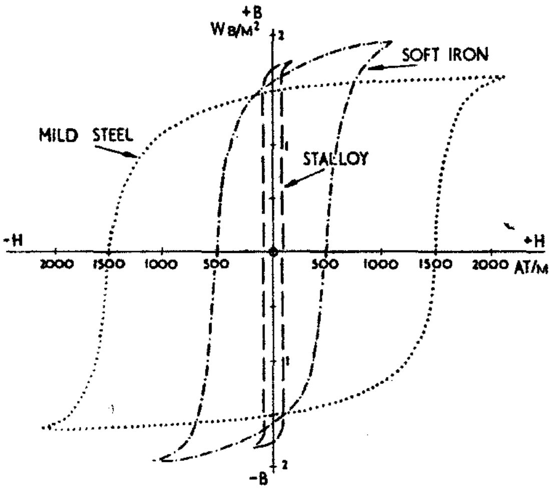

Fig. 18.—HYSTERESIS LOOPS FOR MATERIALS (c), (d), AND (e).

(f) **Permendur**
- (i) High value of μ_r (40,000).
- (ii) Low hysteresis loss.
- (iii) Low saturation values of B and and H : d.c. magnetisation must be kept small.

(*g*) **Permalloy**
- (i) High value of μ_r (100,000).
- (ii) Low hysteresis loss.
- (iii) Very low saturation values of B and H : cannot be used in d.c. circuits.

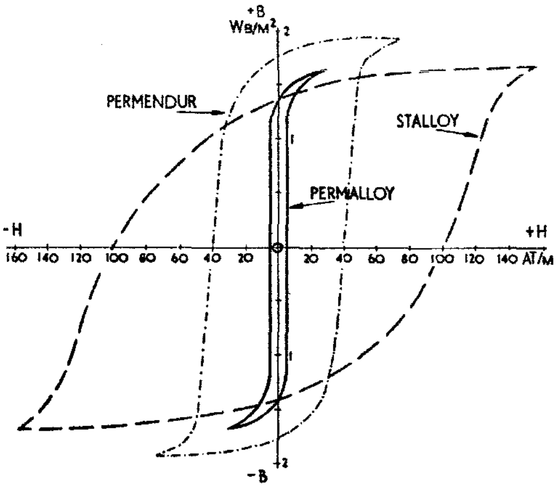

Fig. 19.—HYSTERESIS LOOPS FOR MATERIALS (e), (f), AND (g).

Note. The above graphs have been sketched to different scales. To assist in the comparison of the materials some loops have been repeated in successive graphs.

SECTION 2

CHAPTER 2

ELECTROMAGNETIC INDUCTION

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ELECTROMAGNETIC INDUCTION

Introduction

1. Before such devices as the transformer and the electric generator can be explained, it is necessary to understand the principles of electromagnetic induction. Two laws state the theory of electromagnetic induction very concisely :—

(*a*) **Faraday's law.** When the magnetic flux through a circuit is changing an induced e.m.f. is set up in that circuit, and its magnitude is proportional to the rate of change of flux.

(*b*) **Lenz's law.** The direction of an induced e.m.f. is such that its effect tends to oppose the change producing it.

Verification of Faraday's and Lenz's Laws

2. Fig. 1 shows a coil connected to a galvanometer : when a bar magnet is moved up and down inside the coil, the following observations can be made :—

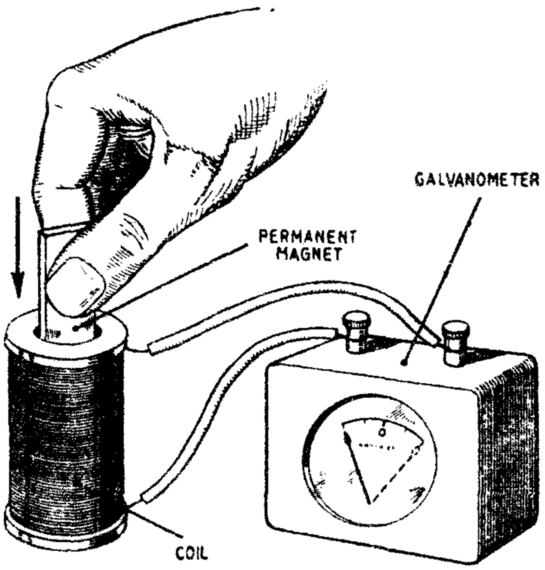

Fig. 1.—VERIFICATION OF FARADAY'S LAW

(*a*) **Relative motion** between the field (magnet) and the conductor (coil) is essential before an e.m.f. (indicated by a deflection in the galvanometer) is induced in the conductor.

(*b*) The greater this relative motion, the greater is the deflection in the galvanometer. Thus, the **rate** at which the flux is changing relative to the conductor determines the magnitude of the induced e.m.f.

(*c*) A bar magnet with a stronger field will give a larger induced e.m.f. for a similar movement.

(*d*) If the magnet is placed in a position at right angles to the axis of the coil (Fig. 2), no e.m.f. will be induced when the magnet is moved towards the coil, or *vice versa.* The lines of flux are now parallel with the turns in the coil so that the conductor is not being "cut" by the flux lines. The conductor must **cut,** or **be cut by,** lines of flux before an e.m.f. is induced in the conductor.

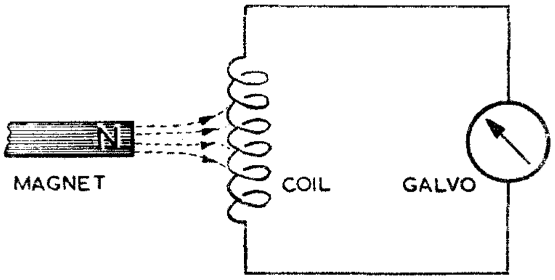

Fig. 2.—ZERO FLUX LINKAGE.

3. The **direction** of the induced e.m.f. in a conductor may be obtained by applying Lenz's law. Fig. 3 shows a conductor in a magnetic field. If this conductor is moved

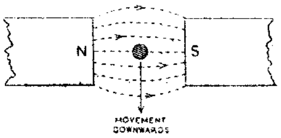

Fig. 3.—CONDUCTOR MOVING IN A MAGNETIC FIELD.

downward through the field it is cutting lines of flux. From Faraday's law, an e.m.f. will be induced in the conductor, the magnitude of the e.m.f. depending on the rate at which the conductor has been moved and on the flux density of the field. From Lenz's law, the direction of the induced e.m.f. will be such that its effect will tend to oppose that downward motion of the conductor producing it—i.e., it is a "**back e.m.f.**" Thus, the direction of the induced e.m.f. will be such that it tends to move the conductor *upward.* For this to happen, the direction of the current in the conductor must be "out of paper" (see Sect. 3).

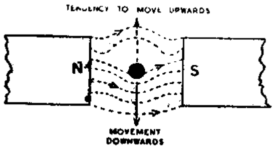

Fig. 4.—ILLUSTRATING LENZ'S LAW.

4. The direction of the induced e.m.f. in a conductor can be found directly by applying **Fleming's RIGHT-hand rule** :—

> "*The thumb, the first finger, and the middle finger of the right hand are held at right angles relative to each other. With the thuMb pointing in the direction in which the conductor has been Moved, and the First Finger in the direction of the Field (N. to S) the mIddle finger will indicate the direction in which current I would flow in the conductor ; this, in turn, gives the direction of the induced e.m.f.*"

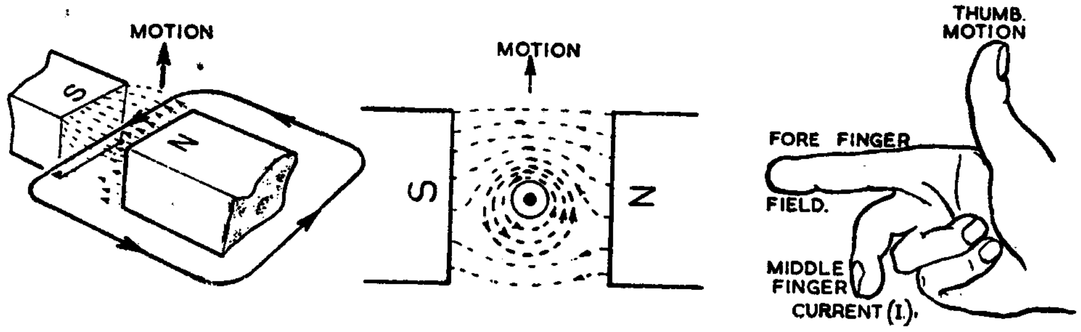

Fig. 5.—FLEMING'S RIGHT HAND RULE.

5. Both Faraday's law and Lenz's law can be summarized by the expression :—

$$E = -\frac{d\Phi}{dt} \text{ (volts)} \quad \ldots \ldots \ldots \quad (1)$$

The minus sign indicates that E is a *back* e.m.f.; Φ is the flux in Webers; t is the time in seconds ; and $\frac{d\Phi}{dt}$ is the change of flux with respect to time.

6. If the conductor is wound in the form of a coil with N turns, each turn will be cut by the flux to give a contribution to the e.m.f. induced in the coil. The total e.m.f. is then given by :—

$$E = -N \frac{d\,\Phi}{dt} \text{ (volts)} \quad \ldots \ldots \ldots \quad (2)$$

Self-inductance

7. If the current in a conductor changes in any way, the magnetic field associated with that current will also change and, from Faraday's law, an e.m.f. will be induced in the conductor. From Lenz's law, the induced e.m.f. will act in such a direction as to oppose the cause of the induced e.m.f. Thus, it will oppose any **change** in the value of current in the circuit, whether the current tends to increase or decrease.

8. Any circuit which has an e.m.f. induced in it by a change of current through that circuit possesses **self-inductance** (symbol L), and has the property of opposing any change of current in the circuit by virtue of the back e.m.f. Any conductor possesses self-inductance. In order to increase it, the conductor is wound in the form of a coil to increase the total induced back e.m.f. A conductor wound in this manner in order to increase its inductance, is termed an **inductor** and its uses in radio are numerous.

9. The unit of inductance L is the **henry** (symbol H). A circuit has an inductance of one henry if a current in it, changing at the **rate** of one ampere per second, induces an e.m.f. of one volt across it.

10. In any given circuit, the ratio of the induced e.m.f. to the rate of change of

current in the circuit is the constant of self-inductance, L henrys. Thus :—

$$\frac{-E}{\frac{dI}{dt}} = L \text{ (henrys)}$$

$$\therefore E = -L \frac{dI}{dt} \text{ (volts)} \quad \ldots \quad \ldots \quad (3)$$

This expression shows that the magnitude of the back e.m.f. is proportional to the value of inductance and to the change of current with respect to time.

Factors Affecting Self-inductance

11. Equations (2) and (3) each give the e.m f. induced in a coil in terms of different factors. By equating these two expressions and substituting for flux Φ (see Chap. 1), the inductance of a coil is shown to be:—

$$L = N^2 \frac{\mu a}{l} \text{ (henrys)} \quad \ldots \quad \ldots \quad (4)$$

where N is the number of turns in the coil ; μ is the absolute permeability of the circuit ; a is the cross sectional area of the coil ; and l is the length of the coil.

12. Inductors having values of inductance ranging from a few micro-henrys to many henrys are used in radio, the value being determined by the factors given in equation (4). In this connection it should be noted that :—

(*a*) Doubling the turns will increase the inductance *four* times.

(*b*) A core, of high permeability, inserted inside the coil will greatly increase the inductance.

Mutual Inductance

13. If a change of current in one circuit induces an e.m.f. in another circuit, the two circuits possess **mutual inductance** (symbol M). Consider two inductors L_1 and L_2 connected as shown in Fig. 6. On closing the switch, the current and associated magnetic field around L_1 rise from zero. During the time that the current is rising the changing flux will produce a *self-induced* e.m.f. in L_1. In addition, some of this changing flux will cut L_2 to produce a *mutually-induced* e.m.f. in L_2, as indicated by a deflection in the galvanometer. As soon as the current in L_1 has reached its final steady value, the magnetic field will no longer be changing, and both the self-induced e.m.f. in L_1 and the mutually-induced e.m.f. in L_2 will fall to zero. E.m.f.s are induced only at the instants of opening and closing the switch, i.e., when the current is *changing*.

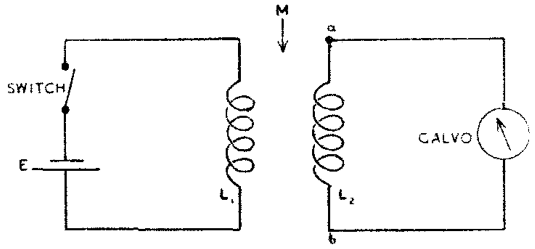

Fig. 6.—MUTUAL INDUCTANCE.

14. If the connections to the coil L_2 are reversed a *reverse* deflection will occur in the galvanometer on closing the switch. The direction of a mutually-induced e.m.f. depends on the direction in which the two coils are wound relative to each other.

15. If the coil L_2 is moved further away from the coil L_1 the deflection in the galvanometer on opening or closing the switch will be reduced. This indicates that fewer flux lines are cutting L_2, i.e., the *flux linkage* is less. The magnitude of the mutually-induced e.m.f. is, therefore, dependant on the *degree of coupling* between the two coils.

16. If the coils L_1 and L_2 are placed with their axes at right angles to each other, as shown in Fig. 7, no e.m.f. will be induced in L_2 ; the flux lines from L_1 will no longer be cutting L_2. This arrangement of coils is sometimes used in practice where it is necessary to prevent an e.m.f. being induced in one circuit from another.

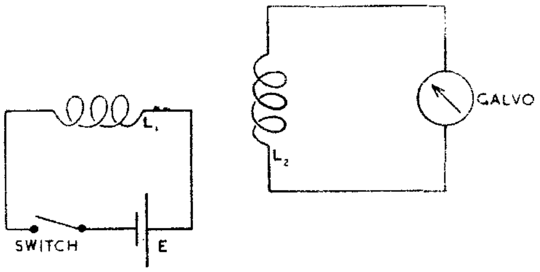

Fig. 7.—ZERO FLUX LINKAGE.

17. The unit of mutual inductance is the **henry** (H). Two circuits have a mutual inductance of one henry if a current in one circuit, changing at the *rate* of one ampere per second, induces an e.m.f. of one volt in the other circuit.

18. With two given circuits, the ratio of the mutually-induced e.m.f. in one circuit to the rate of change of current in the other circuit is the constant of mutual inductance. Thus:—

$$\frac{\pm E_2}{\frac{dI_1}{dt}} = M \text{ (henrys)}$$

$$\therefore\ E_2 = \pm M \frac{dI_1}{dt} \quad \ldots \ldots \ldots \quad (5)$$

The " ± " sign indicates that the induced e.m.f. can be in either direction in accordance with para. 14.

Factors Affecting Mutual Inductance

19. By taking into consideration the factors mentioned in Paras. 14 and 15, equations (2) and (5) can be resolved to give an expression for the mutual inductance of two circuits. Thus :—

$$M = \pm k\ N_1N_2 \frac{\mu a}{l} \text{ (henrys)} \ \ldots \quad (6)$$

where k is a constant, N_1 and N_2 are the number of turns in each coil, μ is the absolute permeability of the circuit, a the mean cross-sectional area, and l the length.

20. Circuits having values of mutual inductance ranging from a few micro-henrys to many henrys are used in practice in radio.

Inductors in Series

21. Fig. 8 shows three coils of self-inductance L_1, L_2, and L_3 connected in series such that the same current I flows through each coil. The e.m.f. induced in each coil is $e = -L\frac{di}{dt}$ and will be the same only if the inductance values are equal. For inductors connected in series, the total inductance is the sum of the individual self-inductances—

$$L_T = L_1 + L_2 + L_3 + \text{ (henrys)} \ \ldots \ldots \quad (7)$$

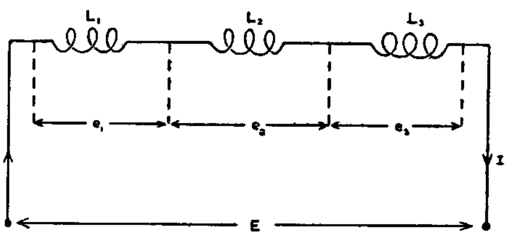

Fig. 8.—INDUCTORS IN SERIES.

Inductors in Parallel

22. Fig. 9 shows three coils of self-inductance L_1, L_2, and L_3 connected in parallel across an e.m.f. E volts such that currents I_1, I_2, and I_3 flow through each. For inductors connected in parallel, the reciprocal of the total inductance equals the sum of the reciprocals of the individual inductances:—

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ \ldots \quad (8)$$

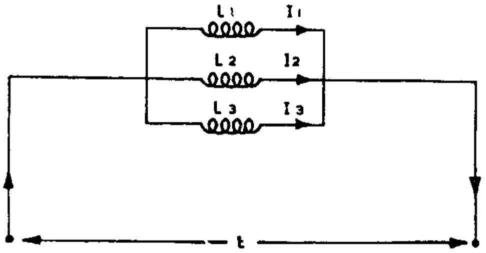

Fig. 9.—INDUCTORS IN PARALLEL.

Inductors in Series with Mutual Inductance between them

23. Consider two coils of self-inductance L_1 and L_2 connected in series across a supply as shown in Fig. 10. A certain mutual inductance M exists between the two coils. Thus, when the current changes, the total e.m.f. induced in L_1 will be the sum of its self-induced e.m.f. and the mutually-induced e.m.f. from L_2. The total e.m.f. induced in L_2 can be calculated in a similar manner. The expressions so obtained can then be resolved to give the total inductance of the circuit :—

$$L_T = L_1 + L_2 \pm 2M \text{ (henrys)} \quad \ldots \ldots \quad (9)$$

The mutual inductance can be either "series aiding" or "series opposing" depending on which way the coils are wound relative to each other.

For series aiding :—

$$L_T = L_1 + L_2 + 2M \text{ (henrys)}$$

For series opposing :—

$$L_T = L_1 + L_2 - 2M \text{ (henrys)}$$

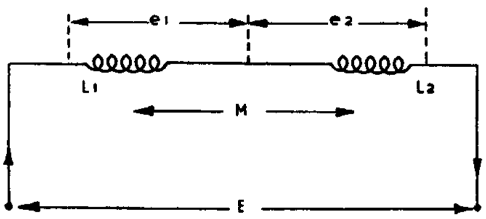

Fig. 10.—SERIES INDUCTORS WITH MUTUAL INDUCTANCE.

Energy Stored in a Magnetic Field

24. In an electric circuit, energy is being expended all the time current is flowing. In a magnetic circuit, energy is expended only in creating the magnetic field. Once the field has been established no further energy is required to maintain it. The original energy expended is stored in the magnetic field in the form of flux and is returned to the source when the field collapses. For purposes of calculation, the energy stored in the magnetic field of a coil is :—

$$W = \tfrac{1}{2} LI^2 \text{ (joules)} \quad \ldots \ldots \quad (10)$$

SECTION 2

CHAPTER 3

INDUCTIVE CIRCUITS

Paragraph

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Experimental study, growth of current 3

General case, growth of current 7

Decay of current in an inductive circuit 12

Time constant 16

Practical example.. 18

Square waves applied to an inductive circuit 20

PART 1, SECTION 2, CHAPTER 3

INDUCTIVE CIRCUITS

Introduction

1. A coil, as well as having a certain value of inductance L henrys must also have a certain value of resistance R ohms. Fig. 1 shows such a coil (where L and R are shown separately) connected to a supply of e.m.f. E volts, via a switch.

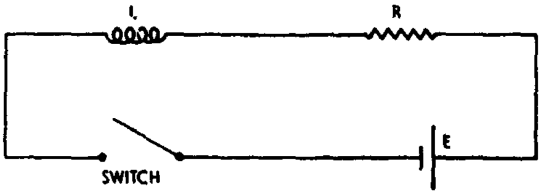

Fig. 1.—SIMPLE INDUCTIVE CIRCUIT.

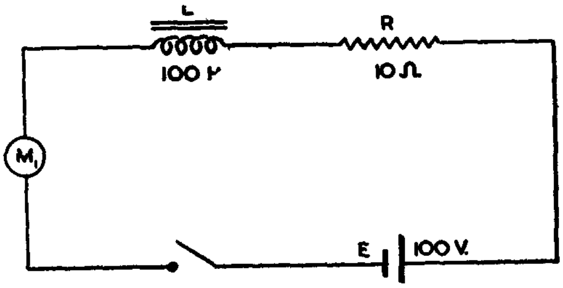

Fig. 2.—EXPERIMENTAL STUDY, GROWTH OF CURRENT.

4. Readings of the current *i* passing through M_1 are taken at intervals of 10 seconds after the closing of the switch. A typical set of readings is given in Table 1, and from these readings the graph of Fig. 3 is plotted.

Time from start (sec)	0	10	20	30	40	50	60
Current (amps)	0	6·32	8·65	9·51	9·82	9·93	9·97

TABLE 1

2. When the switch is closed, the battery will try to pass a current $I = \frac{E}{R}$ amps round the circuit. However, as soon as the current commences to rise, a back e.m.f. is developed across the coil ; from Lenz's law this e.m.f. will oppose the original rise in the current, so that the initial rate of change of current is " slowed up ". The current will continue to rise but at a lower rate of change, and the back e.m.f. will also decrease. This process continues—the current gradually rising to its maximum value of $I = \frac{E}{R}$ amps at a progressively lower rate of change, i.e., it is an *exponential* rise. The back e.m.f. falls exponentially towards zero, reaching this value when the current has reached its maximum steady value.

Experimental Study of the Growth of Current in an Inductor

3. The facts stated in Para. 2 can be verified by means of a simple experiment. In the circuit of Fig. 2, R = 10 Ω, L = 100H, and E = 100V ; M_1 is an instrument for measuring the current in amps ; and a stop-watch is required to give accurate measurement of time in seconds.

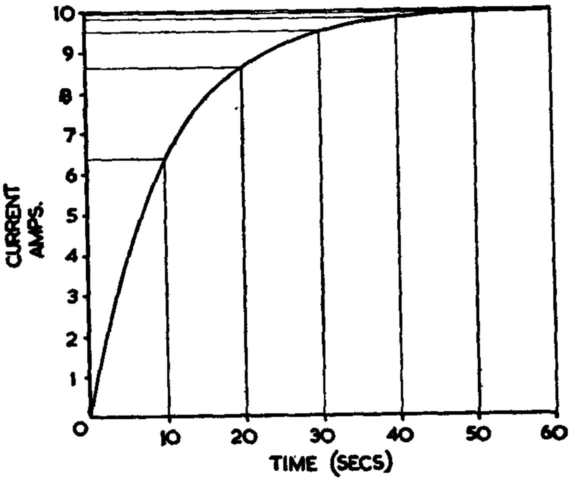

Fig. 3.—VARIATION OF *i* DURING GROWTH.

5. The experiment could be repeated with different values of L and R. It would be found that with a smaller value of L, the current would rise more rapidly. This is to be expected since the back e.m.f. would be smaller. A similar result would be seen with a *larger* value of R. The ratio of $\frac{L}{R}$ is what really determines the rate of growth of the current in the circuit.

6. It is found that whatever the actual values of L and R in such an experiment, the ratio $\frac{L}{R}$ equals the time in seconds for the current in the circuit to rise to 63·2% of its maximum value $\frac{E}{R}$. In the circuit of Fig. 2, $\frac{L}{R} = \frac{100}{10} = 10$ seconds. Thus, in 10 seconds from the start the current rose to 6·32 amps ; this is 63·2% of its maximum value $\frac{E}{R} = \frac{100}{10} = 10$ amps. This quantity $\frac{L}{R}$ is termed the *time constant* of the circuit and is defined in full in Para. 16.

General Case of the Growth of Current in an Inductor

7. Paras. 3 to 6 have dealt with a particular circuit. In the general case, for purposes of accurate calculation, the current in the circuit at any instant t after closing the switch is given by :—

$i = I\,(1 - \epsilon^{\frac{-R}{L}t})$ (amps) .. .. (1)

where, i = the current at any instant.

I = the final current = $\frac{E}{R}$.

ϵ = Napierian log base = 2·718

R = the resistance in ohms.

L = the inductance in henrys.

t = time in seconds after closing the switch.

8. (*a*) From Ohm's law, the p.d. developed across R at any instant after closing the switch is :—

$V_R = i\,R$ (volts) .. .. (2)

(*b*) From Kirchhoff's second law, the sum of the p.d.s across R and across L must at every instant after closing the switch equal the applied e.m.f. E volts.

$\therefore V_R + V_L = E$

$\therefore V_L = E - V_R$ (volts) .. .. .. (3)

9. A graph can be plotted showing the variation in i, V_R, or V_L with respect to the time t after closing the switch. This can be obtained in two ways :—

(*a*) Repeat the experiment of Paras. 3 to 6. Having obtained the values for i, the corresponding values for V_R and V_L follow from equations (2) and (3) respectively.

(*b*) The value of $I = \frac{E}{R}$ amps is calculated and inserted in equation (1), together with the values for R and L. Various instants of time t seconds are inserted in equation (1) and the current i at these instants is evaluated. Having obtained the values for i, the corresponding values for V_R and V_L follow from equations (2) and (3) respectively.

10. In either case, three instants of time are sufficient for most purposes :—

(*a*) *At the instant of closing the switch* ($t = o$).

$i = o \;:\; V_R = o \;:\; V_L = E$

(*b*) *At* $t = \frac{L}{R}$ *seconds after closing the switch*

$i = 0{\cdot}632I : V_R = 0{\cdot}632E : V_L = 0{\cdot}368E$

(*c*) *At* $t = 5\frac{L}{R}$ *seconds after closing the switch*

$i \bumpeq I \;:\; V_R \bumpeq E \;:\; V_L \bumpeq O$

11. These three instants of time are used to plot the graph showing the exponential rise in the current i in the circuit against the time t in seconds after closing the switch (Fig. 4). The graph for V_R will rise in a manner similar to that for i ; that for V_L will fall to zero as i rises to its maximum value.

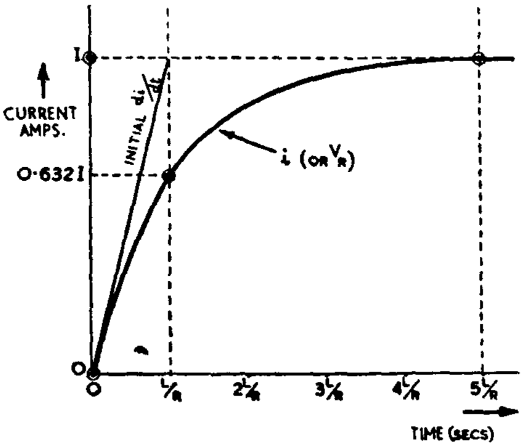

Fig. 4.—GROWTH OF CURRENT.

Decay of Current in an Inductive Circuit

12. Consider the circuit shown in Fig. 5.

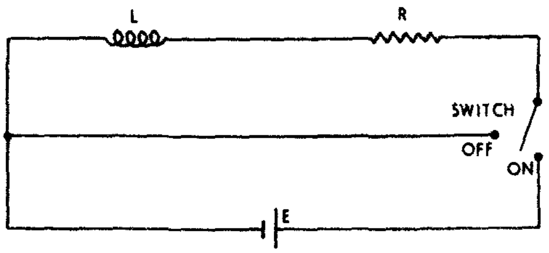

Fig. 5.—SIMPLE INDUCTIVE CIRCUIT, DECAY OF CURRENT.

With the circuit switched on, the current in the coil will rise exponentially towards its maximum value $I = \frac{E}{R}$ amps, reaching this value in a time of approximately $5\frac{L}{R}$ seconds after closing the switch. Assume maximum current to be now established in the coil ; since it is a steady current, the back e.m.f. across the coil will have fallen to zero and the p.d. across the resistance will equal the supply e.m.f. If the circuit is switched off under these conditions, the coil is shunted by the resistance, and the current will tend to fall to zero. It cannot do so instantly, however, because as soon as the current starts to fall a back e.m.f. is produced across the coil ; this back e.m.f. will endeavour to maintain the current in the circuit. Thus, in the same way that the current in an inductive circuit rises in an exponential manner, so it will now fall or decay. The curve of the decay of current can be obtained from an experiment similar to the one described in Paras. 3 to 6. A stop-watch is started at the instant of disconnecting the supply and readings of the current i taken at regular intervals of time. From a set of such readings the required graph can be plotted.

13. (*a*) For purpose of accurate calculation the current in the circuit at any instant t after opening the switch and allowing the current to decay is given by :—

$$i = I.\epsilon^{-\frac{R}{L}t} \text{ (amps)} \quad \ldots \ldots \ldots \quad (4)$$

where all the terms have the same significance as in equation (1)

(*b*) From Ohm's law, the p.d. developed across R at any instant after opening the switch is :—

$$V_R = i\,R \text{ (volts)} \quad \ldots \ldots \ldots \ldots \quad (5)$$

(*c*) From Kirchhoff's second law, the sum of the p.d.s across R and across L must at any instant equal the e.m.f. acting in the circuit. If the supply is disconnected, the e.m.f. is zero. Thus :—

$$V_R + V_L = O$$

$$\therefore V_L = -V_R \text{ (volts)} \quad \ldots \ldots \ldots \quad (6)$$

14. A graph can be plotted showing the variation in *i*, V_R, or V_L with respect to the time t seconds after disconnecting the supply. Either of the two methods described in Para. 9 can be used to obtain such a graph, and again three instants of time are significant :—

(*a*) *At the instant of disconnecting the supply* ($t = o$).

$$i = I \quad : \quad V_R = E \quad : \quad V_L = -E$$

(*b*) *At* $t = \frac{L}{R}$ *seconds after disconnecting the supply.*

$$i = 0{\cdot}368I : V_R = 0{\cdot}368E : V_L = -0{\cdot}368E$$

(*c*) *at* $t = 5\frac{L}{R}$ *seconds after disconnecting the supply.*

$$i \simeq O \quad : \quad V_R \simeq O \quad : \quad V_L \simeq O$$

15. These three instants of time are used to

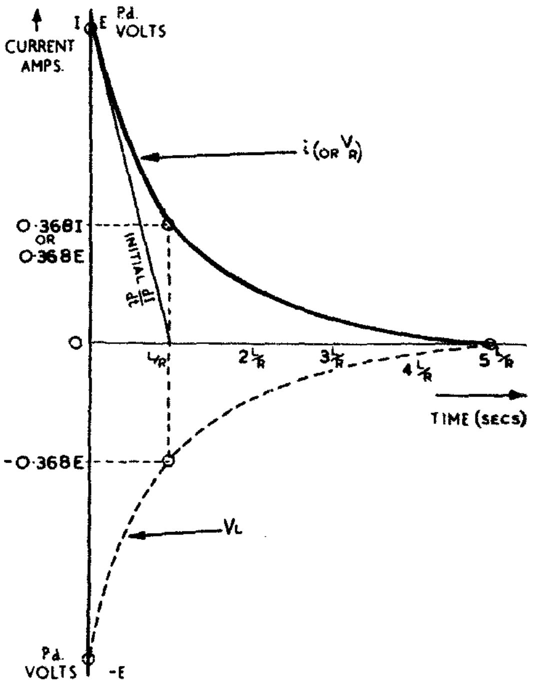

Fig. 6.—DECAY OF CURRENT.

plot the graph showing the variations in i, V_R and V_L with respect to the time t seconds after disconnecting the supply (Fig. 6).

Time Constant

16. The time $t = \frac{L}{R}$ seconds is termed the **time constant** of an inductive circuit and has been referred to in Paras. 6. It is defined as follows :—

> *The time constant* $t = \frac{L}{R}$ *is the time taken for the current in an inductive circuit to rise to* 63·2% (*approximately two-thirds*) *of its maximum value when connected to a supply, or to fall by* 63·2% *of its maximum value when disconnected from a supply. Alternately, it is the time taken for the current to reach its* **maximum** *value in the first case, or to fall to* **zero** *in the other, provided the* **initial** *rate of change of current is maintained.*

(The latter is shown in the graphs although it cannot apply in practice).

17. In theory, the current in an inductor would take an infinitely long time to reach its maximum value or to fall to zero. However, after a time of $5\frac{L}{R}$ seconds the growth or decay is so nearly complete as to be considered so for practical purposes.

Practical Example

18. In the circuit shown in Fig. 7, L = 1H, R = 10 Ω, and E = 10V. The circuit is switched on for one second and then switched off. It is required to sketch a graph to indicate how the current in the circuit varies with time.

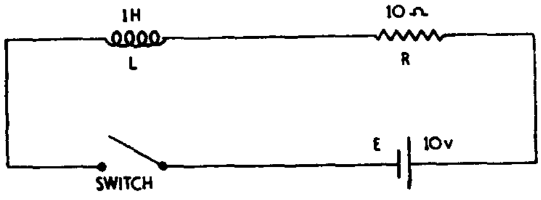

Fig. 7.—EXAMPLE.

19. (*a*) The time constant $t = \frac{L}{R} = \frac{1}{10} = 0{\cdot}1$ second.

(*b*) The maximum current $I = \frac{E}{R} = \frac{10}{10} =$ 1 amp.

	Time (Seconds)	0	$\frac{L}{R} = 0{\cdot}1$	$5\frac{L}{R} = 0{\cdot}5$
Growth of current	i (amps)	0	0·632	1
Decay of current (after 1 sec.)	i (amps)	1	0·368	0

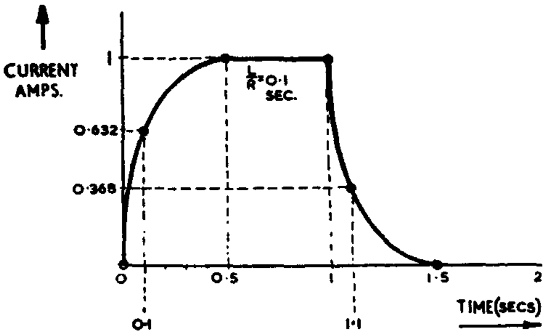

Fig. 8.—GROWTH AND DECAY OF CURRENT.

(*c*) The current at the relevent instants of time is given in the table.

(*d*) The graph of the growth and decay of the current is shown in Fig. 8. The graph of the p.d.s across R and across L could be obtained from equations (2) and (3) during growth, and from equations (5) and (6) during decay.

Square Waves Applied to an Inductive Circuit

20. Although the effect of switching a coil across a d.c. supply is important, in some radio equipments it is more important to consider the effect of applying a square wave of voltage. One simple method of producing such a square wave would be merely to switch the circuit shown in Fig. 9 on and off alternately for definite periods of time. If the periods of time for which the circuit is switched on and off are equal, a *symmetrical* square wave of voltage is produced, as shown. The practical methods for producing square waves are fully dealt with in Part 3.

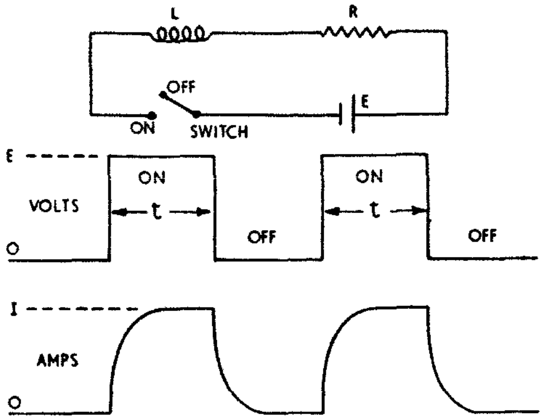

Fig. 9.—SQUARE WAVES APPLIED TO AN INDUCTIVE CIRCUIT.

21. Provided the periods of time for which the supply is switched on and off are long in relation to the time constant of the circuit, the current in the coil will rise and decay exponentially as explained in Para. 19 and as shown in Fig. 9. The corresponding variations in the p.d.s across R and across L are as shown in Fig. 10.

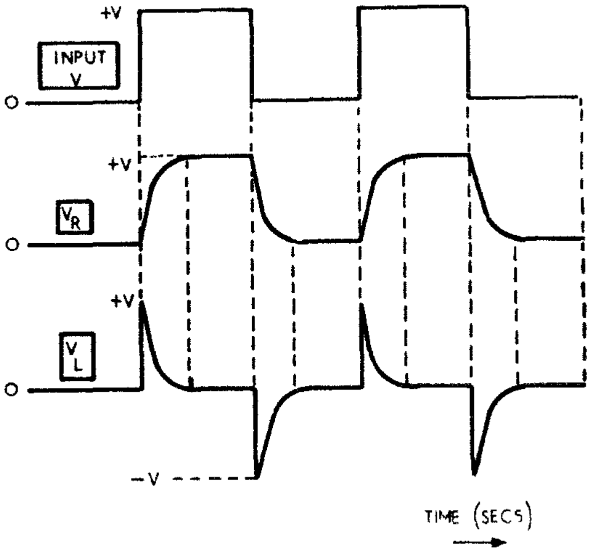

Fig. 10.—VARIATION IN V_R AND V_L WITH TIME

SECTION 2

CHAPTER 4

CONSTRUCTION OF INDUCTORS

Paragraph

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PART 1, SECTION 2, CHAPTER 4

CONSTRUCTION OF INDUCTORS

Introduction

1. Inductors have a wide application in radio ; they vary in size and value from large iron-cored inductors for use at low frequencies to very small inductors for use at the higher frequencies. This Chapter gives an outline of the basic construction of such inductors.

Eddy Currents

2. Consider a coil wound over an iron core. When the current in the coil changes, the magnetic flux linking with the iron core changes, and (according to Faraday's law) an e.m.f. is induced in the *core* as well as in the coil. Such an induced e.m.f. in the core gives rise to circulating currents—these currents being termed **eddy currents.**

3. The direction of the flow of eddy currents in the core will be given by Fleming's right-hand rule, the effective "*motion*" of the core being the reverse of the direction in which the field is moving. In Fig. 1, the current in the coil is assumed to be increasing and the field is moving outwards. Thus, the effective "motion" of the core is *inwards.* Applying Fleming's right-hand rule to the core shows that the eddy currents, at this instant, are circulating in the manner indicated. If the current in the coil is now decreased, the field tends to collapse inwards and the eddy currents will circulate in the reverse direction.

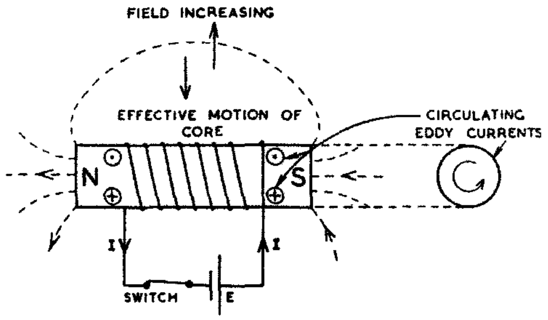

Fig. 1.—EDDY CURRENTS.

4. Eddy currents in the core of a coil have several effects, the two most important being :—

(*a*) The core becomes hot, and the conversion of electrical energy into heat energy constitutes an energy loss termed **eddy current loss.** (This is in addition to any hysteresis loss in the core as discussed in Chapter 1).

(*b*) The eddy currents produce a flux of their own and, since this will be in opposition to the main flux, a reduction in the main flux results.

5. In order to reduce the effects of eddy currents it is usual to "*laminate*" the core. The core is cut up into very thin slices or **laminations** ; each lamination is insulated from the next by a thin film of shellac or other insulator. The path to the flow of eddy currents is thus broken up and the increase in resistance reduces the eddy currents. The thinner the laminations, the smaller is the loss from eddy currents. For the laminations to be effective, they must be in the correct direction relative to the field and to the eddy currents as shown in Fig. 2.

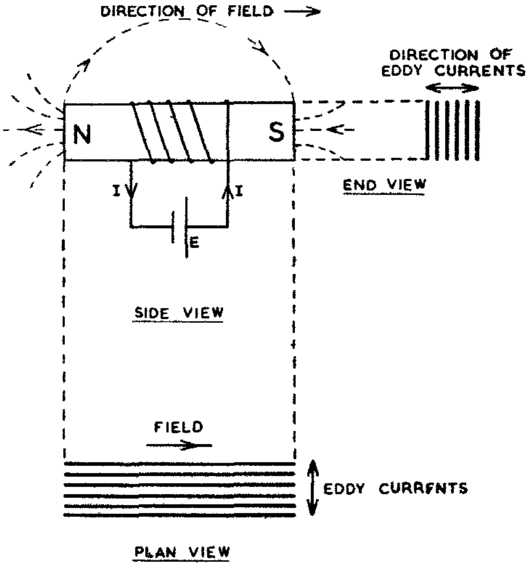

Fig. 2.—LAMINATIONS.

Iron-cored Inductors

6. It was shown in Chapter 2 that the self-inductance of a coil is given by :—

$$L = N^2 \frac{\mu a}{l} \text{ (henrys)}$$

Thus, where a large value of inductance is required, the coil will :—

(*a*) consist of a large number of turns ;

(*b*) be wound over an iron core ;

(*c*) have a high ratio of area to length.

7. Normally the coil will consist of several layers of good quality copper wire, covered by a cotton braiding or by an enamel coating, wound on an iron core which is laminated to reduce eddy current losses. The core will be constructed from a magnetic material, such as Stalloy, which gives a low hysteresis loss.

8. Such coils will give values of inductance up to 100 henrys, and are used to provide effective opposition to slow changes in the current flowing through them—i.e., they are used at low frequencies only, as audio frequency (A.F.) chokes and smoothing chokes. Where a high d.c. value of current is also present, the magnetic circuit will have an *air gap* to prevent magnetic saturation.

9. Laminated iron cores can be used to increase the inductance of a coil only up to frequencies of the order of 20 kc/s. Above this frequency, the high eddy current and hysteresis losses prohibit their use. Above 20 kc/s, iron-dust cores can be used. These are cores in which the iron has been reduced to a very fine dust, mixed with an insulating mica binder, and compressed to form a solid mass. The result is an extremely fine " laminated " core which gives low eddy current losses up to very high frequencies of the order of 60 Mc/s. Above this frequency, the eddy current loss again becomes excessive and air-cored coils are used.

Radio Frequency Chokes

10. Radio frequency (R.F.) chokes are used to provide effective opposition to rapid changes in the current flowing in them—i.e., they are used at high frequencies. Values of inductance from a few micro-henrys to 100 milli-henrys may be required. Air-cored coils are normally used for this. Since the value of inductance depends on the number of turns, the smaller inductors will use a single layer winding for the higher radio frequencies, and the larger inductors multi-layer windings for the lower radio frequencies. Various methods of winding are used.

(*a*) **Single layer.** The coil, of good quality copper wire, is wound on a bakelite or other insulated former, with a considerable spacing between the turns. This gives inductance values up to 100 micro-henrys.

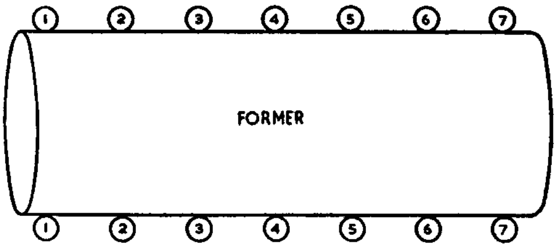

Fig. 3.—SINGLE LAYER COIL

(*b*) **Simple multi-layer.** As shown in Fig. 4, the layers are wound one on top of the other on a bakelite former. Each turn is insulated from the next by a cotton brading or by an enamel coating on the conductor. This method of winding has the disadvantage that turns near each other in adjacent layers (say, 1 and 17) have a high p.d. existing between them ; this may give rise to losses through leakage (capacitive) currents (see Sect. 4). This type of winding gives inductance values up to 100 milli-henrys, but losses are high.

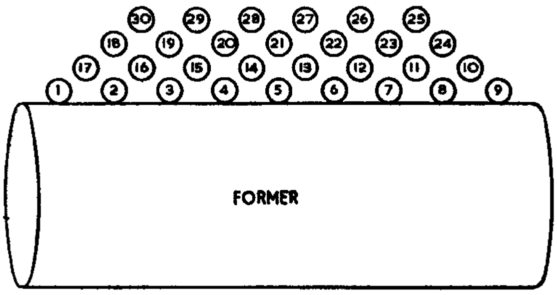

Fig. 4.—SIMPLE MULTI-LAYER COIL.

(*c*) **Bank winding.** This is a multi-layer coil of low loss (Fig. 5). It is wound such that turns near each other in adjacent layers are also near each other in potential. The losses through leakage are then small.

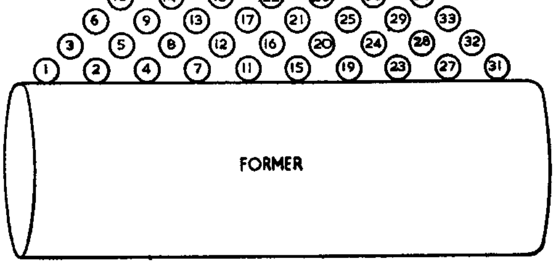

Fig. 5.—BANK-WOUND COIL.

(*d*) **Honeycomb or wave winding.** In this type of multi-layer coil, only a few turns are wound per layer, and the winding is formed in much the same way as a ball of string. The resultant coil is waxed to make it rigid and self-supporting. The p.d. between adjacent turns is small and the losses through leakage are reduced to a minimum to give a low-loss inductor.

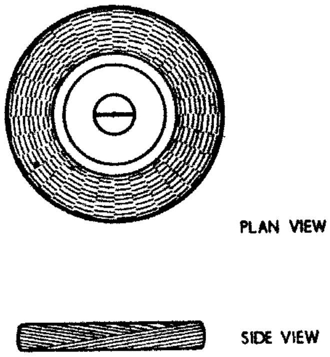

Fig. 6.—HONEYCOMB WINDING.

(*e*) **Pie-winding.** This coil consists of a number of honeycomb windings, all connected in series, and spaced as shown in Fig. 7. The losses through leakage are small, and the result is a low-loss inductor of inductance values up to 100 milli-henrys.

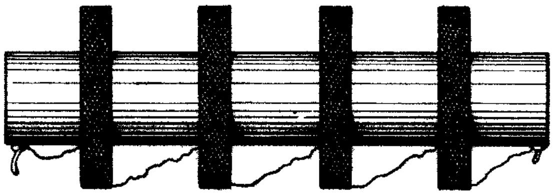

Fig. 7.—PIE-WOUND COIL

Tuned Circuit Coils

11. These are radio frequency coils used in conjunction with a capacitor to form a *tuned circuit* (see Sect. 5). At the higher radio frequencies, small values of inductance are required and a single layer coil will be used ; at the lower frequencies a multi-layer bank-wound coil is normal.

12. It is often necessary to be able to vary the inductance in a tuned circuit. This can be done in two ways :—

(*a*) By " tapping off " the value of inductance required as shown in Fig. 8(*a*).

(*b*) By using a variable iron-dust core inside the coil as in Fig. 8(*b*). When the core is fully " in ", the inductance will be a maximum, and *vice versa*.

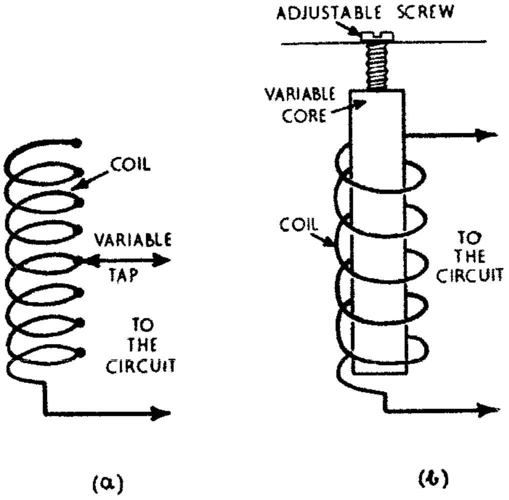

Fig. 8.—VARIATION OF INDUCTANCE.

Skin Effect

13. When a conductor is carrying a direct current, the current is distributed evenly throughout the cross section of the conductor; the magnetic field associated with this current is as shown in Fig. 9. If the current in the conductor is alternating, the changing magnetic field will induce a back e.m.f. in the conductor such as to oppose the change in current. Since all the flux lines build up from, and collapse into, the *centre* of the conductor, the back e.m.f. will be greatest at the centre. The result is that the current tends to flow nearer the surface of the conductor. This is termed **skin effect.**

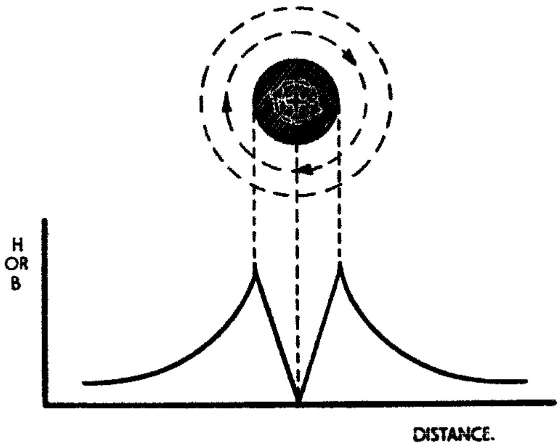

Fig. 9.—CONDUCTOR CARRYING A DIRECT CURRENT.

14. Skin effect becomes more pronounced as the frequency increases because of the increased rate of change of flux. Further, since the effective cross-sectional area of the conductor has been reduced by skin effect, the resistance of the conductor increases with frequency (since $R = \rho \frac{l}{a}$).

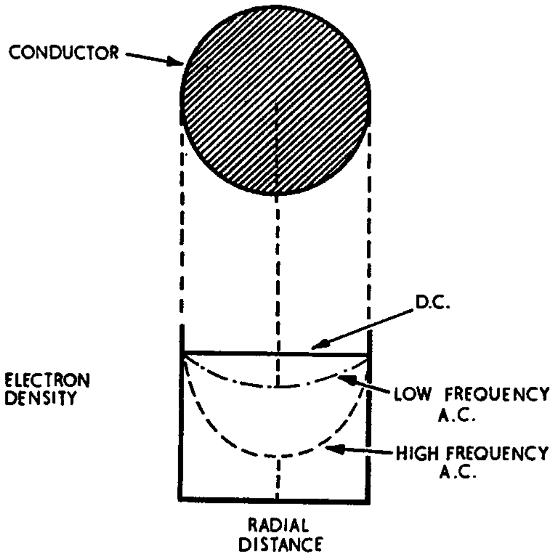

Fig. 10.—SKIN EFFECT.

15. To reduce the high frequency (H.F.) resistance of a conductor, two methods are used :—

(*a*) A special wire, known as "Litzendraht" or "Litz wire" can be employed. This is made up of a number of small diameter, enamelled strands joined in parallel at each end. The strands are thoroughly interwoven so that each strand will, on the average, link with the same number of flux lines as every other strand. Thus, the current divides evenly among the strands ; the effective cross-sectional area is increased and the H.F. resistance decreased. This wire is useful only up to frequencies of 3 Mc/s. Above this, certain leakage effects between the strands give excessive losses.

(*b*) Tubular copper conductors, which have been silver-plated, can be used. The resistivity at the skin is, therefore, reduced and since the electron density is greatest in this region, a reduction in the H.F. resistance results. This method is popular in self-supporting inductors for use at high frequencies.

Proximity Effect

16. When two or more adjacent conductors are carrying current, the current distribution in any one conductor is affected by the magnetic flux produced in adjacent conductors as well as by its own flux. This **proximity effect** is merely an increased case of skin effect and increases the H.F. resistance of inductors used at radio frequencies. Increasing the spacing between turns will reduce proximity effect. Note that maximum current flows in any conductor where it is linked by minimum flux.

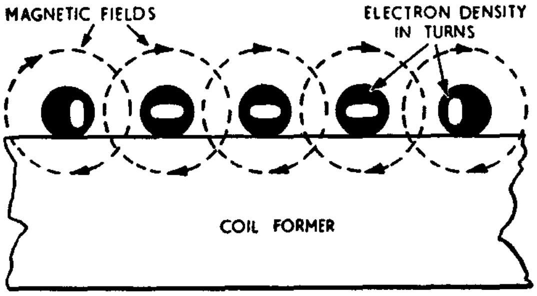

Fig. 11.—PROXIMITY EFFECT.

Non-inductive windings

17. In Sect. 1, Chap. 3 it was noted that wire-wound resistors, being wound in the form of a coil, had inductance as well as resistance. To obtain a pure resistance, the length of insulated wire to be used in the resistor winding is first doubled back on itself and then wound on a former. The current in adjacent turns is flowing in opposite directions so that the magnetic fields around these turns effectively neutralize each other, and the self-inductance becomes negligible.

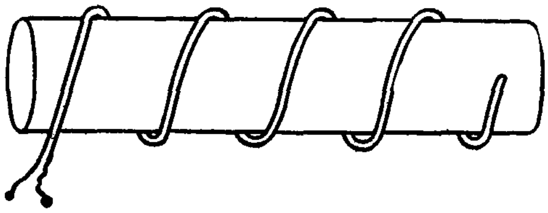

Fig. 12.—A NON-INDUCTIVE WINDING.

Summary

18. Table 1 summarizes the main points on the inductors discussed in this Chapter. Fig. 13 shows a selection of the various types of inductors used in radio.

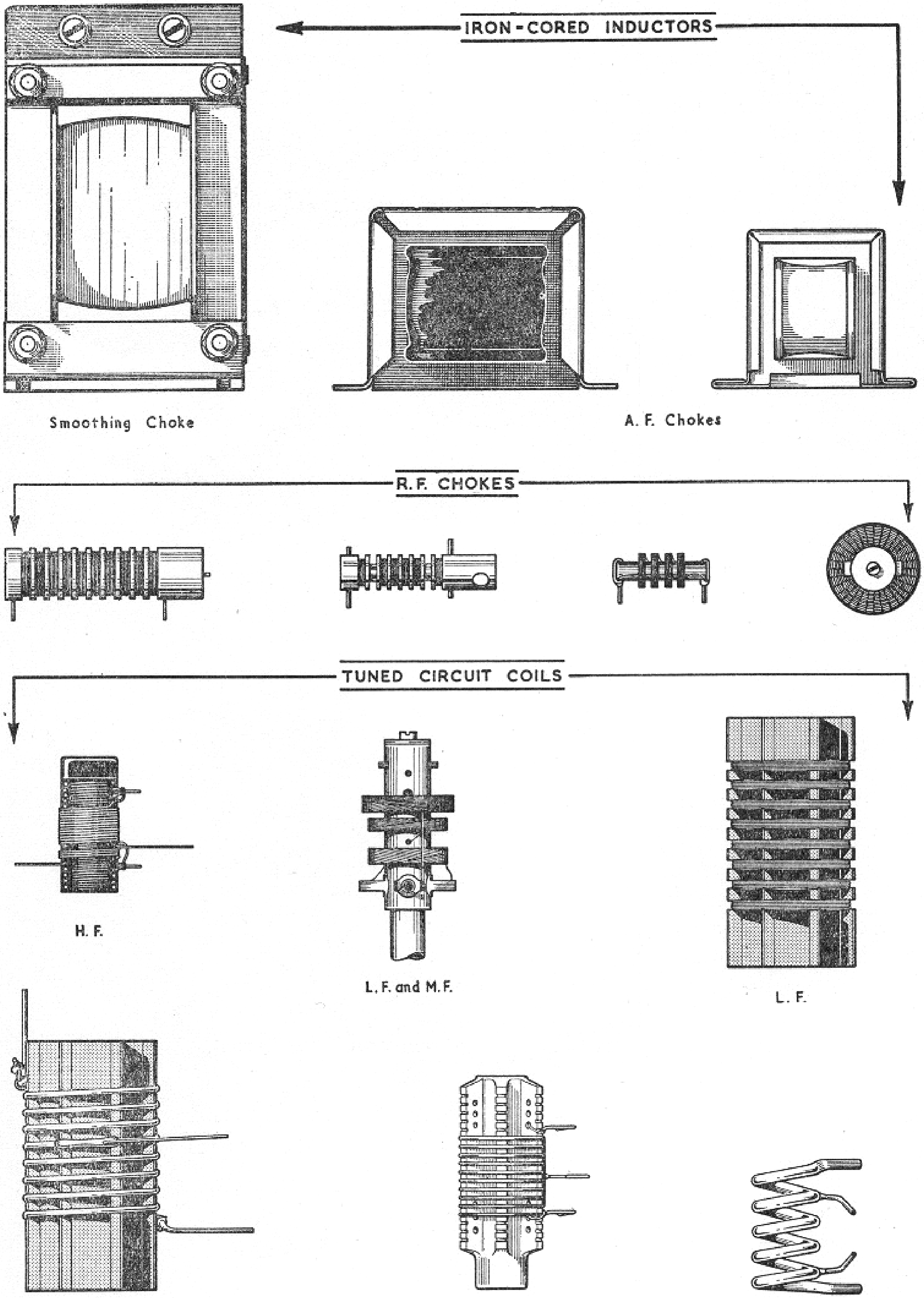
IRON-CORED INDUCTORS
Smoothing Choke
A. F. Chokes
R. F. CHOKES
TUNED CIRCUIT COILS
H. F.
L. F. and M. F.
L. F.
H. F.
H. F.
V. H. F.

Fig. 13.—TYPICAL INDUCTORS USED IN RADIO.

Type	Frequency	Inductance	Winding	Core
Smoothing choke	Power frequencies, e.g., 50 c/s	1H to 100H	Multi-layer	Laminated iron or stalloy with air gap
A.F. choke	20 c/s to 20 kc/s	0·1H to 100H	Multi-layer	Laminated iron or stalloy with air gap
R.F. choke	L.F. and M.F. H.F. V.H.F.	100μH to 100mH 50μH to 100μH 0·25μH to 50μH	Multi-layer Single layer Single layer	Air or iron-dust core Air Air
Tuned circuit coils	← As for R.F. choke →			Normally iron-dust or other core, variable for tuning purposes

TABLE 1—INDUCTORS USED IN RADIO

SECTION 3

D.C. MOTORS AND GENERATORS

Chapter 1 ... ... **The D.C. Generator**
Chapter 2 ... ... **The D.C. Motor**

SECTION 3

CHAPTER 1

THE D.C. GENERATOR

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PART 1, SECTION 3, CHAPTER 1

THE D.C. GENERATOR

Introduction

1. While a circuit is moving in a magnetic field in such a way that the magnetic flux linkage is changing, there is in the circuit, an induced e.m.f. which at each instant is proportional to the *rate of change* of flux linkage (see Sect. 2, Chap. 2). If this e.m.f. is applied to a closed circuit an electric current will be established; mechanical energy has thus been converted to electrical energy. Any machine which does this is known as a generator. The *d.c. generator* is a machine which when driven by mechanical power causes a system of conductors rotating in a magnetic field to generate an e.m.f. and maintain a d.c. output voltage.

The Simple Generator

2. The simplest form of generator consists of a single loop of wire able to rotate freely in the space between the poles of a permanent magnet. Connection is made to the external circuit (or "load") by *brushes* pressing on two *slip rings* which are connected to the ends of the loop (Fig. 1).

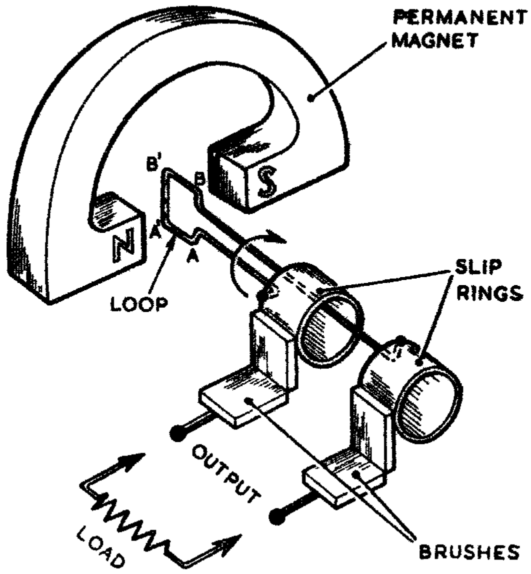


Fig. 1. THE SIMPLE GENERATOR

3. When the loop is caused to rotate, the magnetic flux linkage is changing and e.m.f.s will be induced in each of the straight sides A—A^1 and B—B^1. The *direction* of the induced e.m.f.s given by Fleming's Right Hand Rule, is such that the two e.m.f.s in series are additive and combine to establish a current when the load is connected.

4. The *magnitude* of the induced e.m.f. is proportional to the *rate of change of flux linkage;* thus, assuming the speed of rotation to be constant, the induced e.m.f. will at any instant depend on the position of the loop in the magnetic field. This is illustrated in Fig. 2, which represents the view from the commutator end of the loop.

5.(*a*) In position (*a*) Fig. 2, the conductors A and B are moving parallel to the lines of magnetic flux and are linking maximum flux. However, in a very small period of time dt about the instant shown in (*a*) the change of flux linkage is zero. Thus, the *rate of change* of flux linkage $\frac{d\Phi}{dt}$ is *zero* and since $E = -\frac{d\Phi}{dt}$ volts, the e.m.f. induced in the loop at this instant is zero.

(*b*) Position (*b*) shows the conductors cutting the flux at right angles. The *rate* at which the flux linkage is *changing* is now a *maximum* and the e.m.f. induced in the loop is a maximum, the direction being given by Fleming's Right Hand Rule. At this position, the e.m.f. is arbitrarily assumed to be maximum in a *positive* direction.

(*c*) Position (*c*) represents a position where the rate of change of flux linkage is again zero and no e.m.f. is induced in the loop.

(*d*) Position (*d*) is similar to position (*b*) except that the side of the loop which was previously moving *downwards* (side A) is now moving *upwards* and *vice versa.* The rate of change of flux linkage is again a maximum and Fleming's Right Hand Rule will confirm that in relation to (*b*), the e.m.f. is now a maximum in a *negative* direction.

(*e*) Position (*e*) is identical with position (*a*).

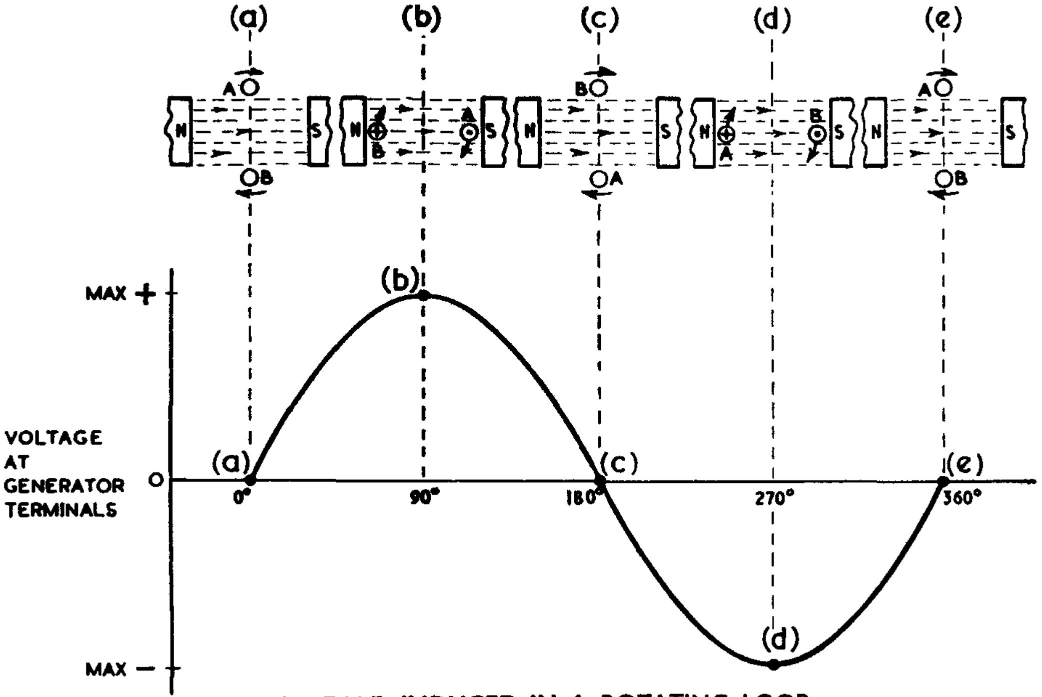


Fig. 2. E.M.F. INDUCED IN A ROTATING LOOP

6. At other positions of the loop the rate of change of flux linkage is intermediate between zero and maximum and so, therefore, is the e.m.f. Thus, during one complete revolution of the loop the voltage at the terminals of the generator will vary in the manner shown in the graph of Fig. 2. This shows *one cycle* of *alternating voltage.*

7. The **frequency** of the alternating voltage at the generator terminals depends on the speed of rotation and on the number of *pairs* of poles in the field magnet system (see Para. 14). Thus :—

$$\text{Frequency } f = \frac{pN}{60} \text{ (c/s)} \quad \ldots \quad \ldots \quad \ldots (1)$$

where p = Number of pairs of poles.
N = Revolutions per minute.

Hence, the frequency of an alternating voltage produced by a 4-pole a.c. generator running at 1500 r.p.m. is:—

$$f = \frac{pN}{60} = \frac{2 \times 1500}{60} = 50 \text{ c/s.}$$

Production of Direct Current

8. Where direct current in the external circuit is required a form of automatic reversing switch (known as a *commutator*) is substituted for the slip rings. The commutator automatically reverses the connections between the loop and the external circuit at the instant that the e.m.f. induced in the loop is zero and reversing, thus maintaining the direction of the current in the external circuit. A simple form of commutator for a single-loop generator consists of the two halves of a split ring, separated from each other by a layer of insulation. Each end of the loop is connected to a *segment* of the commutator, and the external circuit is connected to the loop by brushes bearing on opposite sides of the commutator, as shown in Fig. 3.

9. As the loop rotates, the e.m.f. induced is alternating as described in Para. 6. However, since the commutator rotates with the loop, the brushes bear on opposite segments of the commutator during each half cycle (compare Figs. 3 (*b*) and 3(*d*)). The left-hand brush is always in contact with that segment which is positive, and the right-hand brush with that segment which is negative—the changeover occurring at the instants when the e.m.f. induced in the loop is zero (Figs. 3 (*a*), (*c*), and (*e*)). A uni-directional current is, therefore, established in the external circuit. The variation in brush voltage and the external circuit current during one complete revolution of the loop is illustrated in Fig. 3 (*f*).

10. A more constant brush voltage and a smoother flow of current can be obtained by placing additional loops symmetrically round the axis of rotation. This necessitates additional segments on the commutator, the loops being so arranged that each loop is connected between adjacent segments, the end of one loop being connected to the same segment as the beginning of the next loop as shown schematically in Fig. 4(*a*). For instance, loop A is connected between

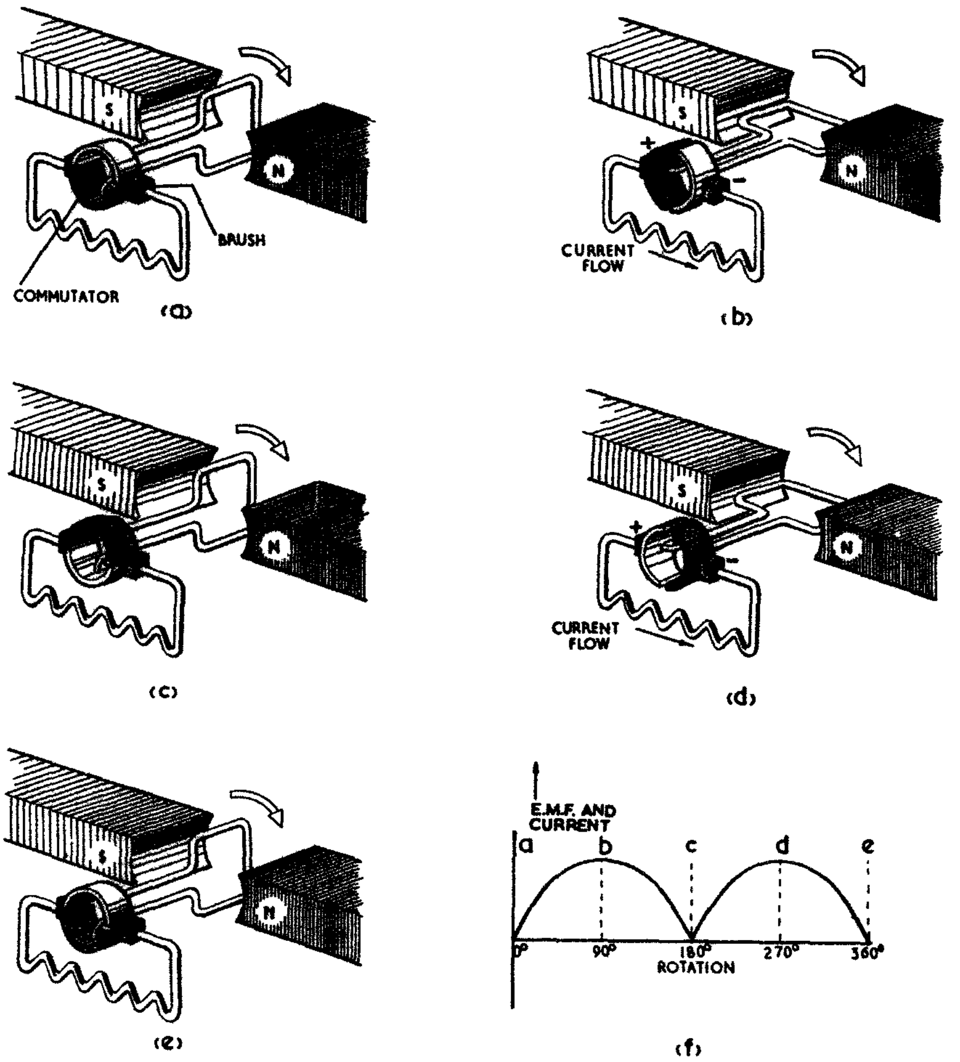


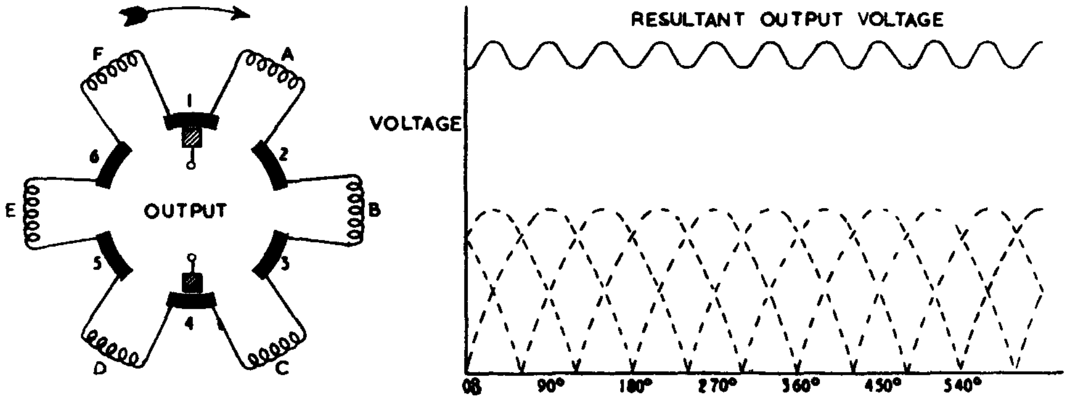
Fig. 3. PRODUCTION OF DIRECT CURRENT

segments 1 and 2, loop B between segments 2 and 3, and so on. With this arrangement, the e.m.f. induced in each loop will reach its maximum value when the e.m.f. in the preceding loop is already decreasing, and that in the succeeding loop still increasing. Thus at the instant in Fig. 4(*a*), if the e.m.f. induced in loop E is a maximum, the e.m.f. in loop F is decreasing and that in loop D increasing. The voltage at the brushes comprises the sum of the e.m.f.s induced in the loops connected in series between the brushes. Thus, in Fig. 4(*a*) loops A, B, and C are in series between the brushes on the right, and loops D, E, and F on the left, the two branches being *in parallel* with each other. The graph showing the resultant voltage between the brushes is shown in Fig. 4(*b*). Only three loops have had to be considered as the arrangement is symmetrical and loops A, B, C, in parallel with loops D, E, F, give the same voltage at the instant shown. As the number of loops is increased the ripple in the brush voltage becomes smaller and the magnitude of the output voltage increases.

Magnitude of Induced E.m.f.

11. From Faraday's and Lenz's laws, the e.m.f. induced in a circuit moving through a magnetic field is $E = -N \frac{d\Phi}{dt}$ volts. The e.m.f. of a generator, therefore, depends on:—

(*a*) The number of conductors connected in series between the brushes.


F
A
1
6
2
E
OUTPUT
B
5
3
4
D
C
RESULTANT OUTPUT VOLTAGE
VOLTAGE
0°
90°
180°
270°
360°
450°
540°

(a) (b)

Fig. 4. RESULTANT E.M.F. PRODUCED BY ADDING MORE LOOPS

(*b*) The speed of rotation.

(*c*) The magnetic flux density.

12. In practice, the number of conductors is fixed and the speed is nominally constant, so that control of the e.m.f. must be obtained through variation of the magnetic flux density. This cannot be done with permanent magnets but by substituting electromagnets the e.m.f. can be controlled by varying the current in the windings (see Sect. 2, Chap. 1, Para. 31).

Basic Construction of a D.C. Generator

13. The arrangement of the main components of a d.c. machine is shown in Fig. 5. Such a machine consists of two main assemblies—the *stator* (or fixed portion) and the *rotor* (or armature assembly). The stator carries the field magnet system and the brush gear. The rotor carries the coils in which the e.m.f. is induced, the commutator, and in many cases a series of fan blades to assist in cooling.

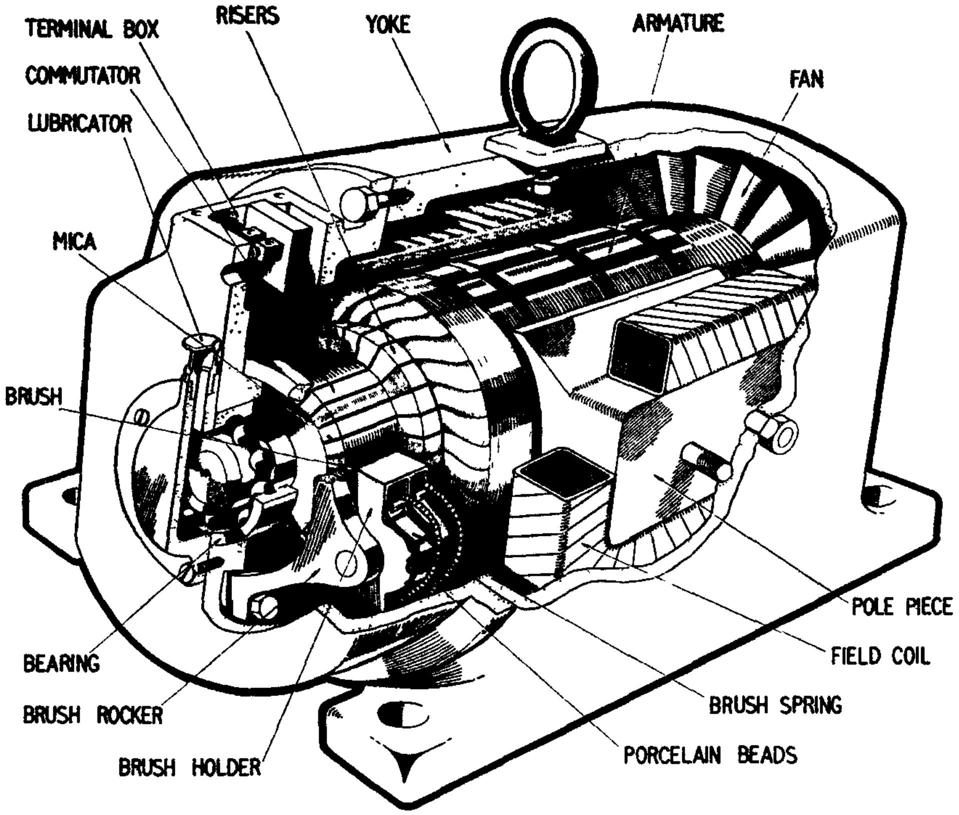

TERMINAL BOX
RISERS
YOKE
ARMATURE
FAN
COMMUTATOR
LUBRICATOR
MICA
BRUSH
POLE PIECE
FIELD COIL
BEARING
BRUSH SPRING
BRUSH ROCKER
PORCELAIN BEADS
BRUSH HOLDER

Fig. 5. CONSTRUCTION OF A TYPICAL D.C. MACHINE

14. **The Field Magnet System.** Except for very small machines the magnetic field is provided by electromagnets in such a way that the armature conductors pass under North and South Poles alternately. Several pairs of poles can be used, depending on the flux required. The number of poles is always even, with 2 poles for very small machines and as many as 20 for large machines. The field magnet system for a 6-pole d.c. machine is shown in Fig. 6. To obtain a strong magnetic field for the minimum expenditure of electrical energy, ferro-magnetic materials of high permeability are used, and the magnetic circuit is so arranged that it has the least possible reluctance.

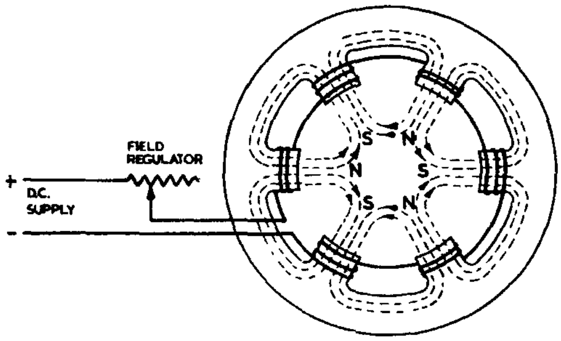


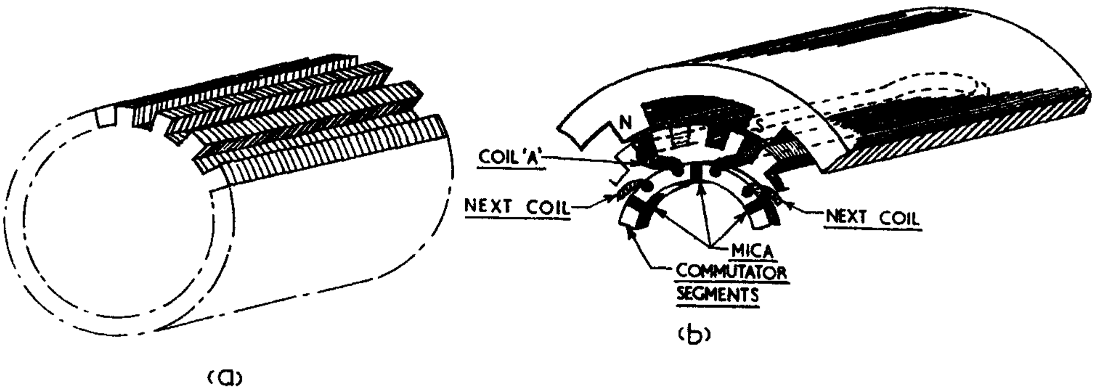
Fig. 6. THE MAGNETIC CIRCUIT OF A 6-POLE D.C. MACHINE

15. **The Armature Assembly.** This consists of the main shaft, the armature core and windings, the commutator, the fan blading, and the bearings. The armature is the practical application of the rotating loop and it is driven by a prime mover—i.e., a heat engine (steam, petrol, or diesel), an electric motor, or a water turbine.

16. (*a*) *Armature.* The armature consists of a soft iron drum, on the surface of which are fixed the conductors to be rotated between the pole pieces of the field magnets. The core is laminated to reduce eddy current losses and the material used has a narrow hysteresis loop. The individual coils which make up the armature winding are identical, and after being taped and impregnated, they are fitted into slots cut in the armature core parallel to the axis of rotation (Fig. 7). The flux from the poles threads through the drum intersecting the sides of each coil as it moves under a pole piece. The coils are not actually wound into adjacent slots but, as shown in Fig. 7, are spread so as to embrace, the angular distance between adjacent poles; that is, if there are four poles spaced round at 90° intervals, the two sides of a given coil accupy slots that are 90° apart on the drum, and so on. The two ends of each coil are connected to separate commutator segments, the finish of one coil being connected to the same segment as the beginning of another coil. Thus, the complete winding forms a closed circuit.

(*b*) *Commutator.* This is mounted towards one end of the shaft and is built up of copper segments insulated from each other by thin sheets of mica. The ends of the coils are soldered into lugs or "risers" on the end of the commutator segments.

17. **The Brushgear.** The brushes are made almost invariably of some form of carbon; they are self-lubricating and cause little commutator wear. In addition, their comparatively high resistance minimizes reactive sparking (see Para. 20). They are carried



(a)

(b)

Fig. 7. ARMATURE AND ARMATURE WINDINGS

in small open-ended boxes termed *brush-holders*, pressure being applied to the top of the brush by an adjustable spring in order to maintain the rubbing contact between the brush and the commutator. Connection to the external circuit from the brushes is normally by "pig-tails" of flexible copper braid. The brush-holders are bolted to *brush-rockers* which can be adjusted to move through a few degrees round the periphery of the commutator to alter the position of the brushes in order to give the best possible commutation (see Para. 21).

Commutation

18. Faulty collection or incorrect commutation at the commutator and brushgear of a d.c. machine produce similar results—the formation of a destructive spark or arc between the trailing edges of the brushes and the commutator surface. Sparking due to faulty collection is usually caused by bad servicing or lack of maintenance. Sparking due to incorrect commutation (usually termed *reactive sparking*) arises from unsuitable positioning of the brushes, or faults in the magnetic circuit.

19. **Reactive Sparking.** The commutator segments connected to the ends of a coil are normally short-circuited by the brush when the sides of the coil are in a magnetically neutral zone and no e.m.f. is being induced in the coil. Consider the ideal case for a 2-pole machine. Immediately before short-circuiting, the coil B would be carrying *half* the total armature current; during the short-circuit period the current in the coil would be zero; and immediately after the short-circuit period the coil would be carrying half the total armature current in the *reverse* direction (Fig. 8).

20. In practice the current cannot change instantly because of the self-induced back e.m.f. (or reactance e.m.f.) in the coil. Thus the current in coil B will *not* have reached its final value in the reverse direction immediately after the short-circuit period. The result is that the *difference* in current between coils B and C will "jump" between the commutator and the brush in the form of a spark.

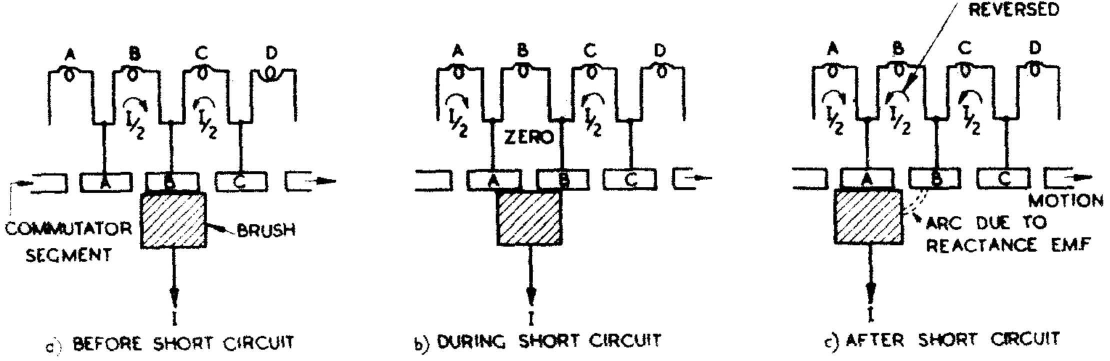
21. **Reduction of Reactive Sparking.**

(*a*) *Resistance Commutation.* By using high resistance carbon brushes the inductive time constant $\frac{L}{R}$ seconds is *reduced*; this ensures a much more rapid build-up of current in the reverse direction in coil B. In addition, the high resistance contact causes more current to go through coil B and assists in attaining the full reversed value of current at the end of the short-circuit period. Resistance commutation suffers from two main dis-advantages:—

(i) Power losses (I^2R watts) are high.

(ii) Slight sparking may still occur at the brushes since the current is still changing although at a greatly *increased* rate.

(*b*) *Brush Position Commutation.* If the brushes of a generator are slightly *advanced* (by means of the brush-rocker) in the direction of rotation, the short-circuiting of the coil will be delayed until it has come under the influence of the next main pole. In these circumstances, an e.m.f. will be induced in the coil considered and this e.m.f. will be in such a direction as to assist the collapse and the build-up of the reversed current in the shorted coil. In this way, reactive sparking can be prevented since the brush position can be so adjusted that the reversed current is a



a) BEFORE SHORT CIRCUIT b) DURING SHORT CIRCUIT c) AFTER SHORT CIRCUIT

Fig. 8. CAUSE OF REACTANCE SPARKING

full value before the end of the short-circuit period. The disadvantage of brush position commutation is that the position of the brushes requires to be adjusted for every variation in "load".

(*c*) *Compoles*. A more satisfactory method of forcibly reversing the current in the shorted coil (so that the current has reached its correct value by the end of the short-circuit period) is by the use of compoles (also known as commutating poles, or interpoles). These are auxiliary poles located midway between the main poles. Their windings are connected in series with the *armature* winding and in generators are so arranged that the compole has the same polarity as the next main pole *ahead* in the direction of rotation (Fig. 9). When the armature coil is in a magnetically neutral zone as far as the main poles are concerned, commutation takes place; the compoles will then induce an e.m.f. in the coil considered, this e.m.f. being in such a direction as to assist in the collapse and the build-up of the reversed current in the shorted coil. Since the compole windings carry the *armature* (load) current, any variation in load is automatically accounted for.

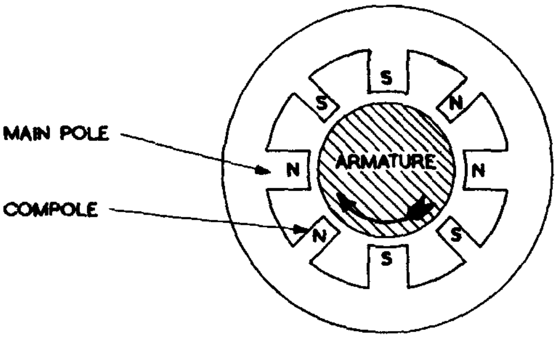


Fig. 9. COMPOLES IN A GENERATOR

Armature Reaction

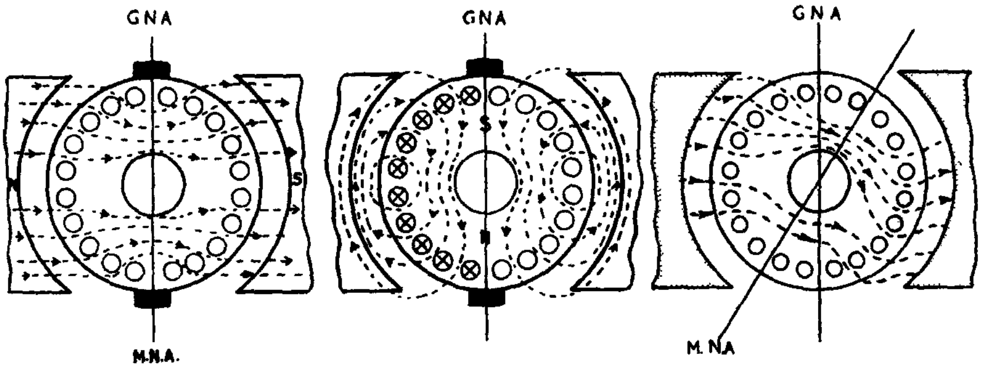
22. **Cause of Armature Reaction.** When a generator is running on load, a magnetic field is created by the current flowing in the *armature* winding. This magnetic field is superimposed on the main field (produced by the current in the *field* winding) to give a resultant field which is distorted and weakened to an extent dependent on the load. This effect is termed *armature reaction* (Fig. 10).

23. A line drawn vertically at a point midway between the poles is termed the *Geometrical Neutral Axis* (G.N.A.). A line joining the two points at which no e.m.f. is induced in a coil is known as the *Magnetic Neutral Axis* (M.N.A.). With the generator on load, the M.N.A. is seen to have *advanced* in the direction of rotation so that it has an angle of lead on the G.N.A.

24. **Effects of Armature Reaction.** Armature reaction, if uncorrected, produces two bad effects:—

(*a*) It causes the M.N.A. to move when the load is varied, thus upsetting commutation.

(*b*) It weakens the main field, causing a reduction in the generated e.m.f.



MAIN FIELD FLUX
Fig. a

ARMATURE FLUX
Fig. b

RESULTANT FLUX
Fig. c

Fig. 10. ARMATURE REACTION IN A GENERATOR

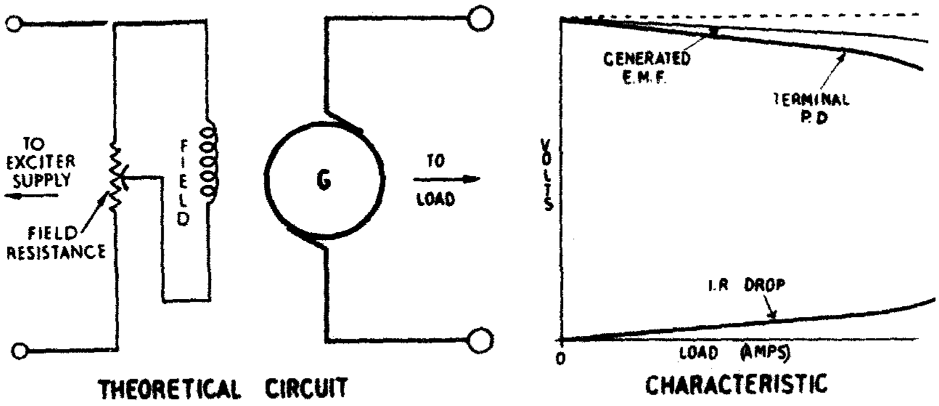


Fig. 11. SEPARATELY-EXCITED GENERATOR

25. **Correction of Armature Reaction Effects.**

(*a*) By moving the brushes through the appropriate angle of *lead* so that their axis coincides with the M.N.A. it is possible to restore good commutation. However, as was shown in Para. 21(*b*), the angle of lead then requires to be adjusted for every variation in load.

(*b*) The effects of armature reaction can be greatly minimized by the use of compoles (although these are primarily intended to provide the necessary commutating e.m.f. as shown in Para. 21 (*c*)). The flux due to the compoles is opposite in direction to the armature flux. By placing the requisite number of turns on each compole, these two fields can be made equal *at all loads*, since both are produced by the armature (load) current. By this means the M.N.A. is made to coincide with the G.N.A. at all times and it is no longer necessary to give the brushes an angle of lead.

Classification of Generators

26. D.C. generators are usually classified according to the method by which the magnetic circuit of the machine is energised. The recognised classes are:—

(*a*) Permanent magnet generators.

(*b*) Separately-excited generators.

(*c*) Self-excited generators.

Permanent Magnet Generators

27. Permanent magnets give the simplest method of producing the magnetic flux in a generator, but their use is confined to relatively small machines because of the difficulty experienced in producing sufficient flux for larger machines.

28. The relationship between the current in the external circuit connected to the terminals of a generator (termed the load current or load), and the p.d. at the generator terminals, is known as the external characteristic of the machine. For a permanent magnet generator, the terminal p.d. falls slightly with increasing load and the machine is said to have a falling characteristic. This fall in terminal p.d. is due to two causes:—

(*a*) Weakening of the main flux by armature reaction.

(*b*) A voltage (IR) drop in the armature windings and in the brushes.

Separately-excited Generators

29. The magnetic field for this class of generator is obtained from electromagnets which are excited by current obtained from an *external* d.c. source. The field and armature windings are not connected in any way. The field winding is usually of fairly high resistance and provision is made for regulating the exciting current, *e.g.*, by a variable resistor (Fig. 11). Complete control of the terminal p.d. over a wide range can then be obtained (see Para. 12).

30. The generated e.m.f. falls slightly with load because of armature reaction, if the latter is not completely neutralized by the use of compoles. The terminal p.d. falls with load to an even greater extent because of the increasing IR drop in the armature windings (Fig. 11).

Self-excited Generators

31. This class of generator uses electromagnets to provide the magnetic flux. The

e.m.f. required to send current through the field coils of the electromagnets is generated *by the machine itself.* Self-excited generators are further classified as follows:—

(*a*) *Shunt winding*, where the field winding is connected directly across the armature (Fig. 12). To avoid unnecessary expenditure of electrical energy, the field current is kept small, the necessary ampere-turns for the required flux being obtained by using many turns of fine wire. The resistance of a shunt field winding is consequently high. Provision is made for regulating the exciting current by a variable resistor in series with the field winding. Complete control of the terminal p.d. over a wide range is then obtained.

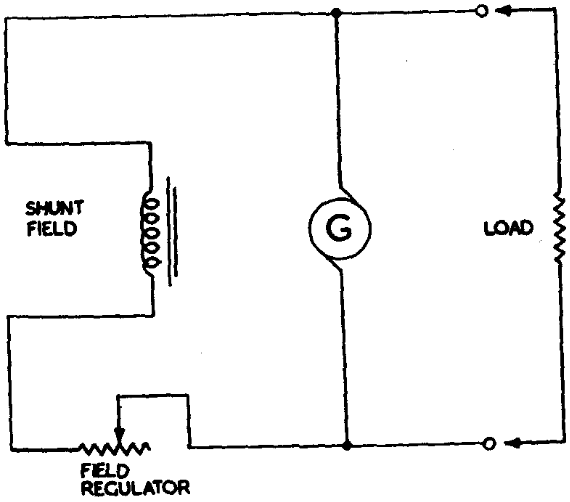


Fig. 12. SHUNT-WOUND GENERATOR

(*b*) *Series winding*, where the field winding is connected in series with the armature and the load (Fig. 13). The field coils are usually made of a relatively small number of turns of thick wire. The terminal p.d. of a series generator may be controlled by a *diverter*, i.e., a variable resistor connected *in parallel* with the field winding so as to adjust the current in the field coils. A *decrease* in resistance *reduces* the current in the field winding thereby reducing the magnetic flux to cause a fall in the generated e.m.f.

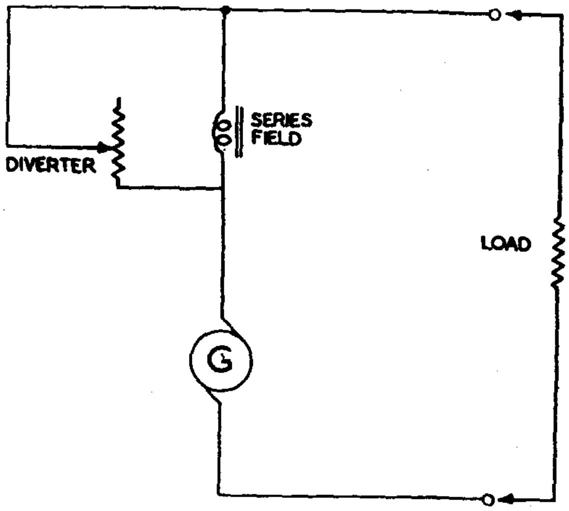
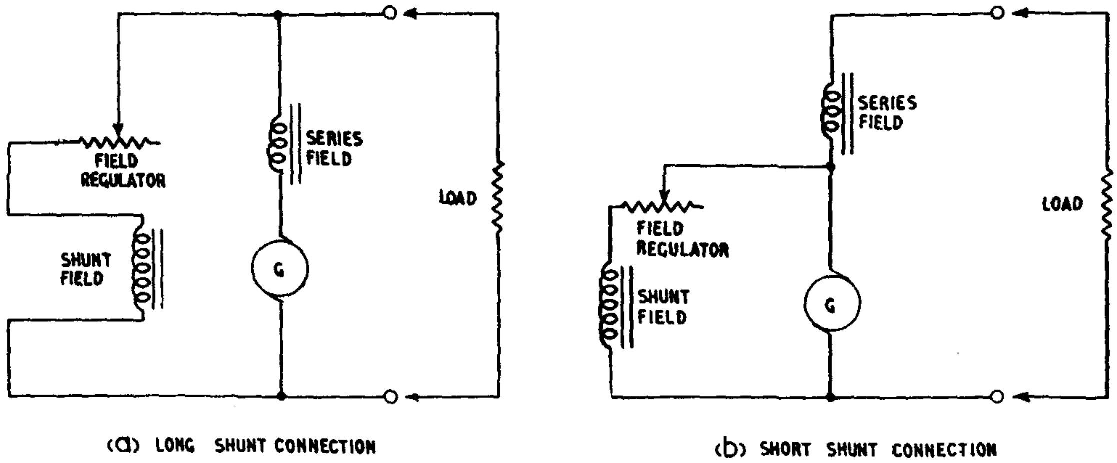


Fig. 13. SERIES-WOUND GENERATOR

(*c*) *Compound winding*, where a combination of shunt and series field windings is used. Each pole piece carries one shunt and one series coil. If the shunt



(a) LONG SHUNT CONNECTION

(b) SHORT SHUNT CONNECTION

Fig. 14. COMPOUND-WOUND GENERATORS

winding is connected across the armature and series windings a *long shunt connection* results (Fig. 14(*a*)). In a *short shunt connection*, the shunt winding is across the armature only (Fig. 14 (*b*)). Control of the terminal p.d. is usually obtained by a variable resistance in series with the shunt winding.

32. **Characteristic of shunt-wound generators.** The initial build-up of field current depends on the retentivity (or residual magnetism) of the magnetic circuit. When the armature is caused to rotate, the conductors cut the weak magnetic flux provided by the residual magnetism. A small e.m.f. is induced in the armature and this is applied across the shunt winding to establish a current in the latter; the magnetic flux increases. In this way, a progressive increase in the induced e.m.f. and in the field current occurs, until the terminal p.d. reaches a steady no-load maximum equal to the generated e.m.f.

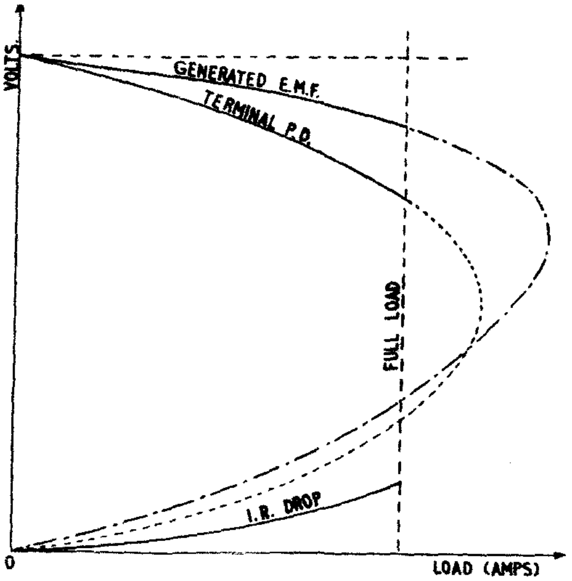


Fig. 15. CHARACTERISTICS OF SHUNT GENERATOR

33. The external characteristic of a shunt generator is similar to that of the separately-excited machine (Para. 30). However, the fall in terminal p.d. with increasing load will reduce the field current, thus weakening the main flux and producing a further fall in the terminal p.d. The total decrease in the terminal p.d. is, therefore, greater than if the machine were separately-excited. If the load is increased beyond the "full load" condition, the terminal p.d. will fall at an increasing rate until the generator shuts down and both the terminal p.d. and the load current fall to zero (Fig. 15). In effect, the field winding is shunted by the low resistance of the external circuit and the main flux collapses. For this reason, a shunt-wound generator should be allowed to build up to its full voltage before connecting the load.

34. **Characteristic of series-wound generators.** In these machines the field current is proportional to the load current. No current flows through the field winding until an external load is connected, and on no-load the only e.m.f. generated is the small amount due to the residual magnetism of the magnetic circuit. When a load is connected, current flows through the armature and field windings in series. As the resistance of the external circuit is decreased the increased load current in the field windings gives an increase of flux so that the generated e.m.f. and the terminal p.d. rise. This is progressive as shown in Fig. 16. Maximum terminal p.d. is attained when the magnetic circuit reaches saturation. Any further increase in load current will not then result in a rise of generated e.m.f. but it will, on the other hand, cause an increased voltage drop in the armature and field windings. It is this IR drop which gives the difference between the generated e.m.f. and the terminal p.d. in Fig. 16.

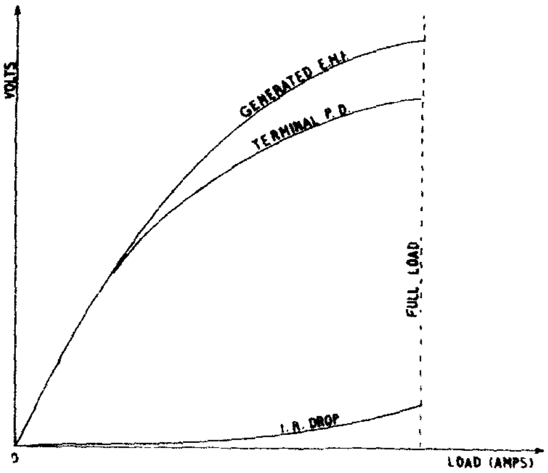


Fig. 16. CHARACTERISTICS OF SERIES GENERATOR

35. **Characteristics of compound-wound generators.** The external characteristics of these generators depend on the relative direction of the shunt and the series windings, and on the number of turns on the series winding.

(*a*) *Cumulative compound winding.* When the series winding is wound to produce the same polarity at the pole pieces as that provided by the shunt winding, the windings are cumulative in their effect. An increase of load current in such a machine will cause an increase in flux due to the action of the series field. This increase of flux may compensate for the weakening of flux which occurs when a load is imposed on a shunt-wound generator (see Para. 33) thus maintaining the terminal p.d. When the effect of the series winding increases the flux to such an extent that the terminal p.d. at full load is the same as that at no load, the machine is said to be *level compounded.* By increasing the number of series turns, the flux due to the load current will increase; the terminal p.d. at full load will then be above the no-load value and the machine is *over-compounded* (Fig. 17).

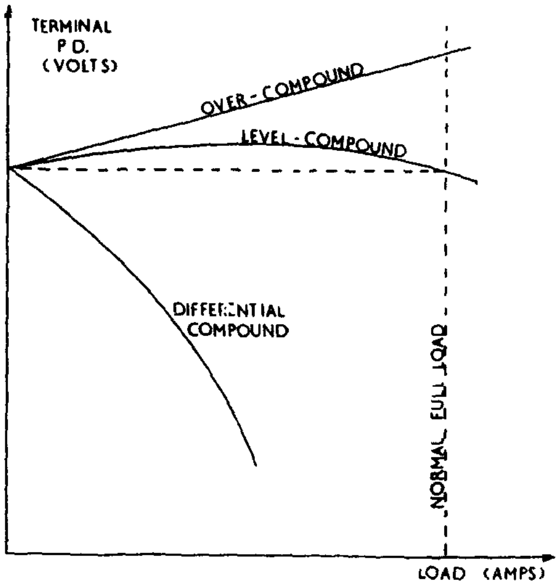


Fig. 17. CHARACTERISTICS OF COMPOUND GENERATOR

(*b*) *Differential compound winding.* These are machines in which the series and shunt fields oppose each other. With increased load, the increase in series field strength, opposing the shunt field, causes a fall in the total flux, resulting in a fall in the terminal p.d.

Generator Losses and Efficiencies

36. **Losses.** The various power losses which occur may be divided into:—

(*a*) *Copper Losses.* These are sometimes referred to as electrical losses and are caused by current passing through the resistance of the various conductors so that a power loss (I^2R watts) results. In general, they increase with the load and to prevent excessive copper losses the generator should not be overloaded.

(*b*) *Iron Losses.* These are sometimes referred to as core losses and are substantially constant at all loads. They include:—

(i) Hysteresis loss in the armature.

(ii) Eddy current loss in the armature.

(*c*) *Friction Losses.* These are purely mechanical losses and include:—

(i) Friction at the bearings and at the commutator.

(ii) Wind resistance of the rotating armature.

37. **Efficiencies.** The energy changes and losses which occur in the transformation of mechanical energy into electrical energy in a generator are shown diagrammatically in Fig. 18. From this it is seen that the electrical power developed in the armature is equal to the applied mechanical power less the iron and friction losses; and the electrical power output is equal to the power developed in the armature less the copper losses.

38. From Fig. 18 it is possible to give three separate efficiencies:—

(*a*) Mechanical efficiency =
$$\frac{\text{Power developed in armature}}{\text{Mechanical power supplied}} \times 100 \text{ (per cent.)}$$

(*b*) Electrical efficiency =
$$\frac{\text{Electrical power output}}{\text{Power developed in armature}} \times 100 \text{ (per cent.)}$$

(*c*) Commercial efficiency =
$$\frac{\text{Electrical power output}}{\text{Mechanical power input}} \times 100 \text{ (per cent.)}$$

Thus, Commercial Efficiency = Mechanical × Electrical Efficiencies.

Note. The Commercial Efficiency is the one more commonly used since it gives the *overall* efficiency of the generator.

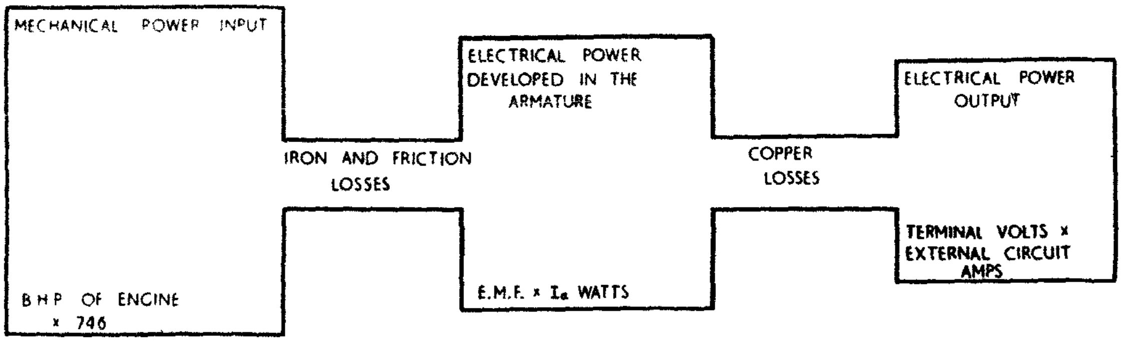

MECHANICAL POWER INPUT
B H P OF ENGINE × 746
IRON AND FRICTION LOSSES
ELECTRICAL POWER DEVELOPED IN THE ARMATURE
E.M.F. × I_a WATTS
COPPER LOSSES
ELECTRICAL POWER OUTPUT
TERMINAL VOLTS × EXTERNAL CIRCUIT AMPS

Fig. 18. STAGES IN THE TRANSFORMATION OF ENERGY IN A GENERATOR

39. The graph of Fig. 19 shows how the commercial efficiency and the losses vary with the load current in a compound generator. From this graph it is seen that, whereas the iron and friction losses are practically constant with load, the copper losses increase with the load current. Because of this the efficiency falls off when overloaded as shown.

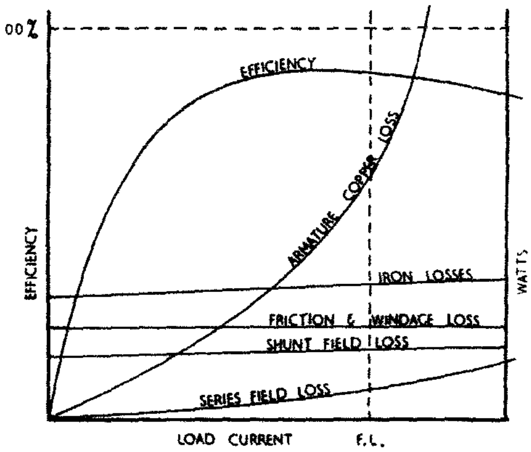


Fig. 19. VARIATION OF LOSSES AND EFFICIENCY WITH LOAD (COMPOUND GENERATOR)

SECTION 3

CHAPTER 2

THE D.C. MOTOR

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PART 1, SECTION 3, CHAPTER 2

THE D.C. MOTOR

The Force Between Two Current-carrying Conductors

1. Consider two parallel conductors each carrying current in the same direction. The direction of the magnetic field round each conductor, given by the Corkscrew Rule, is such that in the space between the conductors, the two magnetic fields are in opposition and tend to neutralize each other. In the space outside the conductors, the two fields assist each other. These two fields cannot exist separately, and the resultant combined field is as shown in Fig. 1. Lines of magnetic flux behave as stretched elastic cords, and the effect is to try to move the two conductors *towards* each other.

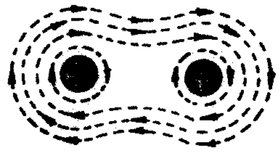

Fig. 1. FORCE OF ATTRACTION BETWEEN TWO PARALLEL CONDUCTORS

2. If the current in one of the conductors is reversed, the combined field will be as shown in Fig. 2. The lateral pressure between the lines of flux exerts a force on the conductors tending to urge them *away* from each other.

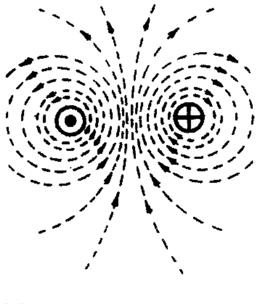

Fig. 2. FORCE OF REPULSION BETWEEN TWO PARALLEL CONDUCTORS

The Force on a Current-carrying Conductor in a Magnetic Field

3. Fig. (3*a*) shows the magnetic field between the poles of a magnetic circuit; Fig. 3(*b*), the magnetic field round a conductor which is carrying current. If the conductor is placed in the space between the two poles of the magnetic circuit, the resultant combined field will be as shown in Fig. 3(*c*). It is seen that the field is strengthened in the space above the conductor (since the lines of flux are in the same direction) and weakened below it. This distorted field acts like stretched elastic cords bent out of the straight, and the flux lines try to return to the shortest paths between the poles, thereby exerting a force on the conductor urging it out of the way.

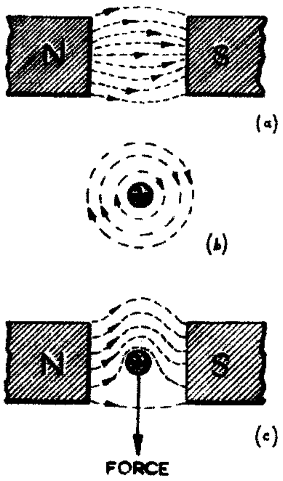

Fig. 3. FORCE ON A CURRENT-CARRYING CONDUCTOR IN A MAGNETIC FIELD

4. If either the current in the conductor or the direction of the magnetic field between the poles is reversed, the force acting on the conductor tends to move it in the *reverse* direction, as shown in Fig. 4.

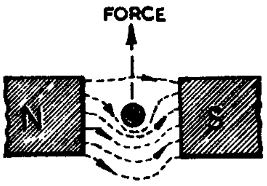

Fig. 4. FORCE ON A CURRENT-CARRYING CONDUCTOR, CURRENT REVERSED

5. The direction in which a conductor which is carrying current tends to move when it is placed in a magnetic field is given by **Fleming's Left Hand Rule**:—

If the first finger, the second finger, and the thumb of the LEFT hand are held at right angles to each other, then with the First Finger pointing in the direction of the Field (N. to S.), and the seCond finger in the direction of the Current in the conductor, the thuMb will indicate the direction in which the conductor tends to Move.

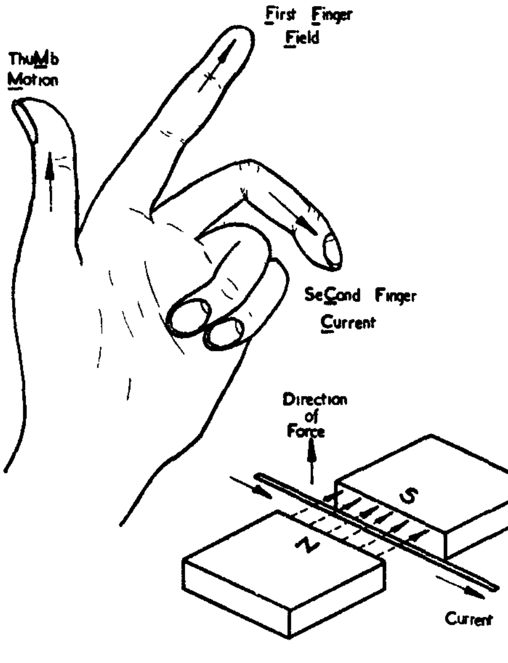

Fig. 5. FLEMING'S LEFT HAND RULE

6. The force acting on a current-carrying conductor when it is placed in a magnetic field is given by:—

$F = BIl$ (newtons) ... ... (1)

where F = Force acting on the conductor in newtons.

B = Flux density of external field in Wb/m^2.

I = Current in conductor in amps.

l = Length of conductor in metres.

7. The fact that a mechanical force acts on such a conductor is used for a number of applications in radio, including certain types of measuring instruments, loud-speakers, telephones, and the electric motor. The latter is considered in this Chapter.

The Electric Motor

8. An electric motor is a machine for converting electrical energy into mechanical energy, its function being the reverse of that of a generator. There is little difference between the construction of generators and motors; both consist of the same essential parts, and the same variations of field-winding connection are found in both types of machines. Provided it is of suitable design as regards brushgear, a d.c. machine can be used either as a motor or as a generator.

The Principles of a Simple D.C. Motor

9. A motor depends for its operation on the force exerted upon current-bearing conductors situated in a magnetic field. Consider a simple permanent magnet motor connected to a battery as shown in Fig. 6.

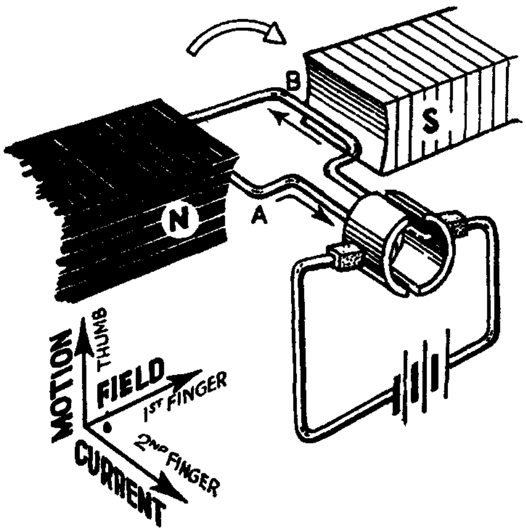

Fig. 6. D.C. MOTOR PRINCIPLE

By applying Fleming's Left Hand Rule it will be seen that side A of the loop (under the N pole) tends to move *upwards*, while side B of the loop tends to move *downwards*. The forces acting on the two sides of the loop are thus cumulative in their effect, and tend to turn the loop in a *clockwise* direction.

10. As the sides of the loop pass through the magnetic neutral position, the commutator reverses the connections of the supply to the loop, and the current in the loop is consequently reversed. Side A of the loop is now coming under the influence of the S pole, with side B coming under that of the N pole. As the current in each side of the loop has been reversed at the instant of transfer from the influence of one pole to that of the other, it follows that the force acting on side A will now be *downwards*, and on side B *upwards*. The mechanical force on the loop is thus continued in the original direction, and rotation continues so long as the supply is connected.

Back E.m.f.

11. When the armature of a motor is rotating, its conductors are cutting the lines of flux of the magnetic field of the machine. An e.m.f. is, therefore, induced in the conductors as in a generator. The direction of the *Back E.m.f.* is given by Fleming's *Right* Hand Rule and is such that it opposes the motion producing it (Lenz's Law). The motion is caused by the external current supply to the armature, and the back e.m.f. will so act as to cut down the current, i.e., it will act in opposition to the applied e.m.f. As was shown in Chap. 1, the value of the back e.m.f. is proportional to the product of the flux and the speed of rotation.

12. There are three voltages to be considered in an electric motor when it is running: the e.m.f. applied to the machine, Ea; the back e.m.f., Eb; and the voltage drop in the armature, Ia Ra. From Kirchhoff's second law, these three voltages are related as follows:—

Applied e.m.f. = Back e.m.f. + Armature voltage drop

$$Ea = Eb + Ia\ Ra \text{ (volts)} \qquad \ldots \quad (2)$$

Armature Reaction

13. When the armature of a motor is carrying current, the main field of the machine is distorted by the magnetic flux resulting from the *armature* current in much the same way as in a generator. In a motor, however, the effect of armature reaction is to *weaken* the main field to an extent dependent upon the value of the armature current. The magnetic neutral axis is moved *backwards* against the direction of rotation (in a generator it is moved forwards); in consequence the brushes are moved slightly *backwards* to reduce reactive sparking (Fig. 7).

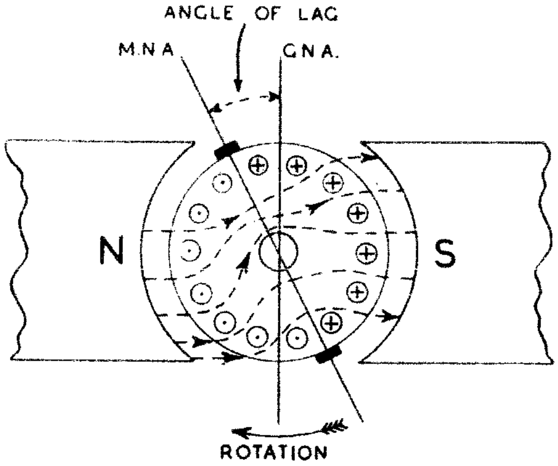

Fig. 7. ARMATURE REACTION IN A MOTOR

14. The objection to moving the brush gear is the same as that for generators, i.e., no one position is completely satisfactory over the whole load range. By the use of *compoles*, motors can be operated with fixed brush positions for all conditions of load. The compoles are connected in series with the armature as in generators, but in this case they are wound so that the magnetic polarity of each compole is *opposite* to that of the next main pole ahead in the direction of rotation.

Torque

15. Torque is the term used to express the turning or twisting effect of a force about an axis. In a d.c. motor each conductor lying in the influence of a pole face exerts a torque tending to turn the armature, the torque of each conductor being determined by the force exerted on the conductor multiplied by the distance of the conductor from the axis of the armature. The sum of these torques is termed the *Armature Torque* and is given by :—

$$\text{Torque, } T = \frac{1}{2\pi} K\Phi IZ \text{ (newton-metres)} \qquad \ldots \quad \ldots \quad \ldots(3)$$

where K = A constant depending on the machine.
Φ = Flux per pole in webers.
I = Armature current in amps.
Z = Number of conductors.

Note. Since the factors K and Z are constant for any given machine it is seen that a variation of torque is obtained by varying either the *flux* Φ or the *armature current* I.

16. The whole of the armature torque is not available for doing useful work. Friction in the bearings, the opposition to magnetic change in the armature core, and wind resistance to the rotating armature all act as a load on the machine, and must be overcome before any useful work can be done. The torque required to overcome these losses is termed the *Lost Torque*, while the difference between the armature torque and the lost torque, i.e. the torque available for useful work, is generally known as the *Shaft Torque*.

17. The power developed by a d.c. motor is proportional to the product of the shaft torque and the speed in revolutions per minute. It follows that *for a given power* any increase in speed can be obtained only at the expense of torque, and *vice versa:* thus, at low speeds the torque will be high, and at high speeds the torque will be low.

18. A mechanical load exerts a torque opposing the motion and this *Load Torque* is constant for a given load, irrespective of the speed at which the load is driven. At low speeds, the shaft torque of the motor will be high and in excess of the load torque. The motor will therefore accelerate and, as its speed rises, the shaft torque will fall until the shaft and the load torques are equal. The motor then continues to drive the load at a steady speed.

19. If the load is increased, the load torque rises to upset the torque balance. The motor will therefore slow down, thereby increasing the shaft torque until the balance is once more restored and the speed is again stablilized. A constant speed with an increased load can be obtained only by increasing the power output of the motor.

Motor Losses and Efficiency

20. **Losses.** As with the generator, the losses in a motor can be classified as:—

(*a*) *Copper losses* due to the generation of heat (I^2R watts) in the armature and field windings; these losses vary with the load.

(*b*) *Iron losses* due to hysteresis losses in every part of the iron through which the flux changes and also to eddy currents induced in the rotating armature.

(*c*) *Mechanical losses* due to friction at the bearings and brushes, and to windage.

21. **Efficiency.** The energy changes and losses which occur in the transformation of electrical energy into mechanical energy in a motor are shown diagrammatically in Fig. 8. From this it is seen that the mechanical power developed in the armature is equal to the applied electrical power less the copper losses; and the mechanical output at the shaft is equal to the mechanical power developed in the armature less the iron and mechanical losses. The overall efficiency of the d.c. motor is:—

$$\text{Efficiency} = \frac{\text{Mechanical Power Output at the Shaft}}{\text{Electrical Power Supplied}} \times 100 \text{ (per cent)}$$

Also,

$$\text{Efficiency} = \frac{\text{Electrical Power Supplied} - \text{Total Losses}}{\text{Electrical Power Supplied}} \times 100 \text{ (per cent).}$$

Speed of a D.C. Motor

22. The back e.m.f. developed in the armature of a d.c. motor when it is running, determines the current in the armature and makes the motor a self-regulating machine in which speed and armature current are automatically adjusted to the mechanical

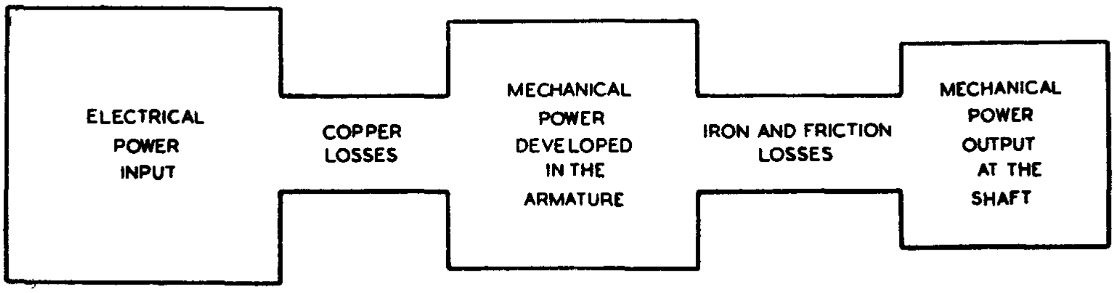
ELECTRICAL POWER INPUT
COPPER LOSSES
MECHANICAL POWER DEVELOPED IN THE ARMATURE
IRON AND FRICTION LOSSES
MECHANICAL POWER OUTPUT AT THE SHAFT

Fig. 8. STAGES IN THE TRANSFORMATION OF ENERGY IN A MOTOR

load. At small values of load the shaft torque exceeds the load torque; the armature therefore accelerates and gives rise to a larger back e.m.f. The increased back e.m.f. cuts down the armature current thus reducing the shaft torque until eventually a state of balance between the two torques is obtained and the speed is stabilized. With increasing load the load torque is increased, exceeding the shaft torque and causing a fall in armature speed. Reduced armature speed results in reduced back e.m.f. and increased armature current; the increase in armature current produces an increase in shaft torque restoring torque balance and stabilizing the speed again. The variation of speed with armature current (i.e. with mechanical load) is known as the *speed characteristic* of the motor.

Control of Speed

23. Assuming constant load, there are two methods commonly used to vary the speed of a d.c. motor:—

(*a*) *Field Control.* By weakening the main flux of a motor the back e.m.f. is reduced, increasing the effective voltage and the armature current. The increased armature current gives rise to an increased shaft torque, causing the motor to accelerate until the back e.m.f., rising with increased speed, restricts the armature current and shaft torque to restore the balance of shaft and load torques. At this point the speed of the motor will stabilize. Conversely, an *increase* in field strength will cause a *reduction* in speed.

(*b*) *Armature Control.* By reducing the voltage across the armature of a motor the effective voltage is reduced, with a corresponding reduction in armature current and shaft torque. The excess of load torque over shaft torque causes the motor to slow down to a point where the reduced back e.m.f. permits sufficient armature current to produce a state of balance between the two torques. At this point the speed of the motor will stabilize.

Types of D.C. Motor

24. D.c. motors are classified according to the method by which the field is excited.

(*a*) The majority of motors are comparable to self-excited generators, i.e., the armature winding and the field winding are supplied from a common source. Their speed and load characteristics vary according to the method of connecting the field winding to the armature, and as a class, they are capable of fulfilling most requirements.

(*b*) Separately-excited motors are used only for special purposes where the more normal types are unsuitable.

(*c*) Permanent magnet motors are employed for certain special purposes, e.g. in small control systems.

Shunt-wound Motors

25. The field winding of the shunt-wound motor is connected in parallel with the armature (Fig. 9). It is thus directly across the supply and must be of fairly high resistance to restrict the current through it. The winding consists of a large number of turns of fine wire. The armature winding is of low resistance to minimize I^2R losses, but unduly heavy current through this winding is prevented by the action of the back e.m.f. when the motor is running.

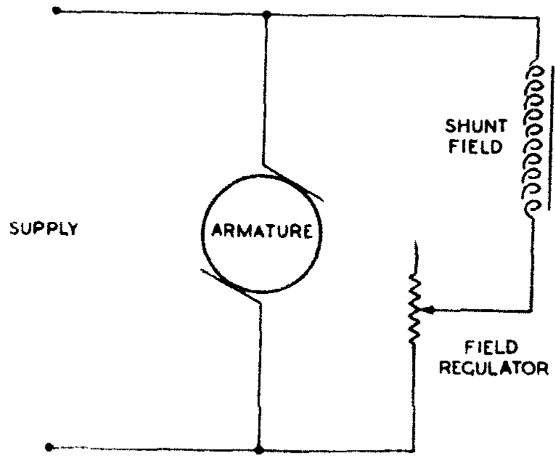

Fig. 9. SHUNT-WOUND MOTOR

26. **Speed Control.** This is normally accomplished by a variable resistor connected in series with the field winding. An increase in resistance will decrease the field strength and increase the speed of the motor (see Para. 23).

27. **Characteristics.**

(*a*) *Speed characteristic.* Whatever the load may be the back e.m.f. adjusts itself to such a value that sufficient armature current can pass to produce a torque equal to the total opposing torque. Further, owing to the low armature resistance, a small decrease in the back e.m.f. (consequent upon a slight reduction in speed) is sufficient to permit full armature current to flow. Reduction in speed from "no load" to "full load" is therefore small (see Fig. 10.) and a shunt-wound motor can be considered as a *constant-speed* machine.

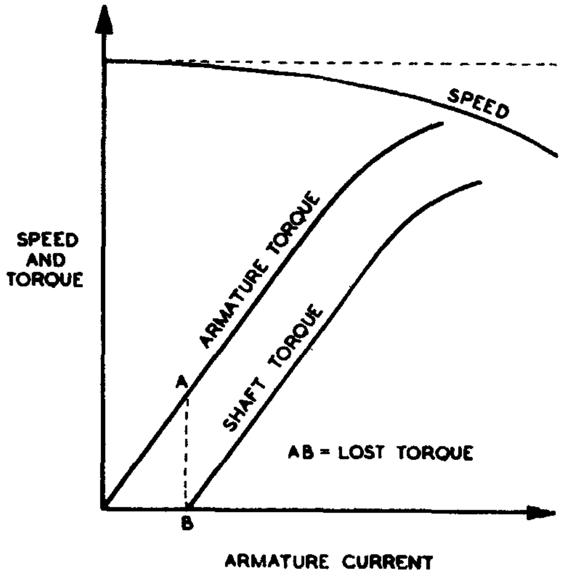

Fig. 10. SHUNT MOTOR CHARACTERISTICS

(*b*) *Torque characteristic.* Since the field current is constant the field strength is also practically constant except for the weakening effect of armature reaction with full load current in the armature. The torque is therefore, proportional to the armature current (see equation (3)) until approaching the full load condition (Fig. 10). The starting torque is small because of the restricted armature current, and shunt-wound motors should, therefore, be started on light load or no load.

28. **Uses.** Shunt-wound motors are suitable for purposes where the speed is required to remain approximately constant from no load to full load, e.g., lathes, drills, and light machine tools generally.

Series-wound Motors

29. The armature winding and the field winding of series-wound motors are in series with each other across the supply (Fig. 11). The armature and field currents are, thus, proportional to each other. Both windings are of low resistance to minimize I^2R losses, the field winding consisting of a few turns of heavy wire. The current in both windings, under running conditions, is restricted by the back e.m.f. induced in the armature.

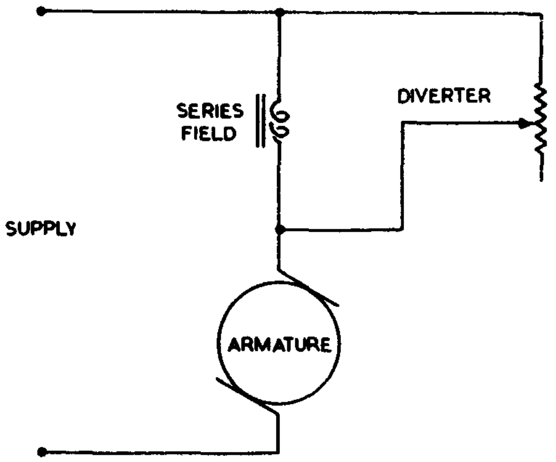

Fig. 11. SERIES-WOUND MOTOR

30. **Characteristics.**

(*a*) *Speed Characteristic.* In a series-wound motor the field flux is proportional to the armature current until the iron of the magnetic circuit approaches saturation. From Para. 23(*a*) it is seen that the speed is inversely proportional to the flux (and hence armature current) up to the point of magnetic saturation, so that the speed decreases with increased load. The speed characteristic is shown in Fig. 12. From

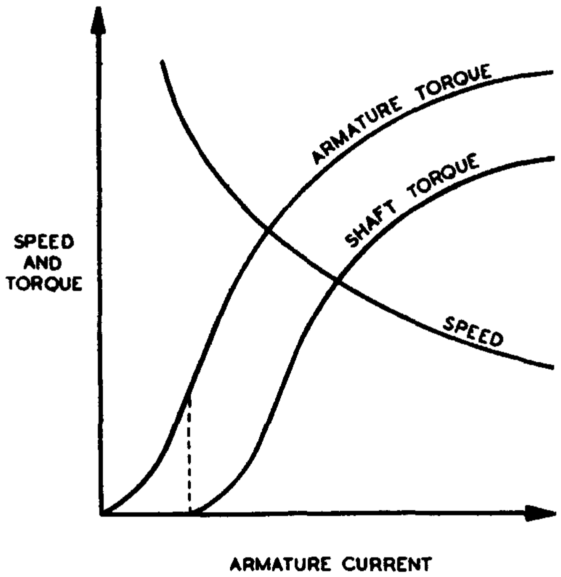

Fig. 12. SERIES MOTOR CHARACTERISTICS

its shape it is seen that the series-wound motor is essentially a *variable speed* motor, the speed being low on heavy load and dangerously high on light load. For this reason the series-wound motor is not run without some mechanical load on it.

(*b*) *Torque Characteristic.* From equation (3) it is seen that the torque is proportional to the product of the flux and the armature current. Further, in a series motor, the field flux is proportional to the armature current so that the torque is proportional to the *square* of the armature current. Hence, the torque increases rapidly as the load is increased until the iron approaches saturation (Fig. 12). The starting torque is high and series motors can be started on full load.

31. **Uses.** Series-wound motors are used where large starting torques are required and where the load is subject to heavy fluctuations. Typical uses include engine starting and traction work.

Compound-wound Motors

32. It is often necessary to modify the characteristics of normal shunt or series motors to meet certain specific requirements. For example, a motor may be required to develop a high starting torque without the tendency to race when the load is removed. Other circumstances may require a motor capable of reducing speed with increased load to an extent sufficient to prevent excessive power demand on the supply, while still retaining the smooth speed control and reliable off-load running of the shunt motor. These and other requirements can be met by suitable compounding. By arranging that part of the field winding is in series with the armature, and part in parallel with it the large starting torque of the series motor can be combined with the steady running under varying load of the shunt motor. This is the compound-wound motor. The windings can be either cumulatively-wound or differentially-wound depending on the requirement (see Chap. 1, Para. 35).

33. **Characteristics.** The torque and speed characteristics for a typical compound-wound motor are shown in Fig. 13. It will be seen that for a small load and armature current the speed is not excessive, and that over a *working range* of load the speed is fairly steady; both of these are "shunt" features. At the same time, there is an appreciable drop in speed between no-load and full-load, and this is a "series" feature.

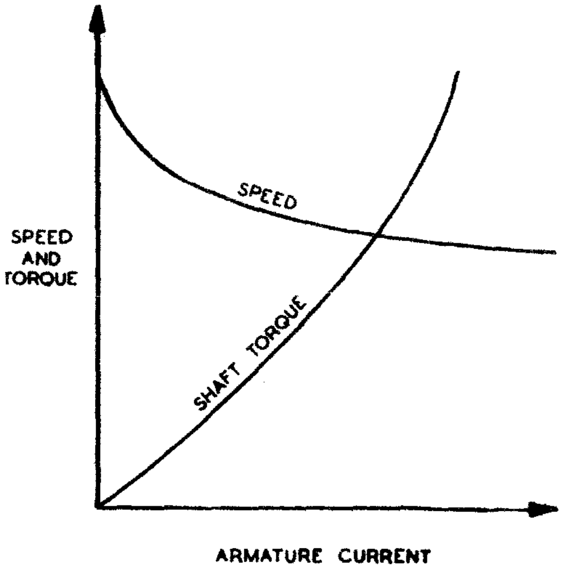

Fig. 13. COMPOUND MOTOR CHARACTERISTICS

Motor Starters

34. Small and medium d.c. motors can be started by connecting the motor terminals direct to the supply, provided the armature windings are of sufficiently high resistance to limit the initial surge of current to a safe level. Direct starting cannot, however, be adopted with larger high power motors having low resistance armature windings. When the armature is stationary there is no back e.m.f. induced in the armature winding so that on connecting the supply to the low resistance armature the current would be excessive. The build-up of flux would be delayed still more by the weakening effect of armature reaction (produced by the excessive armature current). This, in turn, would result in reduced torque and slow acceleration with consequently reduced back e.m.f. The period of excessive current would thereby be prolonged and would undoubtedly lead to damage of the armature winding. Thus, in motors having a low resistance armature winding a *motor starter* must be used.

35. The principles of starting and of the speed regulation of the various types of d.c. motors can be summarized as follows:—

(*a*) *Series Motor.*

(i) The normal starting equipment for low resistance types consists of a variable resistor connected in series with the supply (Fig. 14). The motor is started with all the resistance in circuit to limit the initial surge of armature current; the resistance is then progressively reduced until, at normal speed, all the resistance is out of circuit and the back e.m.f. is such as to keep the armature current at its normal low value. As noted in para. 30 the series-wound motor is started *on load.*

(ii) Speed regulation is obtained by either a variable resistor connected in series with the supply or by a diverter in parallel with the series field winding (see Fig. 14).

(*b*) *Shunt Motor.*

(i) This type is normally started with resistance in series with the *armature* winding only, full voltage being applied at once to the shunt field winding (Fig. 14); the starting resistance is progressively reduced as the speed rises, the increased back e.m.f. then limiting the armature current. The shunt motor is normally started *off load* (see Para. 27).

(ii) Speed regulation is normally by a variable resistor in series with the shunt field winding (see Fig. 14). Increased resistance gives increased speed.

(*c*) *Compound Motor.* Normally as for shunt motors.

Rotary Transformers

36. In many instances of both ground and airborne radio equipments the d.c. power supplies are obtained from a rotary transformer. This is a single machine combining the functions of a d.c. motor and d.c. generator. It consists of a single field system and a single armature, on which are separately

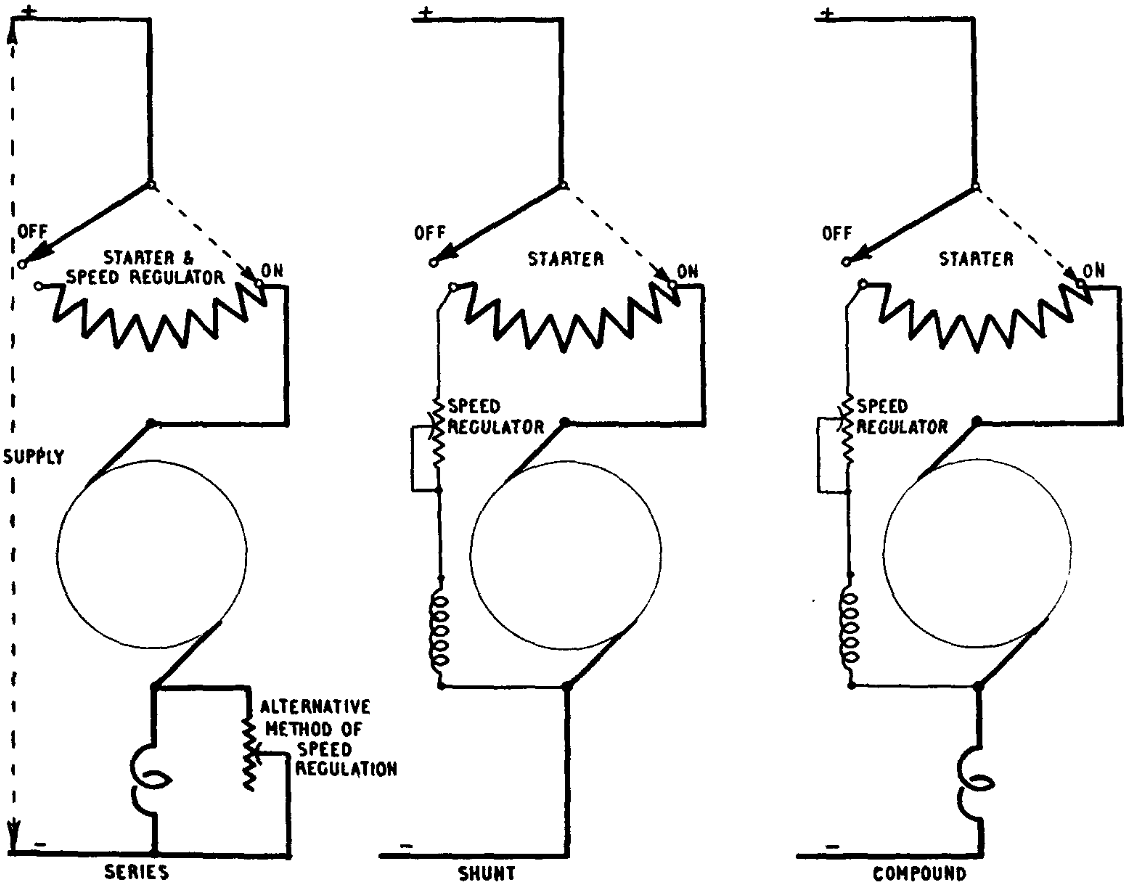

Fig. 14. STARTING AND SPEED REGULATIONS OF D.C. MOTORS

wound the motor and generator windings. On supplying the d.c. input to the field winding and to the motor winding (via the motor commutator and brushes) the armature rotates; since the generator winding on the armature is moving in a magnetic field an e.m.f. is induced in this winding, the d.c. output being taken via the generator commutator and brushgear to the external circuit. In this way it is possible to "step up" or "transform" the input voltage to any required level. For example a typical rotary transformer operates from a d.c. input of 24 volts at 7 amps and gives a d.c. output of 1,200 volts at 100 milli-amps. It should be noted however that in the conversion of energy some losses have occurred (Para. 20) and the *power* output of the machine is always less than the power input.

Rotary Inverters

37. This is the name given to machines which combine the functions of d.c. motor and a.c. generator. A rotary inverter is similar in construction to a rotary transformer with the exception that the generator windings are connected to slip rings instead of to a commutator. In this way the d.c. input to the motor commutator is "inverted" to give an a.c. output at the slip rings of the generator.

Rotary Converters

38. This is the name given to machines which combine the functions of a.c. motor and d.c. generator. It is similar in general construction to the rotary inverter, but the slip rings are now on the input (motor) side and the commutator on the output (generator) side of the machine. The a.c. input to the slip rings of the motor is "converted" to give a d.c. output at the generator commutator.

SECTION 4

ELECTROSTATICS AND CAPACITANCE

Chapter 1 .. .. **Electrostatics**

Chapter 2 .. .. **Capacitors**

Chapter 3 .. .. **Capacitive Circuits**

SECTION 4

CHAPTER 1

ELECTROSTATICS

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PART 1, SECTION 4, CHAPTER 1

ELECTROSTATICS

Introduction

1. It was stated in Sect. 1, Chap. 1 that an electron is the elementary particle of negative electricity or charge. An atom which is deficient of an electron (or electrons) assumes a positive charge and is termed a positive ion. Similarly, *any* body which has a deficiency of electrons is positively-charged; a body with an excess of electrons over its normal complement is negatively-charged. Electrostatics, as the name implies, is primarily the science of electric charges at rest. It is a subject which has many applications in radio (e.g., capacitors, thermionic valves, and cathode-ray tubes); an elementary knowledge of the subject is, therefore, required before its applications can be considered.

The First Law of Electrostatics

2. Much has been known about electrification by friction since early times. For instance, a comb after passing through dry hair attracts the individual hairs, which then themselves tend to stand on end repelling one another. If a glass rod is rubbed with a piece of silk, the silk is then attracted towards the glass. In this case, the silk *removes* electrons from the glass which is thus left with a *positive* charge; the electrons acquired by the silk give it an *equal negative* charge; between the glass and the silk there is a force of attraction. If two glass rods are treated in this manner both become positively-charged, and between the two there is a force of repulsion. From these facts the first law of electrostatics can be stated:—

> "*Like charges repel each other; unlike charges attract.*"

Coulomb's Law

3. The size of the mechanical force of attraction (or repulsion) is greater between large charges than between small ones, and is greater when the charges are close together than when they are more distant. **Coulomb's Law** states that the force between two quantities of electricity Q_1 and Q_2, situated at two points a distance *r* apart is proportional to the product Q_1Q_2, and inversely proportional to r^2. It also depends on the nature of the *medium* between the charges. Thus, the equation for the force F between charges Q_1 and Q_2 coulombs at points separated by a distance *r* metres in free space is:—

$$F = \frac{1}{\kappa_0} \cdot \frac{Q_1Q_2}{4\pi r^2} \quad \text{(newtons)} \ldots \quad \ldots (1)$$

where κ_0 is a factor known as the *permittivity of free space* (see Para. 21).

Potential

4. If an insulated metal sphere A is charged positively and another positively-charged body B is brought up towards it, there is a force of repulsion between them, and the nearer the two approach the greater is this force. *Work* has to be done to overcome the repulsion and bring B nearer to A, and if the repulsion is allowed to take effect it will return this work and move B back again. The nearer B is brought to A, the greater is the *potential energy of the system*. The single word, **potential**, is used in describing this fact; the potential increases as A is approached, there being a *difference* of potential between two points such as X and Y at different distances from A (Fig. 1).

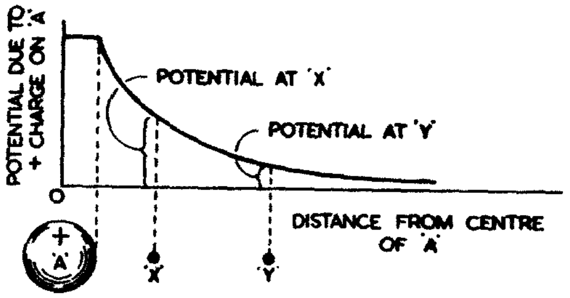


Fig. 1—THE POTENTIAL DUE TO A POSITIVELY-CHARGED BODY

5. Positive charges tend to move in the direction X—Y, from the *higher* to the *lower* potential. The change of potential per unit distance—known as the **potential gradient**—increases as A is approached as Fig. 1 indicates by the change in steepness.

The Electric Field

6. A charged body produces a distribution of electric potential in its neighbourhood, which was not there before it was charged and will disappear when the charge is removed. Thus, around a charged body, there is said to be an **electric field**—the volume of space in which electric effects are experienced.

Alternatively, when space is in such a state that an electric charge in that space experiences a force, the charge is said to be in an electric field.

7. **Electric lines of force** are used to show the distribution of an electric field, each line showing the direction of the force on a free *positive* charge at points along it in the neighbourhood of a charged conductor.

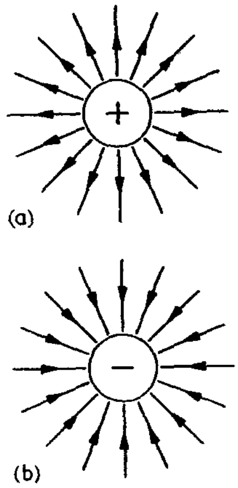


Fig. 2—THE ELECTRIC FIELD DUE TO CHARGED SPHERES.

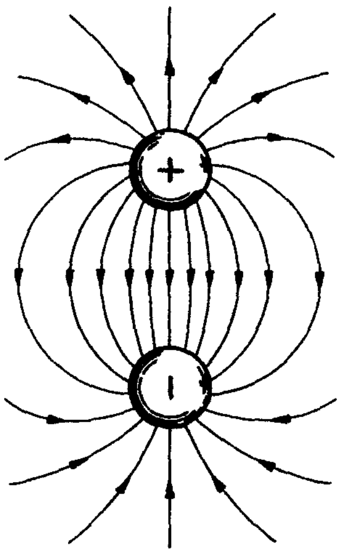


Fig. 3—THE ELECTRIC FIELD IN THE NEIGHBOURHOOD OF OPPOSITELY-CHARGED SPHERES.

The electric fields due to positively-charged and negatively-charged spheres are as shown in Fig. 2(*a*) and 2(*b*) respectively. By convention, the *direction* of an electric field is away from + charges and towards — charges. In Fig. 3 the direction of the field in the neighbourhood of two oppositely-charged spheres is shown by arrow-heads on the electric lines of force.

8. Changes in the spacing of the lines correspond to changes in the strength of the electric field, which is strongest where the lines are most closely spaced. The lines are analogous to invisible stretched elastic strings which tend to contract and also tend to repel one another sideways. Fig. 3 suggests this, for the two charged bodies are acted upon by the attractive forces between the charges, and thin stretched elastic cords could be made to give exactly the same kind of forces on the bodies. It should of course be clearly understood that the lines of force are imaginary, and only a method of representing a field on paper.

9. An electric field will exist between any two points which are at different potentials relative to each other. The strength of the field will depend on the magnitude of the p.d. and on the distance between the two points. The direction of the field will depend on the polarity of the p.d. and, by convention, is from positive to negative.

Capacitor

10. Two metal plates separated by an insulator (or *dielectric*) constitute a **capacitor.** A capacitor is a system of conductors that has the ability or the capacity for storing electricity as an excess of electrons on one plate and a deficiency on the other,—i.e., a capacitor can "hold a charge". When charged, a p.d. will exist between the plates.

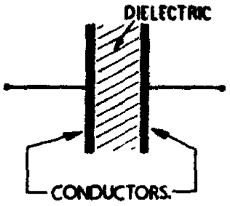


Fig. 4—CAPACITOR.

Charge of Capacitor

11. When the switch is closed in the circuit shown in Fig. 5, current will be established in the circuit. A general movement of electrons away from the top plate of the capacitor and towards the bottom plate results, and the p.d. across the capacitor gradually rises. Since the p.d. across the capacitor is in opposition to the applied voltage the *effective* voltage acting in the circuit falls and with it, the current. This

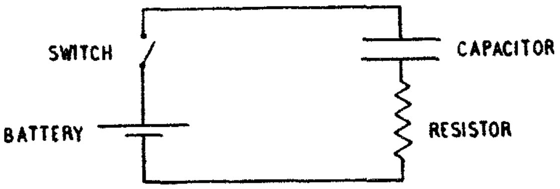


Fig. 5—CHARGE OF A CAPACITOR.

continues until the p.d. across the capacitor equals the applied voltage and the current falls to zero. The graph of Fig. 6 shows how the current in the circuit and the p.d. across the capacitor vary with time. This process will be considered in greater detail in Chap. 3.

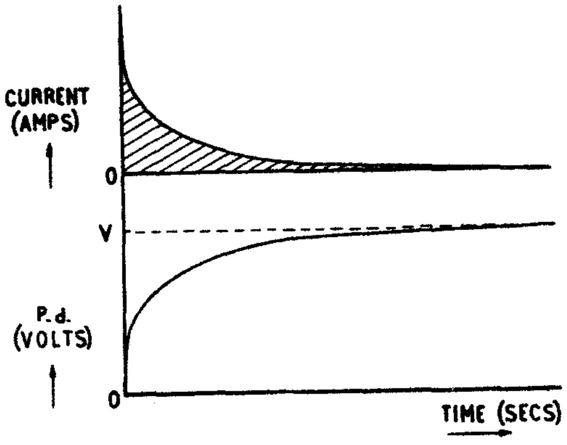


Fig. 6—VARIATION OF CURRENT AND P.D. DURING CHARGE.

12. The shaded area in the graph for current (Fig. 6) represents the product of the average current in amperes and the time in seconds, namely the quantity of electricity or *charge* in coulombs (since Q = It coulombs). This is the quantity of electricity required to charge the capacitor to a p.d. of V volts.

Capacitance

13. If the charge Q coulombs in a capacitor is measured at various fixed values of the p.d. V volts across it, the ratio of Q to V is found to be a constant. This constant is termed **capacitance** (symbol C). Thus, for any given capacitor:—

$$\frac{\text{Charge in coulombs}}{\text{P.d. in volts}} = \text{Capacitance}$$

$$\text{i.e., } C = \frac{Q}{V} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

14. The unit of capacitance is the **farad** (symbol F). *A capacitor has a capacitance of one farad if a charge of one coulomb raises the p.d. by one volt.* In practice, the farad is an inconveniently large unit and capacitance is usually expressed in micro-farads (μF) or pico-farads (pF).

Example. A capacitor of capacitance 500pF is connected across a supply of 100V d.c. The charge on the capacitor is:—

$$Q = CV = 500 \times 10^{-12} \times 100 = 5 \times 10^{-8}$$
$$\therefore \text{ Charge } Q = 0{\cdot}05 \text{ micro-coulombs.}$$

Electric Field Strength

15. Fig. 7 shows a capacitor of C farads connected to a source of p.d. V volts. The capacitor is assumed to be situated in a vacuum; the plates have a surface area of *a* square metres and are separated from each other by *d* metres; and the capacitor is charged to Q coulombs.

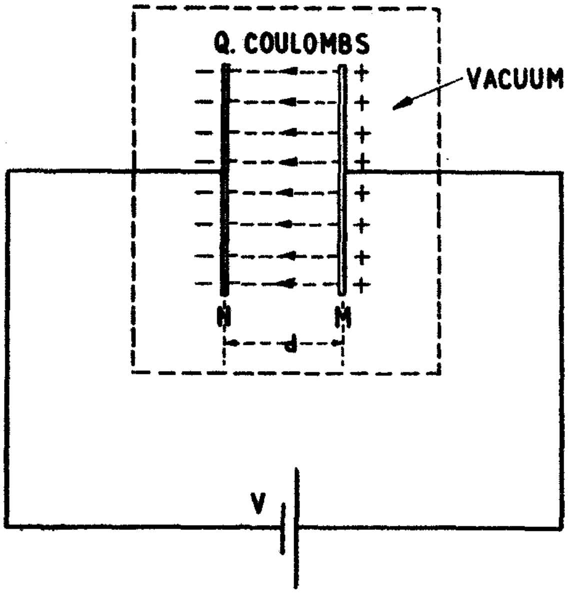


Fig. 7—THE ELECTRIC FIELD BETWEEN THE PLATES OF A CHARGED CAPACITOR.

16. An electric field exists between the plates as shown. The intensity of the field at any point is the **electric field strength** (symbol E) and is the force per unit charge experienced by a small positive charge placed at that point. It is defined as the potential drop per unit length or the *potential gradient* (see Para. 4) and is given by:—

$$E = \frac{V}{d} \text{ (volts/metre)} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

17. If the applied p.d. is 200V and the distance between the plates is one cm., the

electric field strength E will be $\frac{V}{d}$ or $\frac{200}{0.01}$ which equals 20,000 V/m. The graph (Fig. 8) shows how the potential of the electric field varies with distance from N, assuming the field to be uniformly distributed. If the plates are brought closer together, or if the p.d. V is increased, the slope of the graph will rise to indicate an increase in the potential gradient.

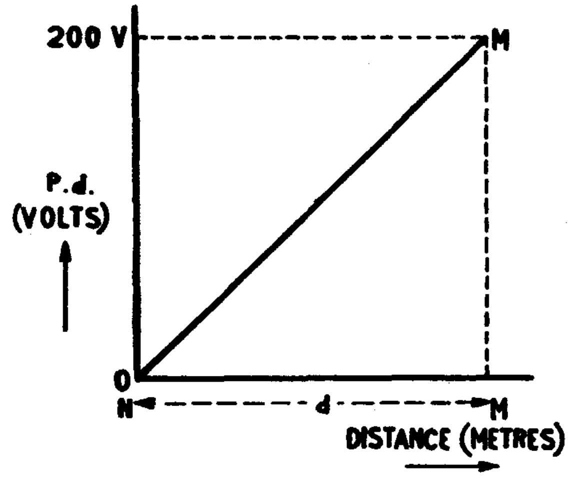


Fig. 8—VARIATION OF POTENTIAL IN A UNIFORM ELECTRIC FIELD.

18. Consider four plates A, B, C, and D as shown in Fig. 9. Plate A is 10V positive with respect to plate C; plate B is 100V positive; and plate D 200V positive. The electric fields existing between the plates are as shown, and their directions should be noted. The graph shows how the potential of the uniform electric field varies in the region between the plates. This type of graph is one which is often encountered in the study of thermionic valves where it is necessary to understand how an electron behaves in an electric field.

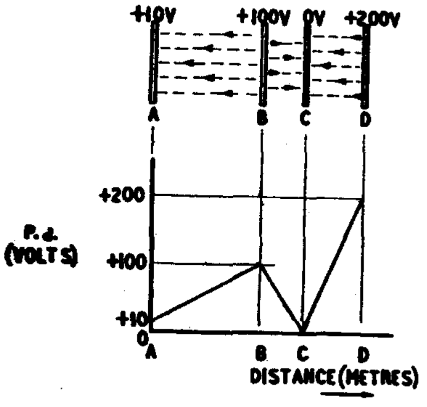


Fig. 9—POTENTIAL GRADIENT BETWEEN PLATES AT DIFFERENT POTENTIALS.

Electric Flux and Flux Density

19. The system of electric lines of force from a charge or through an electric field is termed the **electric flux.** The unit of flux is the flux associated with a charge of one coulomb; so the flux originating from a charge of + Q coulombs is directed *outwards* from it and *equal to Q coulombs.*

20. **Electric flux density** is defined as the amount of flux per unit area falling on a surface taken at right angles to the direction of the flux. If a tube of flux issuing from a charge of Q coulombs has a cross-section *a* square metres, the electric flux density (symbol D) will be:—

$$D = \frac{Q}{a} \text{ (coulombs/square metre)} \quad .. \quad (4)$$

Permittivity of Free Space

21. The ratio of the *electric flux density D* to the *electric strength E* in *free space* is termed the *permittivity of free space* (symbol κ_o). Thus:—

$$\kappa_o = \frac{D}{E} = \frac{Q}{a} \div \frac{V}{d} = \frac{Qd}{Va}$$

$$\text{But } \frac{Q}{V} = C$$

$$\therefore \kappa_o = C\frac{d}{a}$$

$$\text{And } C = \kappa_o \frac{a}{d} \text{ (farads) (in free space)} \quad (5)$$

The value for the constant κ_o is:—

$8 \cdot 85 \times 10^{-12}$ m.k.s. units.

Dielectric Constant

22. If a slab of insulating material is inserted to fill the space between the plates of the capacitor in Para. 15, the capacitance will increase. The ratio of the *capacitance of a capacitor having a certain material as dielectric* to the *capacitance of the same capacitor having free space* (*vacuum*) *as dielectric* is termed the **dielectric constant** or the relative permittivity of the material inserted. The symbol is κ_r.

23. It follows from equation (5) that when the plates of a capacitor are separated by an insulator of dielectric constant κ_r, the capacitance will be:—

$$C = \kappa_o \kappa_r \frac{a}{d} \text{ (farads)} \quad .. \quad .. \quad .. \quad (6)$$

24. From equation (2), $C = \frac{Q}{V}$. Thus, the

increase in capacitance obtained by inserting a material as dielectric indicates that the p.d. V volts for an original charge Q coulombs applied to the capacitor has decreased; this is because the *dielectric* has become polarized by induction from the plates and partially cancels the charge on the plates. Values of the dielectric constant for some of the more important insulating materials are given in Table 1.

Material	Dielectric Constant κ_r
Air	1·0006
Mica	5·5
Dry Paper	3·7
Porcelain	7
Bakelite	5
Glass	6
Rubber	3
Polythene	2
Ceramics (Barium titanate type)	3,000

TABLE 1. DIELECTRIC CONSTANT

Note. For practical purposes, air can be considered to have a dielectric constant of unity (as for free space).

Parallel Plate Capacitor

25. The capacitor shown in Fig. 10 is termed a parallel plate capacitor. The capacitance C in farads of such a capacitor with plates of known area *a* square metres and distance *d* metres apart is given by equation (6) :—

$$C = \kappa_o \kappa_r \frac{a}{d} \text{ (farads),}$$

where $\kappa_o = 8{\cdot}85 \times 10^{-12}$.

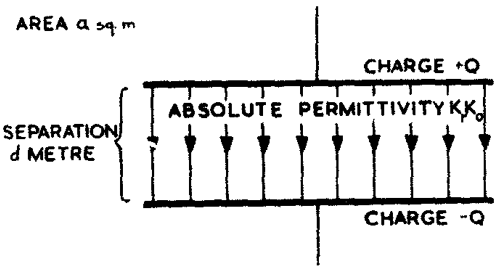


Fig. 10—A PARALLEL PLATE CAPACITOR.

26. In practice, units larger than micro-farads are never used and very frequently, particularly in electronic equipment, the micro-micro-farad (or pico-farad) is sufficiently large. In terms of micro-farads the above formula becomes:—

$$C = \frac{8{\cdot}85 \times 10^{-6} \times \kappa_r \times a}{d} (\mu F)$$

In pico-farads:—

$$C = \frac{8{\cdot}85 \times \kappa_r \times a}{d} \quad (pF)$$

Capacitance of a Multi-plate Capacitor

27. Fig. 11 shows a capacitor made up of seven parallel plates, four being connected to A and three to B. Each side of the three plates connected to B is in contact with the dielectric. Of the plates connected to A, only *one* side of the outer plates is in contact with the dielectric. Consequently, the opposing surface area of each set of plates is the same, namely *6a* square metres. Hence for *n* plates, the effective surface area is *(n — 1) a* square metres, and from equation (6) the capacitance of a multi-plate capacitor is:—

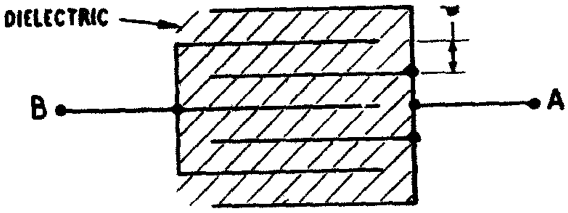


Fig. 11—A MULTI-PLATE CAPACITOR.

$$C = \frac{8{\cdot}85 \times 10^{-6} \times \kappa_r \times (n - 1)a}{d} (\mu F) \quad (7)$$

where κ_r = the dielectric constant
n = total number of plates
a = surface area of one plate in square metres
d = thickness of dielectric in metres.

Capacitors in Parallel

28. Consider three capacitors of capacitance C_1, C_2, and C_3 respectively connected in parallel across a p.d. of V volts as shown in Fig. 12. For circuits in parallel the current through each component may be different and is given by I_1, I_2 and I_3 respectively. Further, since Q = It, the charge Q_1, Q_2 and Q_3 on each capacitor may be different.

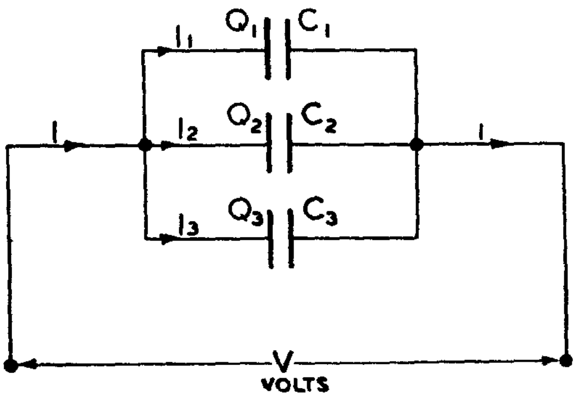


Fig. 12—CAPACITORS IN PARALLEL.

29. From Kirchhoff's first law:—

$$I = I_1 + I_2 + I_3$$

$$\therefore It = I_1t + I_2t + I_3t$$

$$\therefore Q = Q_1 + Q_2 + Q_3$$

Divide throughout by V.

$$\therefore \frac{Q}{V} = \frac{Q_1}{V} + \frac{Q_2}{V} + \frac{Q_3}{V}$$

$$\therefore C = C_1 + C_2 + C_3 \text{ (farads)} \quad .. \quad (8)$$

Thus, the equivalent capacitance of capacitors connected in *parallel* is the *sum* of the individual capacitances. It is easy to see that this must be so, for if three identical sets of plates are connected in parallel the result is to make a larger capacitor of which the total area is the sum of the individual areas (Fig. 13). Thus, capacitors of 1μF, 2μF, and 3μF respectively in parallel are equivalent to a single capacitor of $1 + 2 + 3 = 6\mu$F.

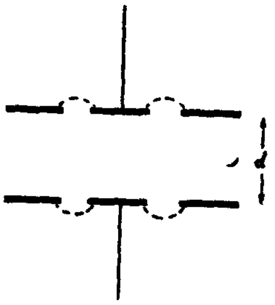


Fig. 13.—CAPACITORS IN PARALLEL REGARDED AS A SINGLE CAPACITOR

Capacitors in Series

30. Consider three capacitors of capacitance C_1, C_2, and C_3 respectively connected in series across a p.d. of V volts as shown in Fig. 14. For a series circuit, the current is the same at all points and equals the total current I. Further, since $Q = It$ *the charge on each capacitor will be the same and equal to the total charge Q in the circuit*, i.e.,

$$Q = Q_1 = Q_2 = Q_3.$$

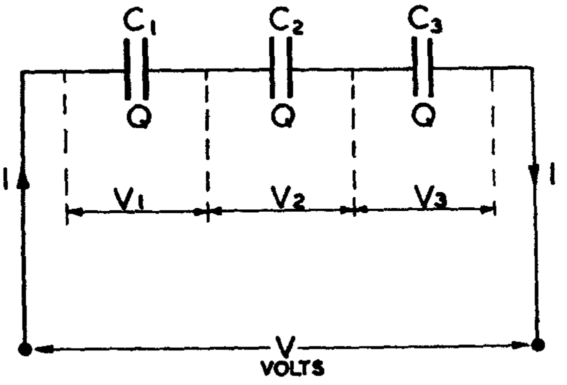


Fig. 14—CAPACITORS IN SERIES.

31. The p.d. across each component in a series circuit may be different, the sum of all the p.d.s (from Kirchoff's second law) being equal to the applied voltage.

i.e., $V = V_1 + V_2 + V_3$

$$\therefore \frac{Q}{C} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

$$\therefore \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

Divide throughout by Q.

$$\therefore \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad .. \quad (9)$$

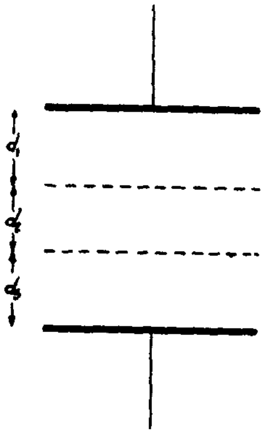


Fig. 15—CAPACITORS IN SERIES REGARDED AS A SINGLE CAPACITOR.

Thus, for capacitors in *series*, the reciprocal of the total capacitance equals the sum of the reciprocals of the individual capacitances. Capacitors in series may be regarded, as a single capacitor of greater plate separation, as shown in Fig. 15.

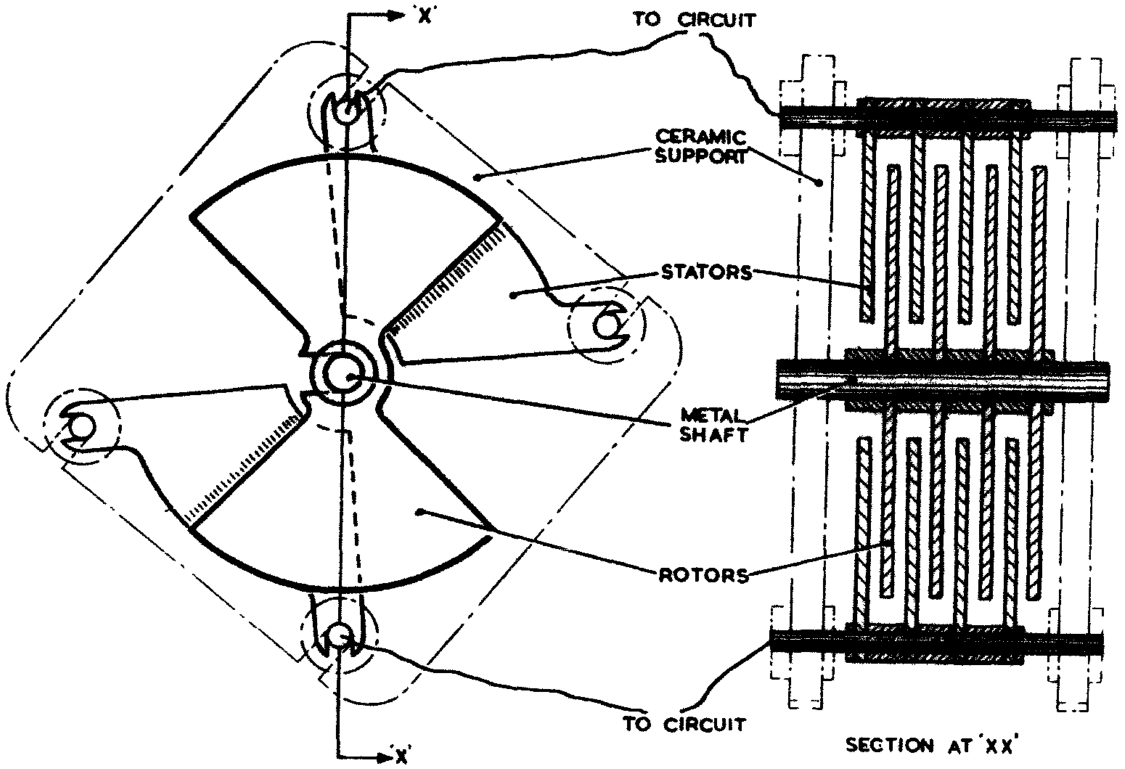

'X'
TO CIRCUIT
CERAMIC SUPPORT
STATORS
METAL SHAFT
ROTORS
TO CIRCUIT
'X'
SECTION AT 'XX'

Fig. 10.—CONSTRUCTION OF A SPLIT-STATOR CAPACITOR.

the bottom set of stators and the rotors; the whole constitutes two variable capacitors in series, where $C = \frac{C_1 \times C_2}{C_1 + C_2}$ farads (Fig. 11). For capacitors in series, the total capacitance is less than the smallest individual capacitance, so that an extremely small value of capacitance can be obtained with this type of capacitor. This is an advantage in some circuits.

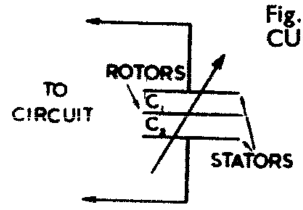

TO CIRCUIT
ROTORS
C_1
C_2
STATORS

Fig. 11—EQUIVALENT CIRCUIT FOR A SPLIT-STATOR CAPACITOR.

Trimmer Capacitor

22. These are normally placed in parallel with the main variable capacitor, and they can be adjusted to give a slight variation to the final capacitance range. The arrangement is as shown in Fig. 12. Trimmer capacitors are constructed in various ways. In general, they are similar to a variable air-dielectric capacitor, but to a smaller scale. Some trimmers incorporate a solid dielectric, such as ceramic. Most are adjusted by a screw. Capacitance values range from 2pF to 150pF at working voltages of up to 350V d.c.

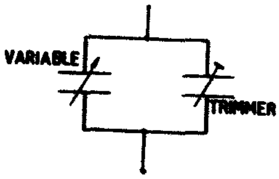
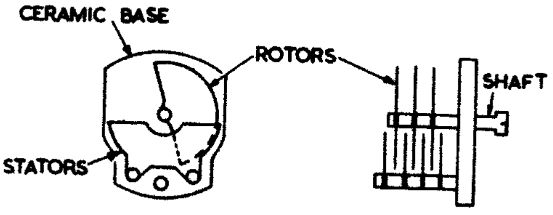
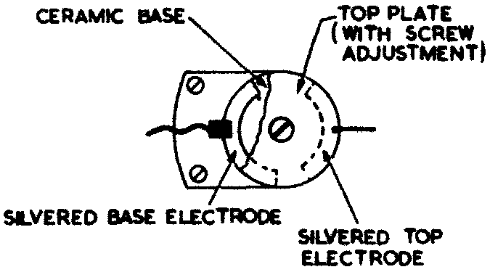

VARIABLE
TRIMMER

Fig. 12—USE OF A TRIMMER CAPACITOR.


CERAMIC BASE
ROTORS
SHAFT
STATORS

(a) VARIABLE AIR TRIMMER


CERAMIC BASE
TOP PLATE (WITH SCREW ADJUSTMENT)
SILVERED BASE ELECTRODE
SILVERED TOP ELECTRODE

(b) CERAMIC TRIMMER

Fig. 13—CONSTRUCTION OF TRIMMER CAPACITORS.

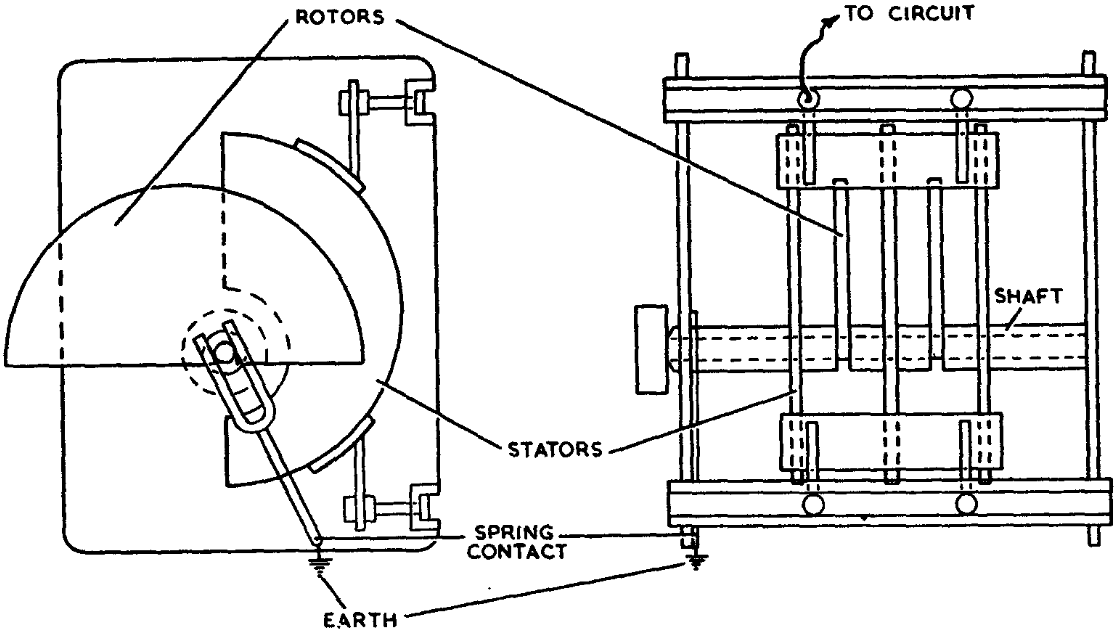


Fig. 7—CONSTRUCTION OF VARIABLE CAPACITOR.

area is nil; when fully "in mesh" the effective area is a maximum. In this way, the capacitance is varied. Although the effective area can be reduced to zero, the capacitance cannot. Even when the plates are fully out of mesh, "fringing" of the electric field between the edges of the plates gives a finite value of capacitance. A typical variation of capacitance is from 50pF to 500pF. In many cases, some of the vanes of the capacitor are *slotted* so that the final variation of capacitance can be adjusted by bending portions of the vanes in one direction or the other.

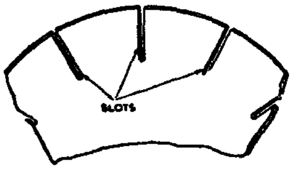


Fig. 8—SLOTTING OF CAPACITOR VANES.

19. The *shape* of the capacitor vanes will determine the manner in which the capacitance varies for angular rotation of the shaft and will depend on the particular requirement of the circuit considered. The symbol for a variable capacitor is as shown in Fig. 9.

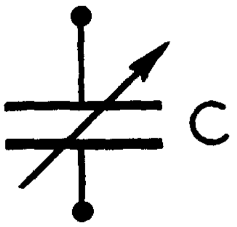


Fig. 9—SYMBOL FOR A VARIABLE CAPACITOR.

Split-stator Capacitor

20. This is a special type of variable, air-dielectric capacitor which is used in some radio equipments. The stators are split to form *two* sets of plates, the rotors being moved between the stators by a shaft as shown in Fig. 10.

21. There is no electrical connection to the rotors; these vanes are insulated from the chassis by the ceramic supports. The advantage thus obtained is that *no contact noise* is introduced into the circuit such as there is in a conventional variable capacitor where the rotors are earthed via a spring contact bearing on the metal shaft. The two stators are insulated from the chassis by ceramic supports, connection to the external circuit being made from each set of stators. Two equal capacitances exist between the top set of stators and the rotors, and between

electrodes to which are soldered metal end caps. The ceramic rod type is as shown in Fig. 5. Where a ceramic tube is used, the silver electrodes can be deposited on the outer and inner surfaces of the tube. The capacitor is finished with an enamel or wax coating, or placed inside another ceramic tube. Capacitance values range from 0·5 pF to 0·005μF at working voltages of up to 500V d.c.

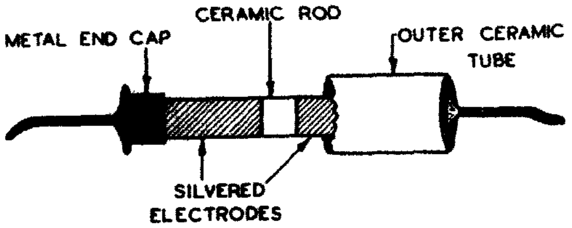


Fig. 5—CONSTRUCTION OF CERAMIC CAPACITOR.

Polystyrene Type Capacitor

12. This type is very similar in construction to the stacked mica capacitor described in Para 9, with polystyrene strips in place of mica. Capacitance values range from 10pF to 4μF at working voltages of up to 500V d.c.

Electrolytic Capacitor

13. If a p.d. is applied across two aluminium plates immersed in a solution of ammonium borate, a current flows through the electrolyte. This current rapidly diminishes to a small value because of the formation of a thin insulating film of aluminium oxide on the positive electrode. Because of the extreme thinness of this dielectric film (e.g. $0{\cdot}01 \times 10^{-3}$ cm.) a high capacitance will exist between the position electrode and the electrolyte (since $C \propto \frac{1}{d}$). The solution is in direct contact with the *negative* electrode, so that between the two electrodes a very large value of capacitance can be obtained in a small volume.

14. The "wet" electrolytic type so described has been almost entirely replaced by the "dry" electrolytic type, where the electrodes take the form of two long strips of aluminium foil, the dielectric film having been previously formed on the positive electrode. The two electrodes are separated by strips of cotton gauze impregnated with the electrolyte, the whole then being rolled up very tightly in a manner similar to that for a paper type capacitor. The roll is mounted in an aluminium or a bakelite case and connections from each metal foil are taken out to the two terminals.

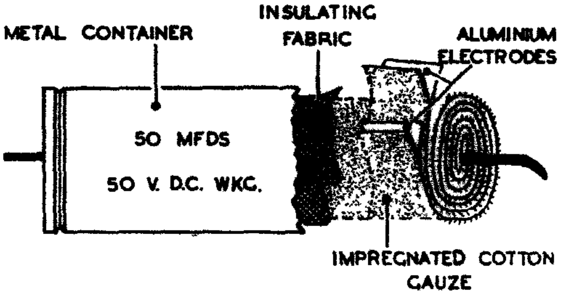


Fig. 6—CONSTRUCTION OF DRY ELECTROLYTIC CAPACITOR.

15. With an electrolytic capacitor, the dielectric film on the positive electrode must be formed and *maintained* before a capacitance is obtained. Because of this, electrolytic capacitors can be used only in those circuits where a polarizing d.c. voltage exists. The terminals are marked "positive" (red) and "negative" (black) to ensure the correct polarity. The capacitance depends on the "forming" voltage, the values ranging from 2μF to 32μF at working voltages of up to 600V d.c., and from 50μF to 3,000μF at working voltages of 50V to 6V d.c. The leakage current required to maintain the dielectric film is of the order of 0·2 milliamp.

Variable Capacitor

16. In many circuits in radio equipment it is necessary to be able to vary the capacitance. Variable capacitors used for this are normally of the air-dielectric, parallel-plate type. One set of plates, the *stators*, is fixed; the other set, the *rotors*, is controlled by a metal shaft such that the rotors can be moved into and out of mesh with the stators, as shown Fig. 7.

17. The stators are insulated from the chassis by ceramic posts, all the stator vanes being connected in parallel and a lead taken from them to the external circuit. The rotors are normally connected in parallel via the metal shaft which, in turn, is earthed to the chassis by a spring contact.

18. As the rotors are moved relative to the stators the area of overlap of each set of plates is varied. When the rotors are fully "out of mesh" with the stators the effective

Paper Type Capacitor

8. Two long, thin strips of aluminium foil (about 0·002 inches thick), with similar strips of dry paper are assembled as shown in Fig 2(*a*). The whole is then rolled up very tightly as in Fig. 2(*b*) to form a capacitor of small volume. The metal foil electrodes are extended at several points along their length to form connecting tabs, the tabs common to each electrode being soldered together and taken out to the two terminals. The capacitor so assembled is placed inside a sealed container of waxed cardboard, metal, ceramic, or plastic; the shape is either tubular or rectangular. Such capacitors have capacitance values ranging from 0·0002 μF to 12μF at working voltages of up to 150kV d.c. and temperatures up to 100°C. One end of the capacitor is sometimes marked with a black band to indicate the outer electrode connection. Since the paper type capacitor is constructed in the form of a coil, its self-inductance gives rise to certain difficulties at high frequencies; this will be discussed in later Sections.

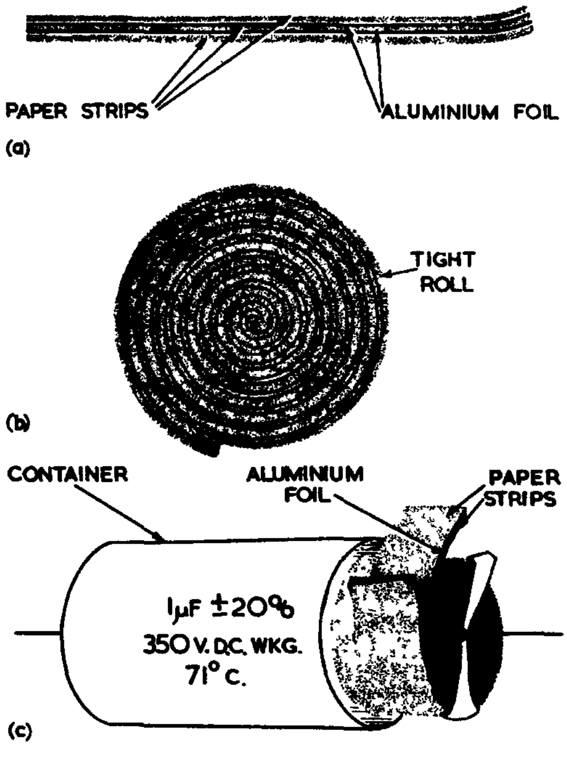


Fig. 2—CONSTRUCTION OF PAPER TYPE CAPACITOR.

Stacked Mica Type Capacitor

9. This type consists of several layers of metal foil interleaved with layers of mica, with alternate foils joined together and taken out to the two terminals as shown in Fig. 3. The assembly is placed inside a moulded plastic or metal case. Such capacitors have capacitance values ranging from 50pF to 0·25μF at working voltages of up to 2kV d.c.

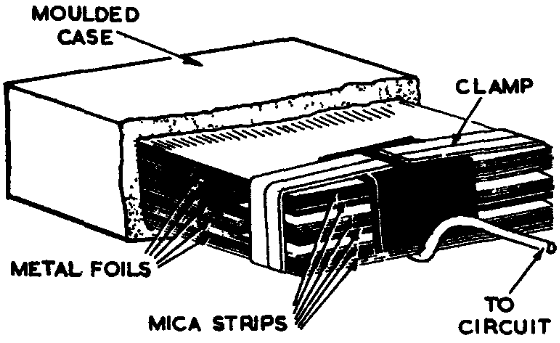


Fig. 3—CONSTRUCTION OF STACKED MICA CAPACITOR

Silvered Mica Type Capacitor

10. This type is similar in construction to the stacked mica type except that in place of the metal foils, silver is deposited directly on one side of each mica strip. A more compact and more stable capacitor results. After assembly, the whole is given a coating of wax or placed inside a plastic case. Capacitance values range from 10pF to 0·01μF at working voltages of up to 750V d.c.

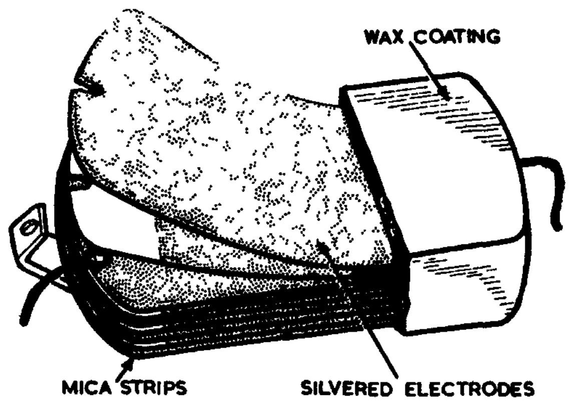


Fig. 4—CONSTRUCTION OF SILVERED MICA CAPACITOR.

Ceramic Type Capacitor

11. This type of capacitor comprises a low-loss ceramic rod or tube with silvered

lags behind the electric field strength E ($= \frac{V}{d}$ volts/metre). In other words, the charge Q lags on the applied voltage V, and if the voltage is alternating rapidly, the capacitor will never receive the full charge. Further, since $C = \frac{Q}{V}$, the effective capacitance will have decreased. Dielectric hysteresis (or dielectric absorption) gives rise to a loss of energy in a manner similar to that for magnetic hysteresis.

Capacitator Efficiency

6. The efficiency of a capacitor is given as:—

$$\text{Efficiency} = \frac{\text{Energy supplied on charging}}{\text{Energy obtained on discharging}} \times 100 \text{ (per cent.)}$$

The efficiency is always less than 100% because of:—

(*a*) The *dielectric losses* (Para. 5).

(*b*) *Skin effect* in the connecting leads and plates (see Sect. 2, Chap. 4, Para. 13).

(*c*) *Brush discharge;* this is an intermittent discharge (similar to a lightning discharge) from sharp corners of a capacitor plate into the surrounding air when the capacitor is charged to a high potential. For this reason, many high voltage capacitors have the corners of their plates rounded off.

Comparison of Capacitor Dielectrics

7. Table 1 gives a comparison of some of the more important dielectrics used in capacitors.

Dielectric	Losses	Dielectric Constant	Dielectric Strength	Remarks	Use
Air	Almost loss-free	Nearly 1	3kV/mm.	—	Variable tuning capacitors and trimmers.
Mica	Low losses	5·5	100kV/mm.	Can withstand high temperatures but expensive	Small fixed capacitors of good quality
Paper	Very high losses	3·7	15kV/mm.	The paper is absorbent. Very cheap and relatively compact	Large fixed capacitors where losses are of little importance
Polystyrene	Very low losses	2·5	75kV/mm.	Temperature limited to 60°C. Bulkier than paper equivalents.	A wide range of fixed capacitors. Suittable for rapid discharge circuits.
Ceramics	Very low losses	100	60kV/mm.	Can have positive or negative temperature coefficients	Small fixed capacitors. Used considerably in temperature compensating circuits.
High Dielectric Constant Ceramics	Low losses	3,000	5kV/mm.	Wide variation of capacitance with temperature	Miniature and sub-miniature capacitors

TABLE 1—COMPARISON OF DIELECTRICS

PART 1, SECTION 4, CHAPTER 2

CAPACITORS

Introduction

1. Capacitors are used for a wide variety of purposes in radio. In their construction, consideration must be given to the dielectric to be used, and in the choice of dielectric three factors are important:—

(*a*) The *dielectric constant* (see Chap. 1, Para. 22).

(*b*) The *dielectric strength.*

(*c*) The *losses* associated with the dielectric.

Dielectric Strength

2. This is a measure of the p.d. required to break down the dielectric. It is quoted as the voltage required to break down a one millimetre thickness of the dielectric and is given in *kilovolts per millimetre* (kV/mm). In any given dielectric the breakdown voltage will depend on:—

(*a*) The *material* from which the dielectric is made.

(*b*) The *thickness* of the dielectric; the thicker this is, the greater is the voltage required to break it down (although not proportionally).

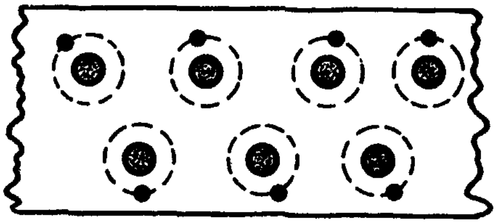
(*c*) The *temperature* of the dielectric; an increase in temperature gives a reduction in the breakdown voltage.

(*d*) The *frequency* of an applied alternating voltage. A higher frequency causes the electrons in the dielectric to alternate more rapidly and the resultant increase in temperature gives a reduction in the breakdown voltage. For example, a certain capacitor withstood a p.d. of 10,000V when the frequency was 100c/s; at a frequency of 10Mc/s, the capacitor broke down at 200V.

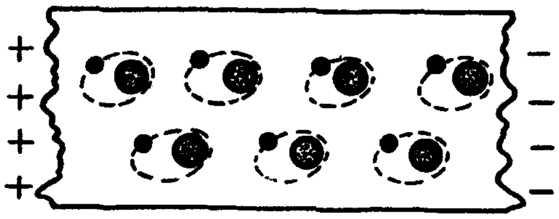
3. Because of the factors given in Para. 2, the voltage which a capacitor is capable of withstanding is sometimes given as a d.c. voltage at a certain temperature—e.g., "350V d.c. Working, 71°C".

Displacement Current

4. This occurs in dielectrics where, under certain conditions (such as that shown in Fig. 1) the orbital electrons will be attracted towards a point which is positive and the electron orbit becomes distorted. During the time that the electron orbit is changing there is a general movement of electrons in the dielectric. This constitutes a *displacement current.* If the applied external force is excessive, the electrons will be "pulled away" from their parent nuclei, resulting in breakdown of the dielectric.



NORMAL ELECTRON ORBIT



INSULATOR UNDER "STRAIN"

ELECTRON ORBIT DISTORTED

Fig. 1—DISPLACEMENT CURRENT.

Losses in the Dielectric

5. Of the energy supplied to a capacitor, a certain proportion is expended in the dielectric. The factors determining this energy loss are given below.

(*a*) No dielectric is a perfect insulator. Thus, when a p.d. is applied across a dielectric, a leakage current results.

(*b*) If the voltage applied to a capacitor is alternating, the continual alternate displacement of the electrons in the dielectric develops heat. Thus, some of the original energy supplied to the capacitor has been "lost" in heat energy.

(*c*) Some dielectrics exhibit a form of hysteresis similar to magnetic hysteresis. In *dielectric hysteresis* the electric flux density D ($= \frac{Q}{a}$ coulombs/square metre)

SECTION 4

CHAPTER 2

CAPACITORS

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Energy Stored in a Charged Capacitor

32. If a capacitor having a capacitance C farads is charged at a *constant* rate of I amps for t seconds, the charge Q = It coulombs. During charge, the p.d. across the capacitor will have risen from zero to V volts at a constant rate, so that the average p.d. during charge is $\frac{V - O}{2} = \frac{V}{2}$ volts.

33. The average power is the product of the average p.d. and the current.

i.e. $P = \frac{V}{2} \times I$ (watts)

The energy expended in charging the capacitor is stored in the charged capacitor and is given by:—

$$\text{Energy, } W = P \times t = \frac{V}{2} \times I \times t = \frac{VQ}{2}$$

$$\therefore \quad W = \tfrac{1}{2}VQ = \tfrac{1}{2}\frac{Q^2}{C} = \tfrac{1}{2}CV^2 \text{ (joules)} \quad (10)$$

Electrostatic Screening

34. In many cases in radio it is not desirable that an electric field be established between two components in a circuit since mutual interference may result. An *earthed* copper can is placed round the component whose electric field has to be confined; a field will then be established between the component and the can, and no electric flux from the component will exist outside the can. Similarly, no external lines of flux will reach the component which is screened, all such lines terminating on the earthed can.

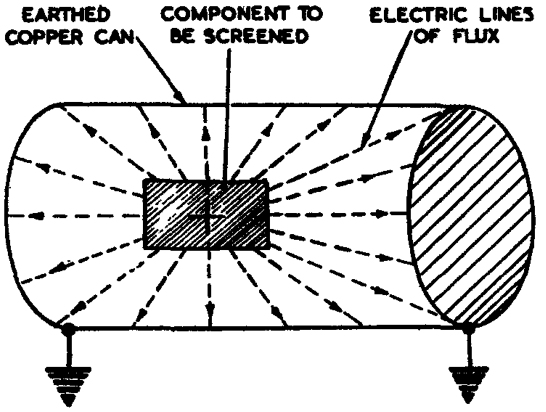

Fig. 16—ELECTROSTATIC SCREENING CAN.

Summary of Capacitors

23. Table 2 gives the main points on the capacitors discussed in this Chapter. Fig. 14 shows a selection of typical capacitors used in radio.

Type	Capacitance Values	D.c. Working Voltage	Remarks
Paper	0·0002μF to 12μF	up to 150kV	Used in circuits where losses are not important; cheap
Stacked Mica	50pF to 0·25μF	up to 2kV	Used in low-loss circuits: expensive
Silvered Mica	10pF to 0·01μF	up to 750V	Used in low-loss, precision circuits
Ceramic	0·5pF to 0·005μF	up to 500V	Used in low-loss, precision circuits where miniaturisation is important or where temperature compensation is required
Polystyrene	10pF to 4μF	up to 500V	Superior to mica type, but more expensive and bulky.
Electrolytics	2μF to 32μF 32μF to 3,000μF	up to 600V up to 50V	Used where losses are not important, e.g., Smoothing. A polarizing d.c. voltage must be operative in the circuit.
Variable	15pF to 500pF	up to 2kV	Used for circuit tuning
Trimmer	2μF to 150pF	up to 350V	Used for circuit alignment.

TABLE 2—SUMMARY OF CAPACITORS

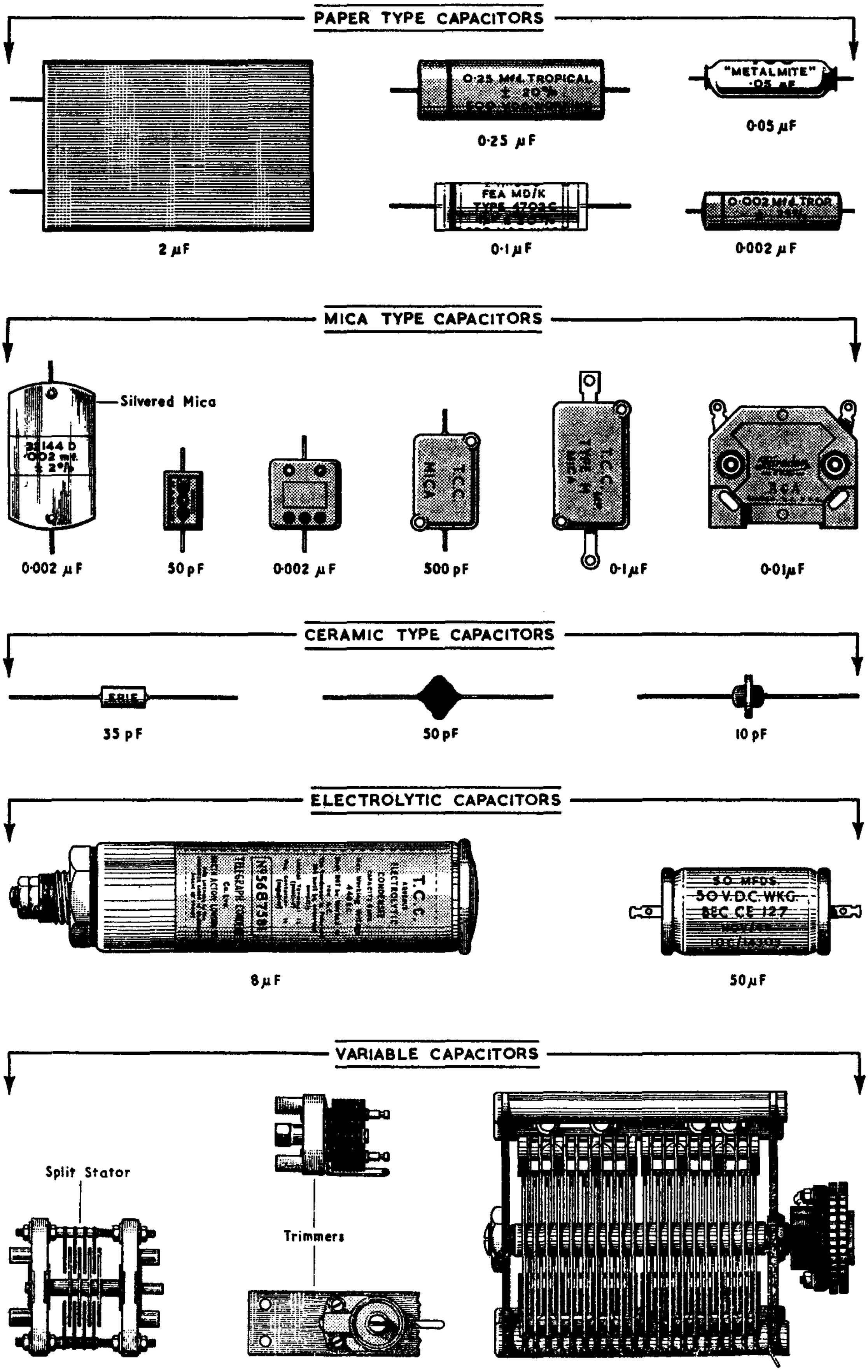
PAPER TYPE CAPACITORS
2 µF
0·25 Mf4 TROPICAL
± 20%
0·25 µF
"METALMITE"
·05 µF
0·05 µF
FEA MD/K
TYPE 4702C
0·1 µF
0·002 Mf4 TROP
0·002 µF
MICA TYPE CAPACITORS
Silvered Mica
·002 mf.
± 2%
0·002 µF
50 pF
0·002 µF
T.C.C.
MICA
500 pF
T.C.C.
TYPE
MICA
0·1 µF
T.C.C.
0·01 µF
CERAMIC TYPE CAPACITORS
ERIE
35 pF
50 pF
10 pF
ELECTROLYTIC CAPACITORS
T.C.C.
ELECTROLYTIC
CONDENSER
TELEGRAPH CONDENSER
8 µF
50 MFDS.
50 V.D.C. WKG.
BEC CE 127
50 µF
VARIABLE CAPACITORS
Split Stator
Trimmers

Fig. 14—TYPICAL CAPACITORS USED IN RADIO.

SECTION 4

CHAPTER 3

CAPACITIVE CIRCUITS

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PART 1, SECTION 4, CHAPTER 3

CAPACITIVE CIRCUITS

Introduction

1. Fig. 1 shows a capacitor of capacitance C farads connected in series with a resistor of resistance R ohms across a supply of e.m.f. E volts, via a switch.

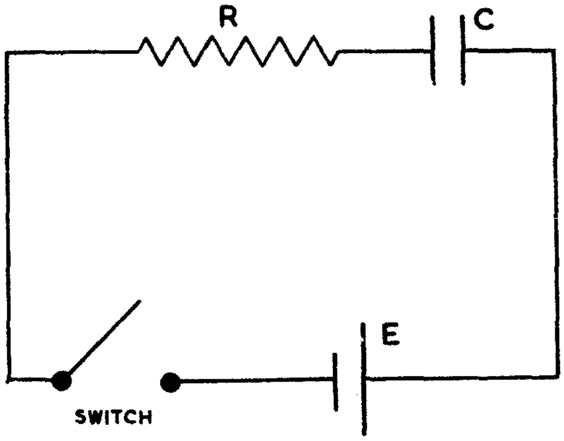

Fig. 1.—CIRCUIT TO SHOW THE CHARGE OF A CAPACITOR.

2. (a) The capacitor is initially uncharged and has zero p.d. across its terminals. From Kirchhoff's second law, on closing the switch, the p.d. across the resistor is E volts, and the current through the resistor $I = \frac{E}{R}$ amps. Thus, at the instant of closing the switch, the instantaneous current *i* is a maximum, the p.d. across the resistor V_R is a maximum, the p.d. across the capacitor V_C is zero — the sum of the p.d.s across R and across C being at all times equal to the applied voltage. Since the capacitor has no effect at this instant, it can be looked upon initially as a virtual short circuit.

(*b*) With a current established in the circuit, the capacitor is charging and V_C is rising. From Kirchoff's second law:—

$$E = V_R + V_C$$
$$\therefore \quad V_R = E = V_C$$

Thus, V_R is falling; and since $i = \frac{V_R}{R}$, the current in the circuit is falling too.

(*c*) V_C, V_R, and *i* vary in an *exponential* manner, the process continuing until the capacitor is fully charged; V_C will then equal the supply voltage, and V_R and *i* will be zero.

Experimental Study of the Charge of a Capacitor

3. The facts stated in Para. 2 can be verified by means of a simple experiment. In the circuit of Fig. 2, $C = 2\mu F$, $R = 5M\Omega$, and $E = 120V$; M_1 is an instrument for measuring the current in micro-amps, and M_2 an instrument for measuring the p.d. across the capacitor in volts. A stop-watch is required to give accurate measurement of time in seconds.

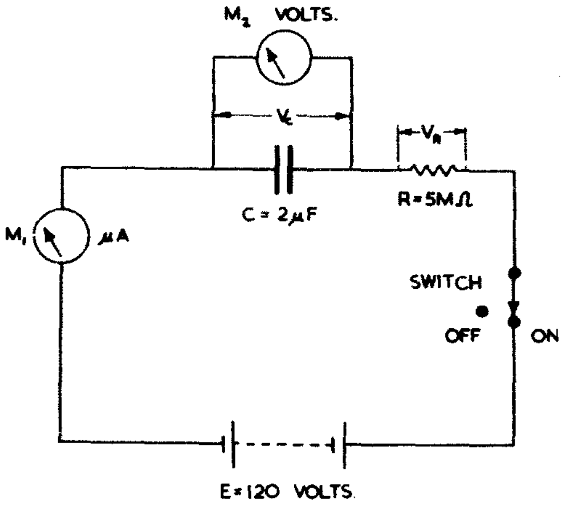

Fig. 2.—CIRCUIT FOR EXPERIMENT.

4. Readings of the current *i* passing through M_1, and the p.d. across the capacitor V_C (measured by M_2) are taken at intervals of 10 seconds after the closing of the switch. A typical set of readings is given in Table 1, and from these readings the graphs of Fig. 3 and Fig. 4 are plotted.

Time from start of charging (sec)	0	10	20	30	40	50	60
Current *i* (μ A) ..	24	8·5	3·2	1·2	0·6	0·2	Very small
P.d. V_c (volts) ..	0	76	104	114	117	119	nearly 120

TABLE 1

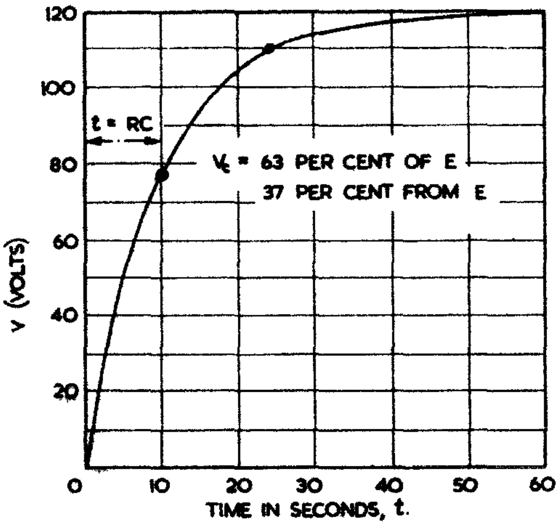

Fig. 3—VARIATION OF V_c DURING CHARGE.

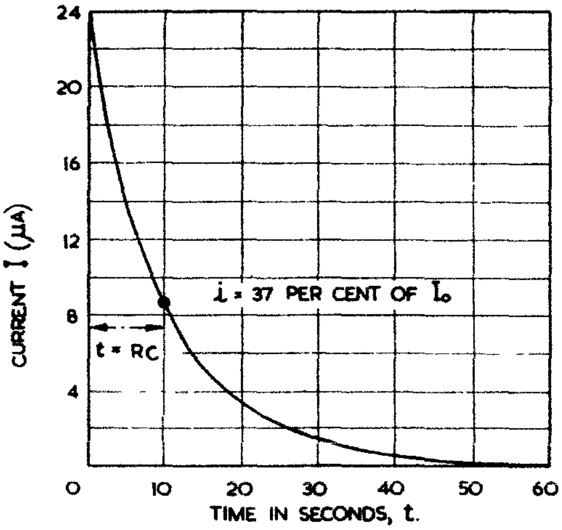

Fig. 4—VARIATION OF i DURING CHARGE.

5. The experiment could be repeated with different values of C and R. It would be found that with a *smaller* value of R, the 2μF capacitor would charge more rapidly. This is to be expected since the initial current $\frac{E}{R}$ would be larger. A similar result would be seen with a *smaller* value of C and the 5 MΩ resistor, for then the total charge required to produce a given voltage across C would be less (Q = CV). Reducing both C and R together would speed up the operation for both these reasons. It is seen that the product CR is what determines the rate of growth of the p.d. across the capacitor and so its rate of charging.

6. It is found that whatever the actual values of C and R in such an experiment, the product CR equals the time in seconds for the voltage across the capacitor to rise to 63·2% of the applied voltage. In the circuit of Fig. 2, $CR = 2 \times 10^{-6} \times 5 \times 10^{6} =$ 10 sec. Thus, in 10 seconds from the start the p.d. across the capacitor V_C rose to 76 volts; this is 63·2% of 120 volts. The figures for the charging current show that after time CR = 10 seconds the current has fallen by 63·2% from 24μA, to about 8·5μA which is 36·8% of the initial value. This quantity CR is termed the *time constant* of the circuit and is defined in full in Para. 16.

General Case of the Charge of a Capacitor

7. Paras. 3 to 6 have dealt with a particular circuit. In the general case, for purposes of accurate calculation, the p.d. across the capacitor at any instant t after closing the switch is given by;—

$$V_C = E\left(1 - \epsilon^{-\frac{t}{CR}}\right) \text{ (volts)} \quad .. \quad .. \quad (1)$$

where, V_C = the p.d. across the capacitor at any instant

E = the applied voltage
ϵ = Napierian log base = 2·718
t = time in seconds after closing the switch
C = capacitance in farads
R = resistance in ohms

8. (a) From Kirchhoff's second law:—

$$E = V_R + V_C$$

$$\therefore V_R = E - V_C \text{ (volts)} \quad .. \quad .. \quad .. \quad (2)$$

(*b*) From Ohm's law:—

$$i = \frac{V_R}{R} = \frac{E - V_C}{R} \text{ (amps)} \quad .. \quad .. \quad (3)$$

9. A graph can be plotted showing the variation in V_C, V_R or *i* with respect to the time t seconds after closing the switch. There are two ways of doing this:—

(*a*) Repeat the experiment of Paras. 3 to 6. Having obtained the values for V_C and *i*, the corresponding values for V_R follow from equation (2).

(*b*) Insert the values for E, C, and R in equation (1) and thence calculate for V_C at various instants of time t seconds. From equations (2) and (3) respectively, the corresponding values for V_R and *i* can be obtained.

10. In either case, three instants of time are sufficient for most purposes:—

(*a*) *At the instant of closing the switch* ($t = o$)

$$V_C = 0 \ : \ V_R = E \ : \ i = \frac{E}{R}$$

(*b*) *At t = CR seconds after closing the switch*

$$V_C = 0{\cdot}632E \ : \ V_R = 0{\cdot}368E \ : \ i = 0{\cdot}368\frac{E}{R}$$

(*c*) *At t = 5 CR seconds after closing the switch*

$$V_C \simeq E \ : \ V_R \simeq 0 \ : \ i \simeq 0$$

11. These three instants of time are used to plot the graph showing the exponential rise in the p.d. across the capacitor V_C, and the fall in the p.d. across the resistor V_R against the time t in seconds after closing the switch (Fig. 5). The graph for the current *i* will fall in a manner similar to that for V_R.

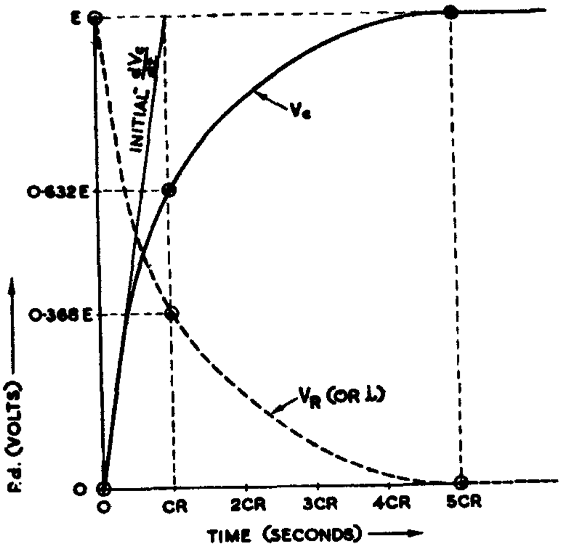

Fig. 5—VARIATION OF V_c AND V_R DURING CHARGE.

Discharge of a Capacitor

12. Consider the circuit shown in Fig. 6. With the circuit switched on, V_C will rise exponentially to its maximum value E volts in a time of approximately 5CR seconds; the p.d. across the resistor and the current in the circuit will both be zero after this time. Assume the capacitor to be now fully charged and the switch put to the off position. The capacitor is now connected across the resistor and will commence to discharge. As soon as the capacitor partly discharges, V_C falls and the current is reduced—i.e., the *rate* of discharge decreases. The curve of the discharge of the capacitor can again be obtained from a simple experiment. A stop-watch is started at the instant of putting the switch to the OFF position and readings of V_C and *i* taken at regular intervals of time. The graphs resulting from such a set of readings will show that the discharge of the capacitor is exponential.

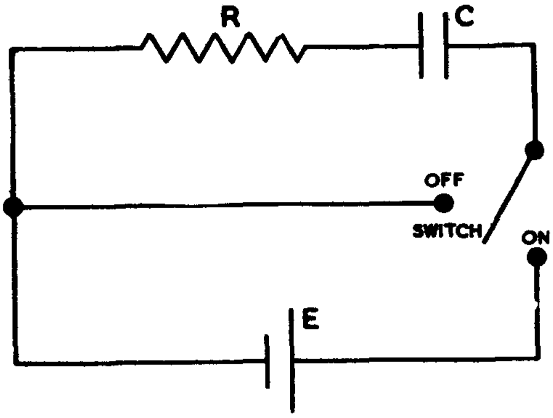

Fig. 6—CIRCUIT TO SHOW THE DISCHARGE OF A CAPACITOR.

13. (*a*) For purposes of accurate calculation, the p.d. across the capacitor at any instant t after disconnecting the supply is given by:—

$$V_C = E.\epsilon^{-\frac{t}{CR}} \text{ (volts)} \quad \dots \quad \dots \quad (4)$$

where all the terms have the same significance as in equation (1).

(*b*) From Kirchhoff's second law, the sum of the p.d.s in the circuit must equal the applied voltage (which is now zero).

$$\therefore V_R + V_C = 0$$

$$\therefore V_R = - V_C \text{ (volts)} \quad \dots \quad \dots \quad (5)$$

(*c*) From Ohm's law:—

$$i = \frac{V_R}{R} = -\frac{V_C}{R} \text{(amps)} \dots \quad \dots \quad (6)$$

14. A graph van be plotted showing the variation in V_C, V_R, or *i* with respect to the time t seconds after disconnecting the supply. Either of the two methods described in Para. 9 can be used to obtain such a graph and again three instants of time are significant:—

(*a*) *At the instant of disconnecting the supply (t = o)*

$$V_C = E \ : \ V_R = -E \ : \ i = -\frac{E}{R}$$

(*b*) *At t = CR seconds after disconnecting the supply*

$$V_C = 0{\cdot}368E \ : \ V_R = -0{\cdot}368E \ : \ i = -0{\cdot}368\frac{E}{R}$$

(*c*) *At* t = *5CR seconds after disconnecting the supply*

$V_C \simeq 0$: $V_R \simeq 0$: $i \simeq 0$

15. These three instants of time are used to plot the graph showing the variations in the p.d.s across the capacitor and across the resistor with respect to the time t seconds (Fig. 7). The graph for the current *i* will vary in a manner similar to that for V_R.

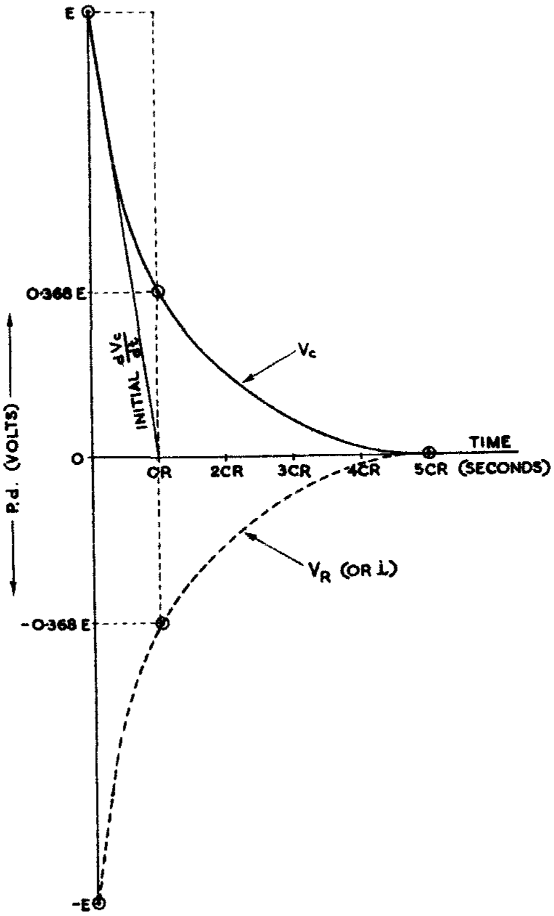

Fig. 7—VARIATION OF V_C AND V_R DURING DISCHARGE.

Time Constant

16. The time t = CR seconds is termed the **time constant** of a capacitive circuit and has been referred to in Para. 6. It is defined as follows:—

The time constant t = CR seconds is the time taken for the p.d. across the capacitor to rise to 63·2% (approximately two-thirds) of its maximum value on charge, or to fall by 63·2% of its maximum value on discharge. Alternately, it is the time taken for the p.d. across the capacitor to rise to its **maximum** *value on charge, or to fall to* **zero** *on discharge, provided the* **initial** *rate of change of voltage is maintained.*

(The latter is shown in the graphs although it cannot apply in practice).

17. In theory, a capacitor would take an infinitely long time to completely charge or discharge. However, after a time of *5CR seconds* the charge or discharge is so nearly complete as to be considered so for practical purposes.

Practical Example

18. In the circuit shown in Fig. 8, C = 2μF, R = 50kΩ, and E = 100V. The circuit is switched on for one second and then switched off. It is required to sketch graphs to indicate how the p.d.s across the capacitor and across the resistor vary with time.

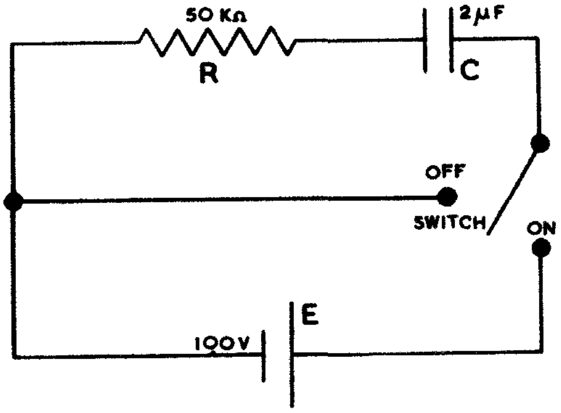

Fig. 8—EXAMPLE.

19. (*a*) The time constant t = CR = $2 \times 10^{-6} \times 50 \times 10^3 = 0{\cdot}1$ seconds.

(*b*) The p.d.s at the relevant instants of time are given in the table.

	Time (seconds)	0	CR 0·1	5CR 0·5
During charge	V_C (volts)	0	+63·2	+100
	V_R (volts)	+100	+36·8	0
During discharge	V_C (volts)	+100	+36·8	0
	V_R (volts)	—100	—36·8	0

(*c*) The required graphs are given in Fig. 9.

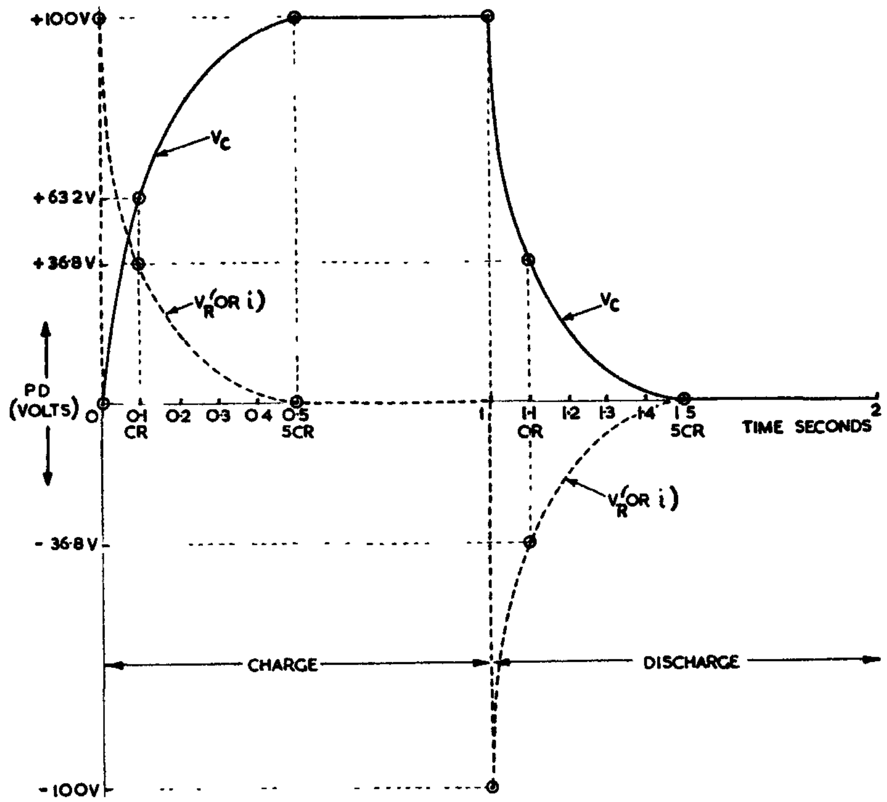

Fig. 9—VARIATION OF V_C AND V_R DURING CHARGE AND DISCHARGE, EXAMPLE.

Square Waves Applied to a Capacitive Circuit

20. The preceeding paragraphs have shown the effect of charging a capacitor from a d.c. supply and then allowing the capacitor to discharge. In some radio circuits it is more important to consider the effect of applying a *square wave* of voltage to a capacitive circuit.

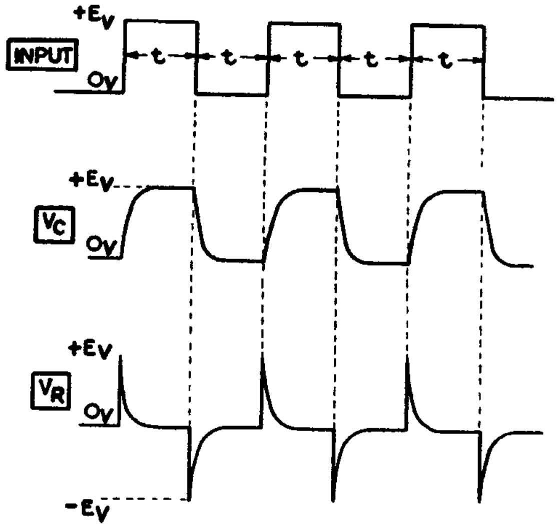

Fig. 10—SQUARE WAVE APPLIED TO A CAPACITIVE CIRCUIT.

21. Consider a capacitor connected in series with a resistor across a square wave input. Provided the time t seconds taken for one half cycle of the square wave input is much *longer* than the time constant of the circuit, the p.d.s across the capacitor and across the resistor will vary in the manner described in Para. 19 and as shown in Fig. 10.

Current Through a Capacitor

22. It has been assumed so far in this Publication that there must be a complete circuit in order that current may flow, yet the dielectric of a capacitor is necessarily an insulator. The process of charging is indicated in Fig. 11, where the charging of the positive plate A is represented by a flow of positive charges towards it. (Really, of course, electrons are led away in the opposite direction, but in terms of conventional current direction the flow of positive charges is considered). A few representative neutral atoms are shown on the other plate B. As positive charges reach A, equal negative charges are *induced* in B, and equal positive charges repelled away from B. That is, during charge, the current flowing away from B is exactly equal to the current flowing towards A, which is just what would happen

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if there were a complete circuit. Actual charges do *not* traverse the dielectric; but the electric flux from the charges on A does so and establishes the field between the plates to induce equal and opposite charges on B. There is also a displacement current in the dielectric (see Chap. 2, Para. 4).

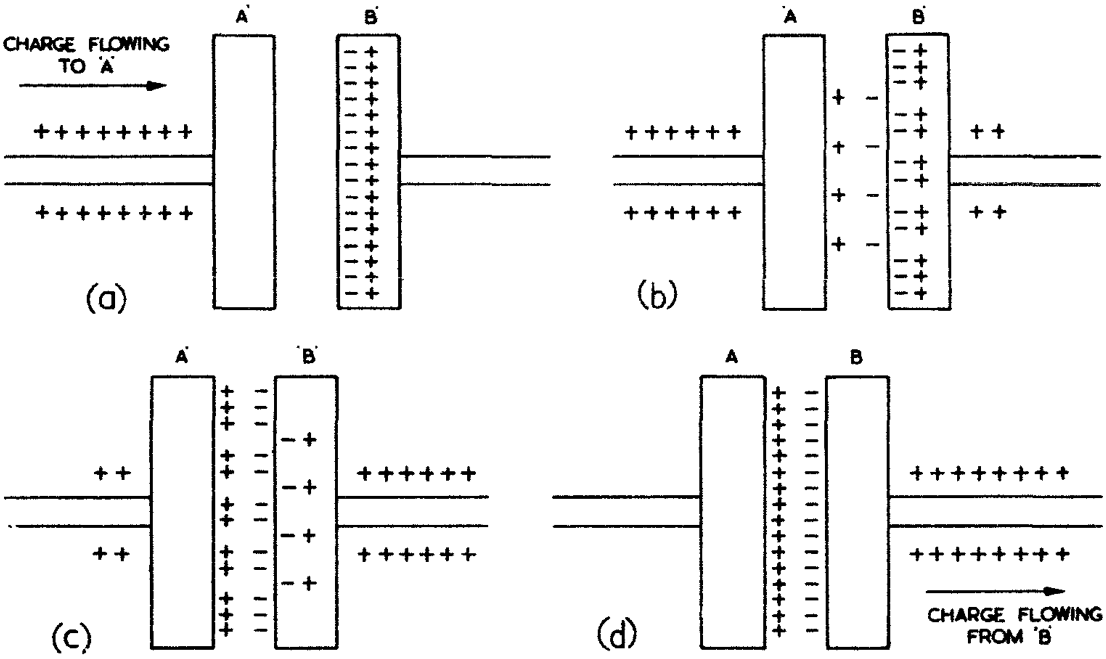
A
B
CHARGE FLOWING TO 'A'
(a)
(b)
(c)
(d)
CHARGE FLOWING FROM 'B'

Fig. 11—CURRENT THROUGH A CAPACITOR.

SECTION 5

A.C. THEORY

Chapter 1 .. .. **Alternating Current Theory**
Chapter 2 .. .. **Series A.C. Circuits**
Chapter 3 .. .. **Parallel A.C. Circuits**
Chapter 4 .. .. **Polyphase A.C. Systems**
Chapter 5 .. .. **The Operator "j"**

SECTION 5

CHAPTER 1

ALTERNATING CURRENT THEORY

Paragraph

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Definitions 2
Alternating Current Values 3
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PART 1, SECTION 5, CHAPTER 1

ALTERNATING CURRENT THEORY

Introduction

1. The production of an alternating e.m.f. by a generator was discussed in Sect. 3, Chap. 1. It was there shown that an alternating current or voltage is one which periodically reverses its direction in a regular recurring manner. Fig. 1 shows the graph of such a waveform, the vertical axis representing voltage and the horizontal axis, time.

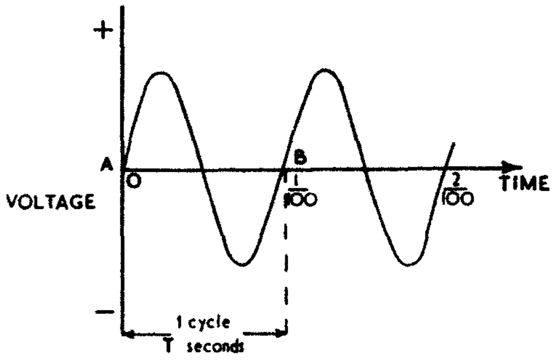


Fig. 1. ALTERNATING VOLTAGE

Definitions

2. In alternating quantities, the following definitions are important (see Fig. 1):—

(*a*) **Cycle.** The complete sequence of events through both positive and negative values, such as from A to B; it is followed by another cycle exactly the same as the first.

(*b*) **Periodic time (or period).** The time taken for the execution of one cycle and denoted by the symbol T. In Fig. 1, $T = \frac{1}{100}$ second.

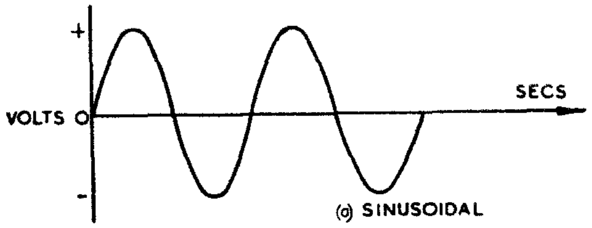
(*c*) **Frequency.**

(i) The number of cycles completed in one second and denoted by the symbol f. Since in Fig. 1, one cycle takes $\frac{1}{100}$ second the frequency is 100 cycles per second (c/s), the frequency and the period being related by the expression:—

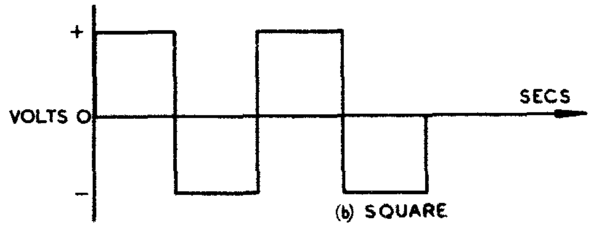
$$f = \frac{1}{T} \text{ (c/s)} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

(ii) The frequency may have any value. For convenience, the range is roughly divided into bands. Frequencies below 100 c/s are normally used for commercial power supply distribution; in Great Britain, a frequency of 50 c/s is used for this. The "audio frequency" (a.f.) band—i.e., the range of frequencies audible to the human ear—lies between approximately 20 c/s and 20,000 c/s (20 kc/s.). Above this range, up to several thousand megacycles per second, is the "radio frequency" (r.f.) band which is used for all forms of radio communication.

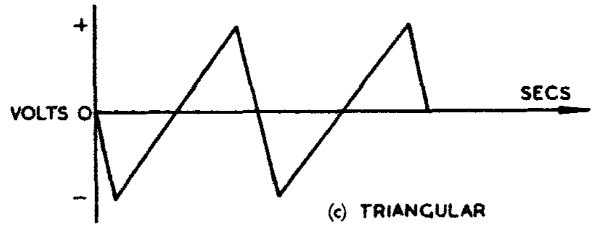
(*d*) **Waveform.** The shape of the alternating voltage or current graph. Fig. 2 shows three voltages that have the same frequency but different waveforms.



(a) SINUSOIDAL



(b) SQUARE



(c) TRIANGULAR

Fig. 2. A.C. WAVEFORMS

A.C. Values

3. Fig. 3 shows a "sinusoidal" waveform with various values annotated. These values are described below:—

(*a*) **Instantaneous value.** The value of an alternating quantity is continuously changing. Its instantaneous value is that at any given instant of time. Instantaneous current is denoted by the symbol *i* and instantaneous voltage by *e*.

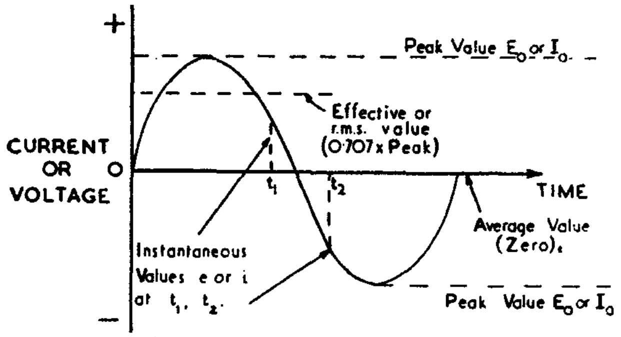

+
Peak Value E_o or I_o
Effective or r.m.s. value (0·707 x Peak)
CURRENT OR VOLTAGE
O
t_1
t_2
TIME
Average Value (Zero).
Instantaneous Values e or i at t_1, t_2.
Peak Value E_o or I_o
—

Fig. 3. A.C. VALUES

(*b*) **Peak value (or amplitude).** The amplitude of an alternating quantity is the maximum value reached (either positive or negative) during a cycle. Peak current is denoted by I_o and peak voltage by E_o.

(*c*) **Average or mean value.** This value, over any number of complete cycles, is zero. The area between the curve and the time axis represents the quantity of electricity that has passed during the interval concerned. As the curve is perfectly symmetrical about this axis, the quantity of electricity that passes one way during the first half-cycle exactly equals that passing the other way during the next half-cycle. The net transfer of electricity, and hence the average current, is zero.

(*d*) **Effective or Root-Mean-Square (R.m.s.) Value.**

(i) The heating effect of an electric current is independent of the direction of the current, and the *effective* value of an alternating current is I amps if it has the same heating effect as an unvarying direct current of I amps.

(ii) It was shown in Sect. 1, when considering d.c. circuits, that the rate of dissipation of energy as heat in a resistor depended upon the *square* of the current or voltage, —i.e., $P = I^2R = \frac{V^2}{R}$ (watts). This applies also to a.c. circuits. Fig. 4 illustrates this graphically. The square of a sinusiodal current plotted against time, gives a curve which is always above the time axis and is symmetrical about the half-way line $\frac{I_o^2}{2}$. The energy dissipated over any time interval is proportional to the shaded area beneath the curve, namely (current)². In this graph, crest *a* will fit into trough *a*, crest *b* into trough *b*, and so on, so that the energy dissipated over any time interval is represented by the area beneath the *half-way line.* This is the same energy as would be dissipated by a steady value $\frac{I_o^2}{2}$.

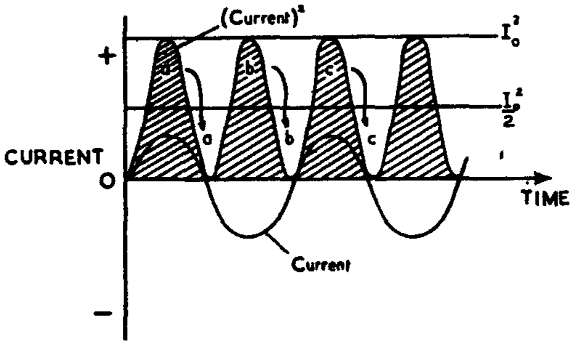


Fig. 4. EFFECTIVE OR R.M.S. VALUE

(iii) Therefore, the value of the alternating current which gives the same heating effect as an unvarying direct current is $\sqrt{\frac{I_o^2}{2}} = \frac{I_o}{\sqrt{2}}$. It is known either as the *effective* value (since it is that value which has the same effect as an equivalent direct current), or as the *root-mean-square* (r.m.s.) value (since it is obtained by finding the mean value of the square of the current and then taking the square root). If I is the r.m.s. value of current in a circuit it is related to its peak value I_o by the expression:—

$$I = \frac{I_o}{\sqrt{2}} = 0{\cdot}707\, I_o \text{ (amps) (2)}$$

Also, $I_o = \sqrt{2}I = 1{\cdot}414\, I$ (amps)

(iv) Alternating currents and voltages are usually measured by their r.m.s. values. For instance, the normal a.c. mains supply has a voltage of 230V; this is an r.m.s. voltage; the peak voltage is $230 \times \sqrt{2}$ which equals 325·3V.

Fourier's Theorem

4. The mathematical aspect of Fourier's theorem is complex and beyond the scope of this Publication. This theorem states that *any recurrent waveform of frequency f can be resolved into the sum of a number of sinusoidal waveforms having frequencies f, 2f, 3f ---- The number of sine waves may be finite or infinite.*

5. For example, any steady note in the audio frequency range can be split up into a *fundamental* and *harmonics*, the fundamental (or first harmonic) having frequency f, the second harmonic 2f, the third harmonic 3f, and so on. This theorem, therefore, reduces any recurrent waveform to a number of sine waves. The square wave for instance, consists of a fundamental and all the *odd* harmonics up to infinity. Suppose the amplitude of the fundamental is 1; then the amplitude of the third harmonic is $\frac{1}{3}$, that of the fifth harmonic is $\frac{1}{5}$, and so on. Fig. 5 shows the result of taking up to the fifth harmonic; it will be seen that this gives a good approximation to a square wave.

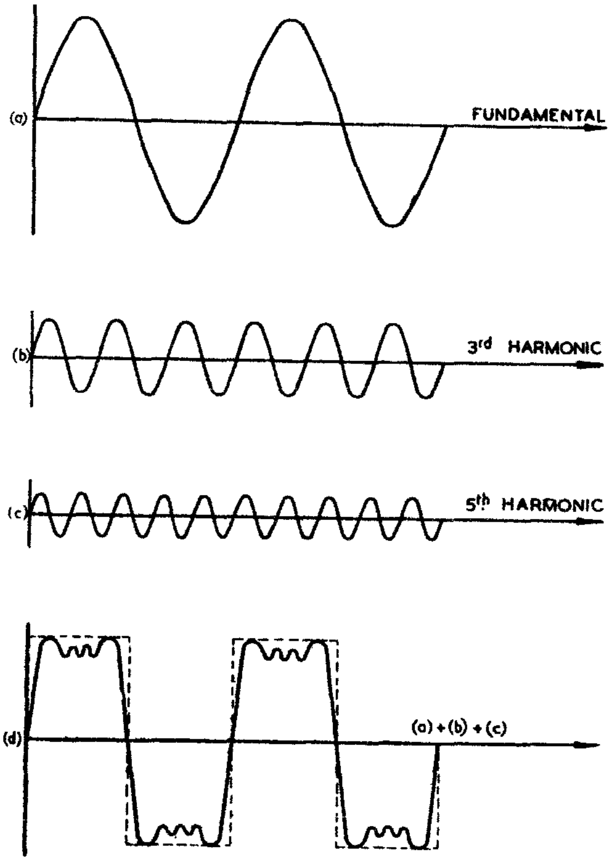


Fig. 5. COMPOSITION OF A SQUARE WAVE.

6. Since any recurrent waveform can be reduced to a number of sine waves, the theory developed in this Section will be based on the sinusoidal waveform.

Sinusoidal Waveform

7. Consider a conductor loop rotating in an anti-clockwise direction at constant speed in a uniform magnetic field, as shown in Fig. 6. It was shown in Sect. 3, Chap. 1 that when such a loop is in the neutral plane, at right angles to the magnetic field ($\theta = 0°$), the e.m.f. induced in the loop at this instant is zero. When the loop is at right angles to the neutral plane ($\theta = 90°$) the e.m.f. is a maximum, its value being dependent on the speed of rotation and on the flux density. At intermediate points, the induced e.m.f. is between zero and its maximum value and depends on the angle which the loop makes to the neutral plane. Thus, the e.m.f. induced in the loop at any instant is proportional to the sine of the angle (sin θ) through which the loop has rotated from the neutral plane. Hence at any instant:—

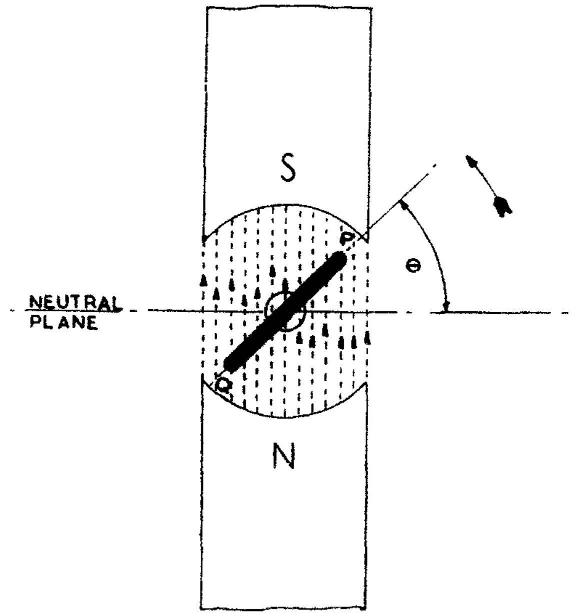


Fig. 6. DEPENDENCE OF E.M.F. ON SIN

$$e = E_0 \sin \theta \text{ (volts)} \ldots\ldots\ldots\ldots (3)$$

For example, when the loop is in the neutral plane, $\theta = 0°$, $\sin \theta = 0$, and $e = E_0 \sin \theta =$ zero. When the loop is at right angles to the neutral plane, $\theta = 90°$, $\sin \theta = 1$, and $e = E_0 \sin \theta = E_0$.

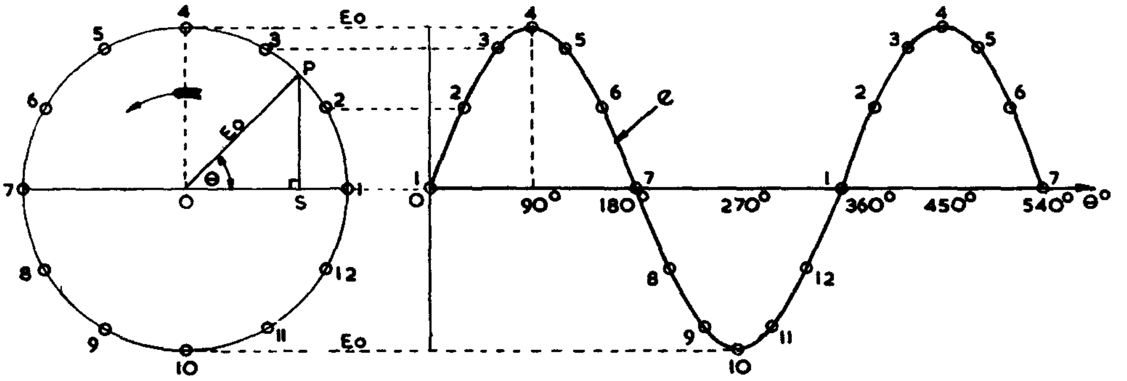


Fig. 7. PLOTTING A SINE WAVE

8. To plot a curve showing the instantaneous value of the e.m.f. for all values of the angle θ, consider Fig. 7. The line OP is assumed to rotate about the point 0 in an *anti-clockwise* direction, its length representing the maximum value of the e.m.f. E_o to any convenient scale. A horizontal line is drawn through the centre of rotation 0, and upon this, a scale of degrees of rotation is set up.

Now:—

$$\sin \theta = \frac{PS}{OP}$$

$$\therefore PS = OP \sin \theta = E_o \sin \theta.$$

Thus, the vertical line PS represents to scale the instantaneous e.m.f. $e = E_o \sin \theta$. As OP rotates, the length of the line PS varies, and plotting this against the various values of the angle θ on the horizontal axis, the graph of the instantaneous e.m.f. is obtained. This curve is termed a *sine curve* and any quantity which varies in this manner is said to have a *sinusoidal waveform.*

Angular Velocity

9. Angle θ can be expressed in such a way that the instantaneous value of the e.m.f. is made to depend upon time. To do this, express the angle in *radians* instead of in degrees. *A radian is the angle at the centre of a circle subtended by an arc of the circumference equal in length to the radius of the circle* (Fig. 8). The circumference of a circle contains 360 degrees and its length is 2 π times its radius. Thus, rotation through 360 degrees is the same as rotation through 2π radians, so:—

$$360° = 2\pi \text{ radians; } 180° = \pi \text{ radians;}$$

$$90° = \frac{\pi}{2} \text{ radians; and one radian is}$$

$$\frac{360}{2\pi} = \frac{360}{6 \cdot 28} = 57 \cdot 3°.$$

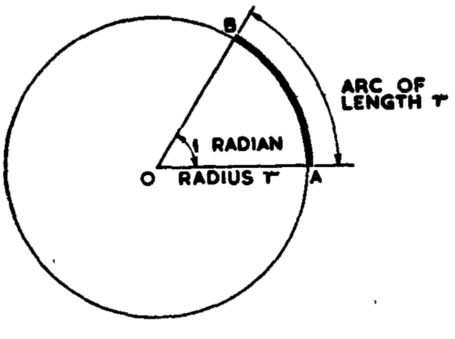


Fig. 8. RADIAN MEASURE

10. In performing one revolution the loop passes through 360° or 2π radians. If it rotates at f revolutions per second it passes through 2πf radians per second; this is termed the *angular velocity* of the loop and is denoted by the Greek letter ω. Thus:—

$$\text{Angular velocity} = \omega \text{ (radians/sec)} \quad .. \ (4)$$

After an interval of t seconds from the commencement of rotation the loop has rotated through an angle θ equal to 2π f t = ωt radians, and the e.m.f. at this instant is:—

$$\epsilon = E_o \text{ Sin } \theta$$

$$= E_o \text{ Sin}\omega t \quad \text{(volts)} \quad .. \quad .. \ (5)$$

Similarly, with an alternating current in a circuit, the value of the current at any instant t is:—

$$i = I_o \sin \omega t \text{ (amps)} \ .... (6)$$

Phase Difference

11. When two alternating quantities of the *same frequency* pass through corresponding points in a cycle at the *same instant* of time they are said to be *in phase* with each other (Fig. 9(*a*)). If they pass through corresponding points at *different* instants of time there is a *phase difference* between them and one is said to be *leading* or *lagging* on the other by a certain *phase angle.* For

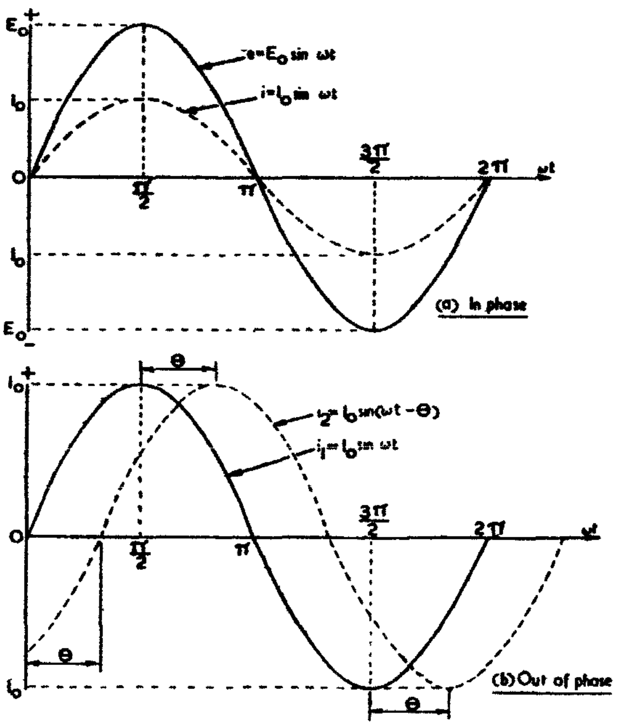


Fig. 9. PHASE DIFFERENCE BY GRAPHS

example, in Fig. 9(*b*), i_1 is leading i_2 by θ radians (or i_2 is lagging i_1 by θ radians). Thus i_1 reaches its maximum value, θ radians before i_2.

12. The information in the graph of Fig. 9(*b*) can also be conveyed by trigonometrical equations. For instance:—

$$i_1 = I_0 \sin \omega t$$
$$i_2 = I_0 \sin (\omega t - \theta).$$

indicates that the two currents have the same amplitude I_0 and the same angular velocity ω, but i_2 is lagging i_1 by θ radians.

Vectors

13. **Introduction.** Neither the graphical nor the trigonometrical method of representing alternating quantities is entirely satisfactory, particularly when relative phase has to be shown. Further, the process of adding two sine waves graphically is tedious as will be seen from examination of Fig. 10 in which two components of current i_1 and i_2, having the same frequency but different amplitudes and phases, are acting together in a circuit. The resultant current $(i_1 + i_2)$ is obtained by adding the individual values of the two curves (a) and (b) at each instant of time. A more simple, speedy and accurate method is to represent alternating quantities by *vectors.*

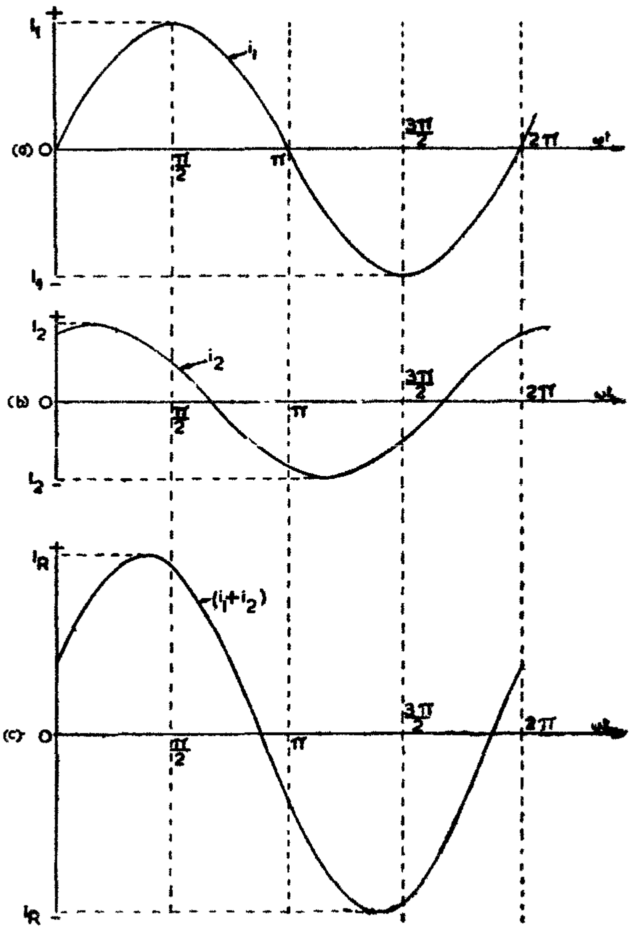


Fig. 10. GRAPHICAL ADDITION OF SINE WAVES

14. **Scalars and vectors.** All measurable quantities can be classified either as *scalars* (having magnitude only) or as *vectors* (having both magnitude and direction). Thus, energy is a scalar quantity, whereas force is a vector quantity. The addition and subtraction of scalars follows the ordinary rules of arithmetic. If a system having 3 joules of energy is given an additional 4 joules, the resultant energy is 7 joules. Vectors however, are added and subtracted geometrically. For instance, if two forces act on a body, one being 4 newtons due North and the other 3 newtons due East the resultant force is *not* 7 newtons, but 5 newtons at an angle θ from due North. This is shown in Fig. 11 where the line OA of length 4 units is the vector representing the force of 4 newtons acting due North on a body situated at O. The line OB of length 3 units is the vector representing the force of 3 newtons acting due East on the body. The line OC is the resultant vector. This is obtained from the *parallelogram of forces rule* which states:—

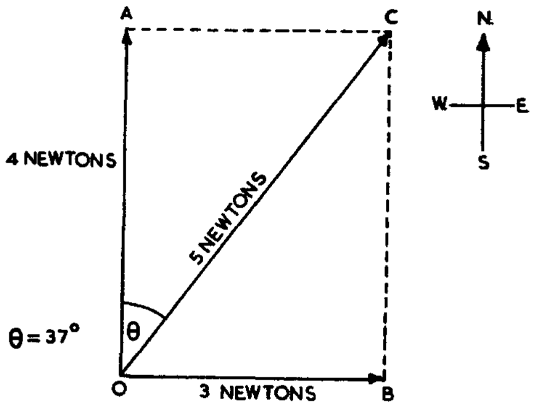


Fig. 11. VECTOR ADDITION

If OA and OB represent in size and direction two forces acting at a point O, their resultant is represented in size and direction by the diagonal OC of the parallelogram OACB.

Vector OC is found to be 5 units long and represents the resultant force of 5 newtons acting on the body in a direction angle θ (= 37°) from due North.

15. **Subtraction of vectors.** The result of subtracting a vector OB from a vector OA can be obtained by regarding "OA — OB" as "OA + (−OB)" as in Fig. 12. Thus, draw the vectors OA and OB; then draw a vector equal in magnitude to OB but contrary in direction (i.e. making 180° to its true direction). This vector is equal to (− OB). Finally, perform the addition of the vectors OA and (− OB) by the parallelogram of forces rule to give the resultant vector OC.

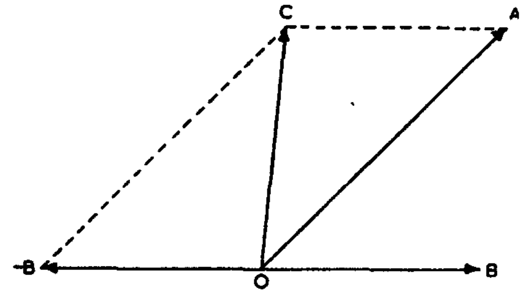


Fig. 12. SUBTRACTION OF VECTORS

Vector Representation of Alternating Quantities

16. In the special application of vectors to alternating quantities, the vector is assumed to be fixed at its origin and to rotate at the frequency of the alternation, the length of the vector being equal to the peak value of the alternating quantity. For example, consider a voltage $e = E_0 \sin \omega t$. This voltage may be represented by a vector of length corresponding to E_0 rotating in an *anti-clockwise* direction with reference to a datum or reference line with angular velocity ω radians per second. This is shown in Fig. 13 where, after time t seconds, the vector has rotated through an angle of ωt radians. The instantaneous value of e is shown to the same scale as the vector E_0 by the line PQ (see Para. 8).

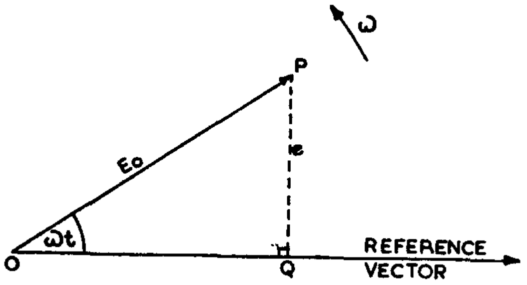


Fig. 13. VECTOR REPRESENTATION OF A.C.

17. While the voltage E_0 throughout one cycle would be represented by a complete series of lines OP, OP_1, OP_2, etc. (Fig. 14), it is usually sufficient to show one position of the vector, for its rotation with angular velocity ω is understood. It is usual to take the instant t = O, when OP in Fig. 14 lies along the reference line to represent the voltage $e = E_0 \sin \omega t$.

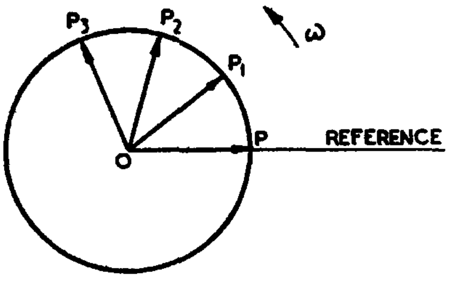


Fig. 14. VECTOR CONVENTION

Phase Difference on the Vector Diagram

18. The vector diagram is particularly useful in showing phase differences. Fig. 15(*b*) shows the graphs for a voltage of peak value E_0 and a current of peak value I_0, the voltage leading the current by θ radians. Such graphs could be drawn by considering two vectors of length E_0 and I_0 respectively rotating in an anti-clockwise direction as shown in Fig. 15(*a*). To introduce the

20. An *impedance triangle* can be constructed from the vector diagram of Fig 12 by dividing each voltage vector by I. The result is shown in Fig. 13 where the base represents the resistance, the perpendicular the reactance, and the hypotenuse the impedance. The phase angle of Z is given by $\tan\theta = \frac{X_L}{R}$.

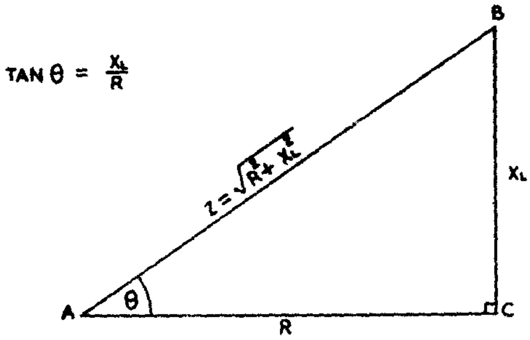


Fig. 13. IMPEDANCE TRIANGLE, R AND L IN SERIES

21. **Summary.** In a circuit containing resistance and inductance in series:—

(*a*) The *magnitude* of the current is $I = \frac{V}{\sqrt{R^2 + X_L^2}} = \frac{V}{Z}$.

(*b*) The *phase* of the applied voltage relative to the current is found from $\tan\theta = \frac{X_L}{R}$. The angle is positive in an inductive circuit indicating that V *leads* I.

(*c*) The *impedance* of the circuit is $Z = \sqrt{R^2 + X_L^2}$.

Resistance and Capacitance in Series

22. With a current of r.m.s. value I established in the circuit of Fig. 14, components of the applied voltage V appear across R and across C. The vector diagram (Fig. 15) is constructed as follows:—

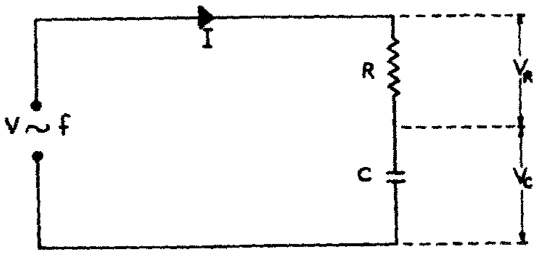


Fig. 14. R AND C IN SERIES

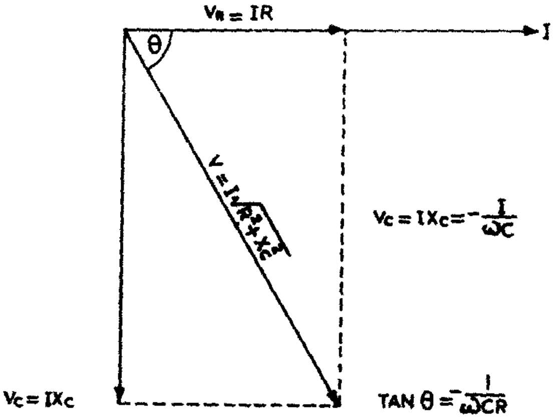

$V_R = IR$
I
θ
$V = I\sqrt{R^2+X_C^2}$
$V_C = IX_C = -\frac{I}{\omega C}$
$V_C = IX_C$
TAN $\theta = -\frac{1}{\omega CR}$

Fig. 15. PHASE RELATIONSHIPS, R AND C IN SERIES

(*a*) The current I is common to R and C and the vector I is the reference vector.

(*b*) The p.d. across R is $V_R = IR$ and is *in phase* with I.

(*c*) The p.d. across C is $V_c = IX_c = \frac{-I}{\omega C}$ and it *lags* the current by 90°. Its magnitude decreases as the frequency increases.

(*d*) The resultant applied voltage V is found by applying the parallelogram of forces rule. This voltage is then seen to lag the current by an angle θ.

23. Applying Pythagoras' theorem to Fig. 15:—

$$\begin{aligned} V^2 &= V_R^2 + V_c^2 \\ &= I^2 R^2 + I^2 X_c^2 \\ &= I^2 (R^2 + X_c^2) \\ \therefore V &= I\sqrt{R^2 + X_c^2} \quad \ldots \quad \ldots \quad (6) \end{aligned}$$

Also, V lags I by an angle θ where:—

$$\tan\theta = \frac{V_c}{V_R} = \frac{X_c}{R} = -\frac{1}{\omega CR}$$

$$\therefore \theta = \tan^{-1} - \frac{\text{Reactance}}{\text{Resistance}} \quad \ldots \quad \ldots \quad (7)$$

Note than the angle is *negative* in a capacitive circuit, indicating that V *lags* I.

24. The impedance of this circuit is:—

$$Z = \frac{V}{I} = \sqrt{R^2 + X_c^2} \quad \ldots \quad \ldots \quad (8)$$

The corresponding impedance triangle is shown in Fig. 16.

25. **Summary.** In a circuit containing resistance and capacitance in series:—

(*a*) The *magnitude* of the current is $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + X_c^2}}$.

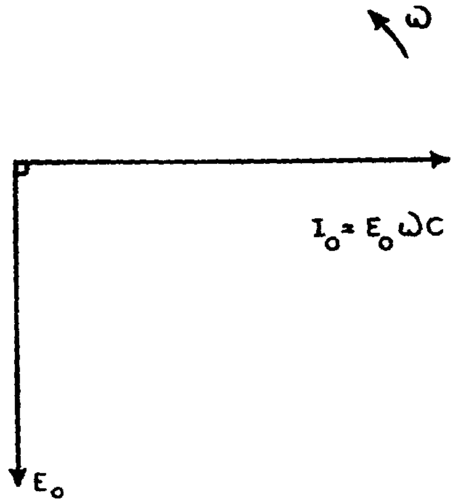


Fig. 9. PHASE OF CURRENT AND VOLTAGE IN A PURE CAPACITANCE

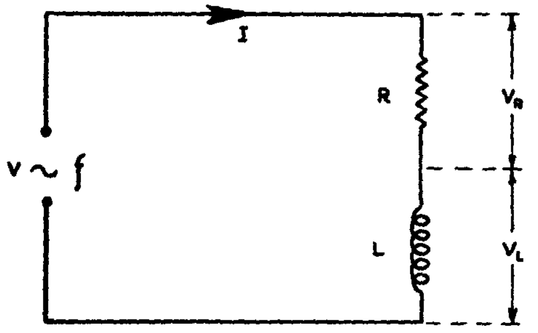


Fig. 10. R AND L IN SERIES

17. The vector diagram for this circuit is shown in Fig. 11 and is constructed in the following manner:—

(*a*) The current vector I is drawn to form the reference line.

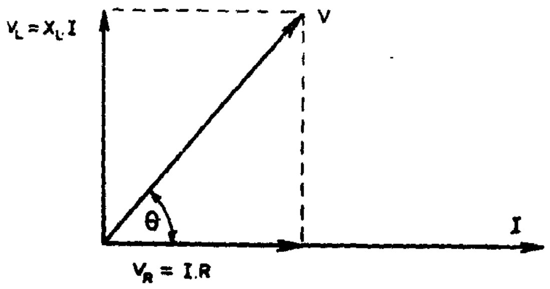


Fig. 11. PHASE RELATIONSHIPS, R AND L IN SERIES

(*b*) The p.d. across R is the product of the current and the resistance. It is *in phase* with the current and has an r.m.s. value $V_R = IR$. This vector is drawn to any convenient scale in line with the current vector.

(*c*) The p.d. across L is the product of the current and the inductive reactance X_L. It *leads* the current by 90° and has an r.m.s. value $V_L = IX_L$ which increases with frequency. This vector is drawn to the same scale as V_R, leading the current by 90°.

(*d*) The applied voltage V is found by applying the parallelogram of forces rule. This voltage is seen to lead the current by an angle θ.

18. The magnitude and phase of the applied voltage V is found by considering the triangle ABC in Fig. 12. This is a right-angled triangle to which Pythagoras' theorem applies. This states that *the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides*. Thus, from Fig. 12:—

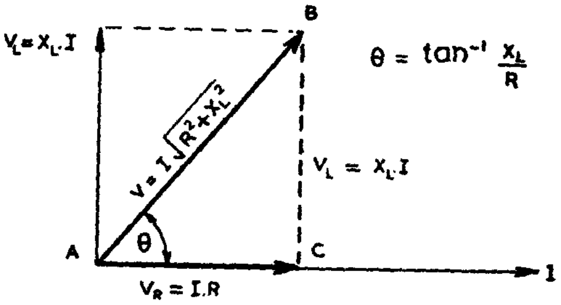


Fig. 12. MAGNITUDE AND PHASE OF V

$$\begin{aligned} V^2 &= V_R^2 + V_L^2 \\ &= I^2R^2 + I^2X_L^2 \\ &= I^2(R^2 + X_L^2) \\ \therefore V &= I\sqrt{R^2 + X_L^2} \quad \ldots \quad \ldots \quad (3) \end{aligned}$$

Also, V leads I by an angle θ where:—

$$\operatorname{Tan} \theta = \frac{V_L}{V_R} = \frac{X_L}{R}$$

$$\therefore \theta = \tan^{-1} \frac{\text{Reactance}}{\text{Resistance}} \quad \ldots \quad \ldots \quad (4)$$

19. **Impedance.** The ratio of voltage to current in a circuit containing resistance and reactance in combination is termed *impedance* (symbol Z) and is measured in ohms. Thus, in a series circuit containing resistance and inductance where the voltage V is $I\sqrt{R^2 + X_L^2}$, the impedance is:—

$$Z = \frac{V}{I} = \sqrt{R^2 + X_L^2} \quad \ldots \quad \ldots \quad (5)$$

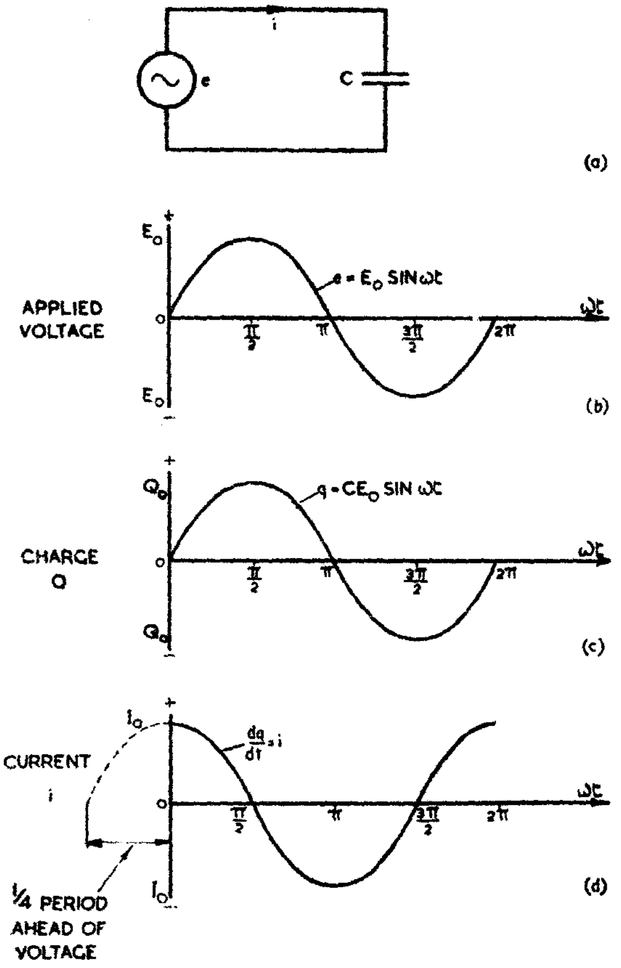


Fig. 7. PHASE RELATIONSHIPS IN A CAPACITANCE

13. **Capacitive reactance.** In a pure capacitance the ratio of voltage to current is:—

$$\frac{E_o}{I_o} = \frac{E}{I} = \frac{1}{\omega C} = \frac{1}{2\pi fC}.$$

Hence, in a circuit having capacitance only, the current is directly proportional to the applied voltage, the frequency and the capacitance. The opposition to the establishment of a current offered by a pure capacitance is termed the *capacitive reactance.* It is denoted by the symbol X_c and is expressed in ohms. Thus:—

$$X_c = \frac{E}{I} = \frac{1}{2\pi fC} \text{ (Ohms)} \quad \ldots\ldots(2)$$

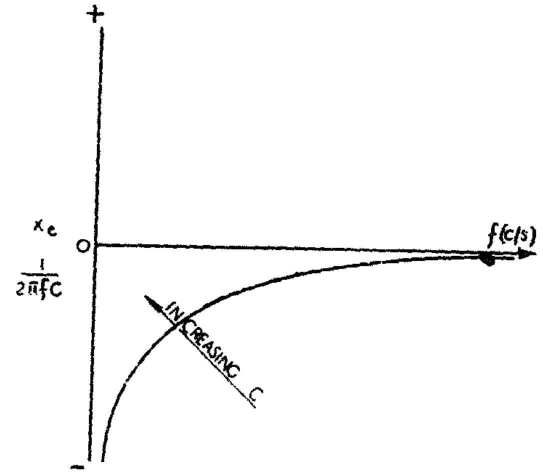


Fig. 8. VARIATION OF X_c WITH FREQUENCY

14. The reactance of a capacitance is inversely proportional to the frequency. A graph of capacitive reactance against frequency is shown in Fig. 8. The reactance of a capacitance is assumed *negative* (opposite to that of an inductance) and is normally given as $X_c = -\frac{1}{\omega C}$; it decreases as the frequency increases. Thus, for a 2μF capacitor:—

When f = 50c/s, $X_c = \frac{1}{2\pi fC} = \frac{10^6}{2\pi \times 50 \times 2}$
$= 1{,}600\,\Omega.$

When f = 1k/cs, $X_c = \frac{10^6}{2\pi \times 10^3 \times 2}$
$= 80\,\Omega.$

When f = 1Mc/s, $X_c = \frac{10^6}{2\pi \times 10^6 \times 2}$
$= 0{\cdot}08\,\Omega.$

15. **Summary.** In a purely capacitive circuit:—

(*a*) The current *leads* the voltage by 90° ($\frac{\pi}{2}$ radians).

(*b*) The ratio of voltage to current gives the capacitive reactance X_c.

These two statements can be shown by drawing a vector diagram for the situation (Fig. 9).

Resistance and Inductance in Series

16. In Fig. 10 a sinusoidal alternating voltage of r.m.s. value V and frequency f is applied across a circuit consisting of resistance R and inductance L connected in series. A corresponding r.m.s. current I is established in the circuit. With components in series, the current is the same at all points in the circuit, and for this reason the *current vector is taken as the reference* when constructing a vector diagram.

$$\frac{E_0}{I_0} = \frac{E}{I} = \omega L = 2\pi fL.$$

Hence, in a circuit having inductance only, the current is directly proportional to the applied voltage and inversely proportional to the frequency and the inductance. The opposition to the establishment of a current offered by a pure inductance is termed the *inductive reactance.* It is denoted by the symbol X_L and is expressed in ohms. Thus:—

$$X_L = \frac{E}{I} = 2\pi fL \text{ (ohms)} \quad .. \qquad .. \qquad (1)$$

8. The reactance of an inductance is proportional to ω and so directly proportional to the frequency f, since $\omega = 2\pi f$. A graph of inductive reactance against frequency is shown in Fig. 5. This graph is known as a *reactance sketch* and for a pure inductance is a straight line through the origin at O. Thus, for a 1mH inductance:—

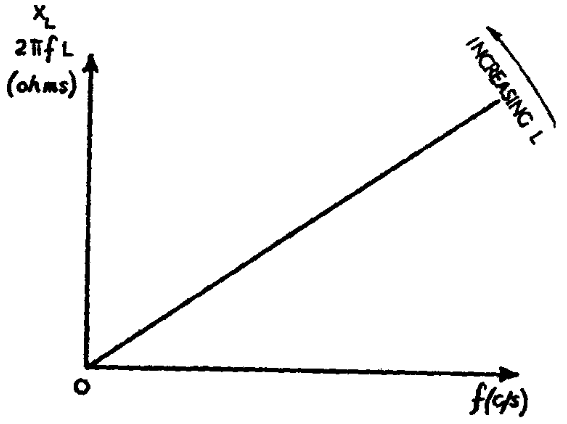


Fig. 5. VARIATION OF X_L WITH FREQUENCY

When $f = 50$ c/s, $X_L = 2\pi \times 50 \times 10^{-3}$
$= 0{\cdot}314\,\Omega$

When $f = 1$ kc/s, $X_L = 2\pi \times 10^3 \times 10^{-3}$
$= 6{\cdot}28\,\Omega$.

When $f = 1$ Mc/s, $X_L = 2\pi \times 10^6 \times 10^{-3}$
$= 6{,}280\,\Omega$.

9. **Summary.** In a purely inductive circuit:—

(*a*) The current *lags* the applied voltage by 90° ($\frac{\pi}{2}$ radians).

(*b*) The ratio of voltage to current gives the inductive reactance X_L.

These two statements can be shown by drawing a vector diagram for the situation (Fig. 6).

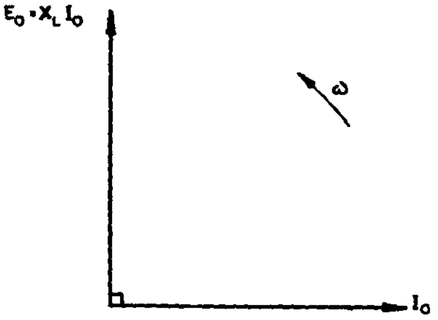


Fig. 6. PHASE OF CURRENT AND VOLTAGE IN A PURE INDUCTANCE

Pure Capacitance in an A.C. Circuit

10. **Rate of change of charge.** It was shown in Sect. 1, Chap. 1 that the current in a circuit is given by:—

$$I = \frac{\text{Charge, Q coulombs}}{\text{Time, t seconds}} \text{ (amps).}$$

To be more specific, the value of current at time t is "the rate of change of charge with respect to time", i.e., $\frac{dq}{dt}$ amps. It is required to consider the rate of change of charge more closely.

11. Fig. 7(*b*) shows the curve for a voltage $e = E_0 \sin \omega t$ applied across the pure capacitance C of Fig. 7(*a*). From the equation $Q = CV$, the charge on the capacitor plates at time t is $q = CE_0 \sin \omega t$. Thus, the charge is *in phase* with the applied voltage as shown in Fig. 7(*c*). The value of the current *i* at time t is the rate of change of charge, i.e. $\frac{dq}{dt}$. Using the same argument as in Para. 5, the curve for the rate of change of a sinusoidal quantity is itself a sine wave leading on the quantity by $\frac{\pi}{2}$ radians. Thus, the current through a capacitor *leads* the charge and therefore the applied voltage by $\frac{\pi}{2}$ radians or 90° (Fig. 7(*d*)).

12. The relative phase of current and voltage in a capacitor and in an inductor can be remembered from the word "CIVIL", where C is the capacitance, I the current, V the voltage, and L the inductance. CIVIL then indicates that in a capacitor the current leads the voltage; in an inductor the current lags the voltage.

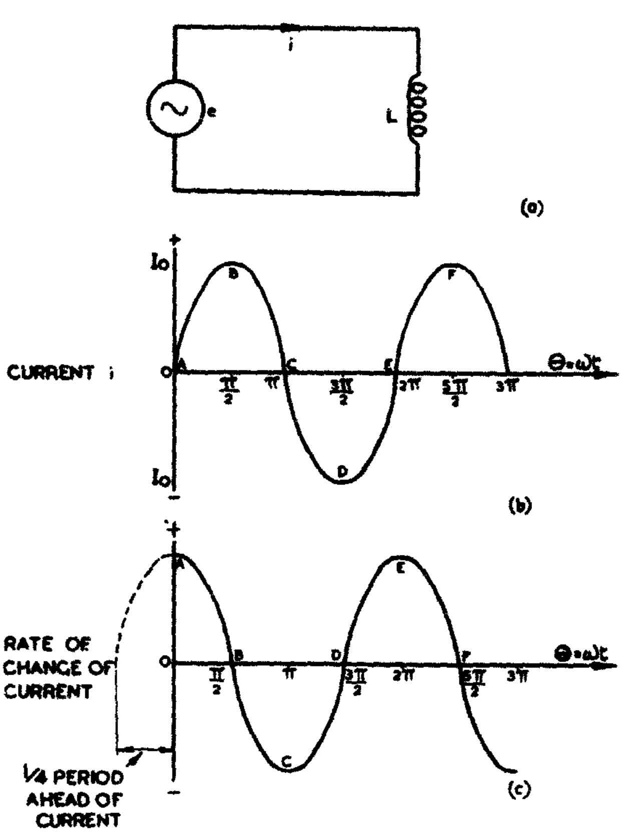

i
e
L
(a)
CURRENT i
Io
A
B
C
D
E
F
O
π/2
π
3π/2
2π
5π/2
3π
θ=ωt
(b)
RATE OF CHANGE OF CURRENT
¼ PERIOD AHEAD OF CURRENT
(c)

Fig. 2. RATE OF CHANGE OF A CURRENT

for the rate of change of current. Such a curve is itself a sine wave *leading* on the current by $\frac{\pi}{2}$ radians or 90°.

6. **Phase of voltage and current in an inductance.** For a pure inductance the back e.m.f. is $e = -L\frac{di}{dt}$ volts; it is in *anti-phase* to the rate of change of current (by virtue of the negative sign). The applied e.m.f. is, however, equal and opposite to the back e.m.f. (Kirchhoff's second law) and is given by $e = +L\frac{di}{dt}$ volts; it is, therefore, *in phase* with the rate of change of current. Hence, since the rate of change of current leads the current by $\frac{\pi}{2}$ radians, the applied e.m.f. is a sinusoidal wave leading the current by $\frac{\pi}{2}$ radians. Therefore, *in a pure inductance the current and voltage are 90° out of phase, the current lagging the applied voltage by $\frac{\pi}{2}$ radians or 90°.* This is shown graphically in Fig. 3 and vectorially in Fig. 4.

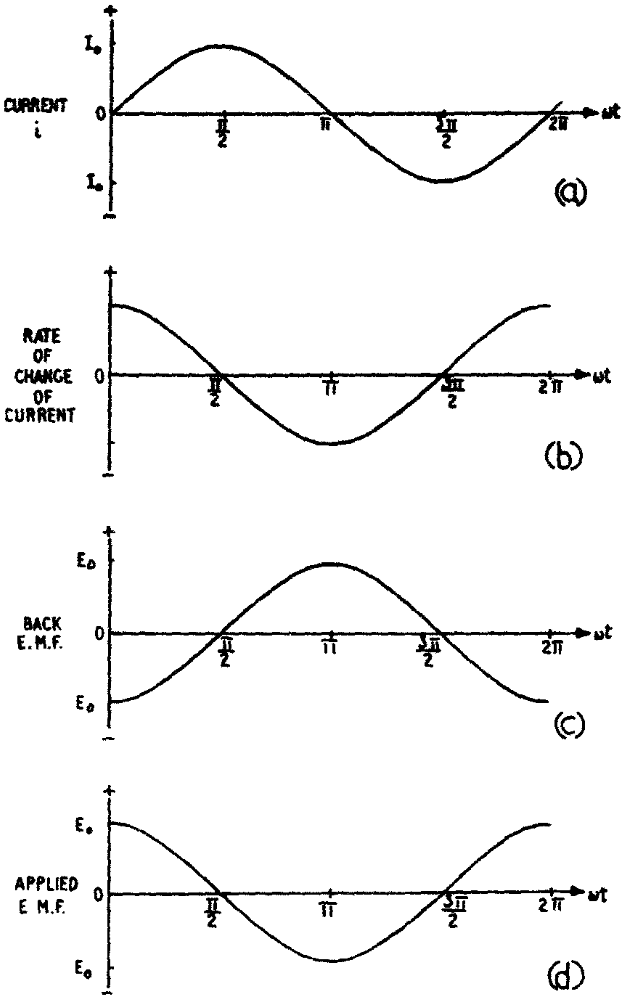


Fig. 3. PHASE RELATIONSHIPS IN AN INDUCTANCE

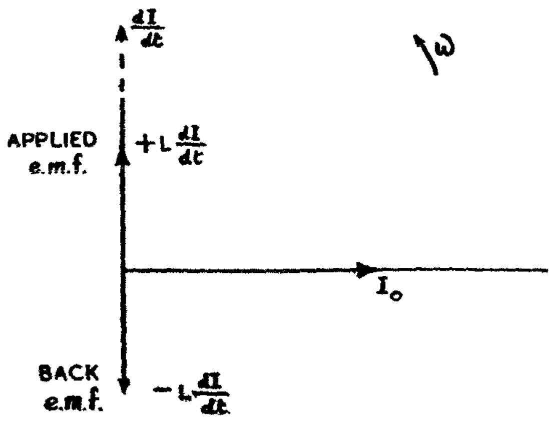

dI/dt
APPLIED e.m.f.
+L dI/dt
I₀
BACK e.m.f.
−L dI/dt
ω

Fig. 4. VECTOR RELATIONSHIPS IN AN INDUCTANCE

7. **Inductive reactance.** In a pure resistance the ratio of voltage to current gives the resistance R. In a pure inductance the ratio of voltage to current is:—

PART 1, SECTION 5, CHAPTER 2

SERIES A.C. CIRCUITS

Introduction

1. This Chapter considers the behaviour of circuits containing combination of resistance, inductance and capacitance *in series.* Voltage and current in such circuits have identical waveforms but not necessarily the same phase.

2. Previous chapters have shown some imperfections of electrical components but in order to simplify treatment, it is helpful to consider *ideal* components before discussing general cases.

Pure Resistance in an A.C. Circuit

3. (*a*) In the circuit shown in Fig. 1(*a*) a sinusoidal voltage $e = E_0 \sin \omega t$ is applied to a resistance R ohms.

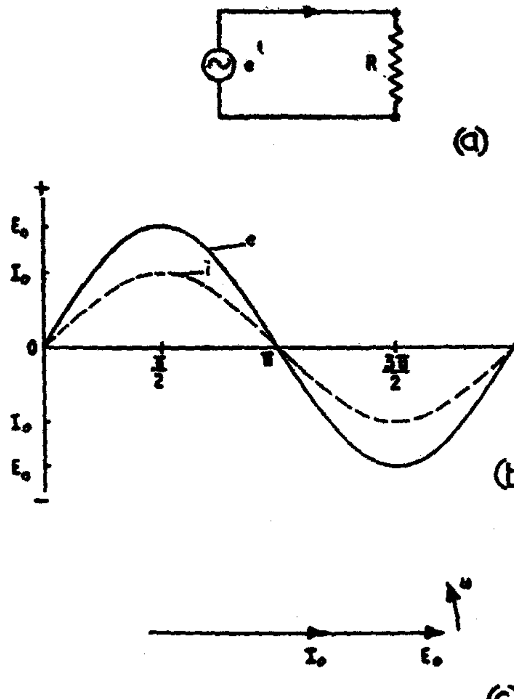


Fig. 1. RESISTANCE IN AN A.C. CIRCUIT

(*b*) Ohm's law applies at any instant. Hence, *i* can be determined from the equation:—

$$i = \frac{e}{R} = \frac{E_0 \sin \omega t}{R} \text{ (amps),}$$

which shows that *i* is a sinusoidal current in phase with *e* and having the same frequency $f = \frac{\omega}{2\pi}$. The peak value of *i* is $\frac{E_0}{R} = I_0$. Thus, in a purely resistive circuit:—

(i) The current and voltage are *in phase.*

(ii) The ratio $\frac{\text{Peak voltage } E_0}{\text{Peak current } I_0} = \frac{\text{R.m.s. voltage E}}{\text{R.m.s. Current I}}$ gives the resistance R ohms.

(*c*) The phase relationship between current and voltage is shown graphically in Fig. 1(*b*) and vectorially in Fig. 1(*c*).

Pure Inductance in an A.C. Circuit

4. **Rate of change of current.** It was shown in Sect. 2, Chap. 2 that the back e.m.f. induced in a coil by a changing current is given by $e = -L\frac{di}{dt}$ volts. The symbol $\frac{di}{dt}$ was there considered to be an abbreviation of the phrase "rate of change of current with respect to time". It is now required to consider the rate of change of a current more closely.

5. Fig. 2(*b*) shows the curve for a sinusoidal current $i = I_0 \sin \omega t$ which is established in the circuit of Fig. 2(*a*). The flattest parts of the curve are at the points B, D, and F when $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$. Therefore, the *rate of change* of current is *zero* at these instants, giving points B, D, and F in Fig. 2(*c*). The curve has maximum and equal gradients at the points A, C, and E of Fig. 2(*b*), when $\theta = 0, \pi, 2\pi$. At these instants the rate of change has some maximum value. At A and E the current is positive-going and the rate of change has a maximum positive value; at C the current is negative-going and the rate of change has a maximum negative value. Joining the points A, B, C, D, E and F in Fig. 2(*c*) gives the curve

SECTION 5

CHAPTER 2

SERIES A.C. CIRCUITS

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vectors are rotating at the same angular velocity they maintain the same *relative* positions as shown in Figs. 17(*b*) and 17(*c*). In a.c. problems it is sufficient to consider vectors in one position only and it is usual to take the instant $t = O$ as depicted in Fig. 17(*a*) when one of the vectors lies along the reference line.

Resultant of Two Alternating Voltages

21. There is a single voltage that is equivalent to two alternating voltages acting across the same circuit at the same time, and this can be found by the parallelogram rule for finding the resultant of two vectors. Thus, in Fig. 18, E_1 and E_2 are the vectors representing two voltages $e_1 = E_1 \sin \omega t$ and $e_2 = E_2 \sin (\omega t - \theta)$ respectively. The resultant voltage is represented by the diagonal of the parallelogram drawn outwards from 0, i.e. the vector E_R. This resultant vector rotates at the same angular velocity ω as E_1 and E_2 and with reference to E_1 lags by an angle $\varnothing$. The resultant voltage is, therefore, $e_R = E_R \sin (\omega t - \varnothing)$.

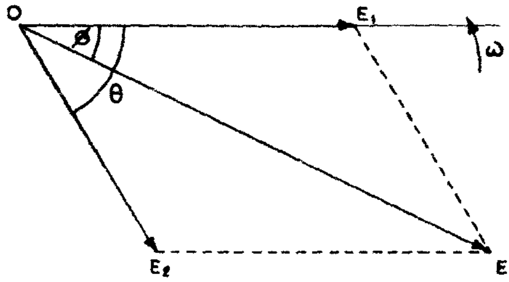


Fig. 18. RESULTANT OF TWO ALTERNATING VOLTAGES

22. Vectors may be used to represent to scale, not only the peak values of alternating quantities, but their *r.m.s. values*. In the latter case, however, the projections perpendicular to the axis do not represent instantaneous values (see Para. 8). Phase differences, as before, are shown by the angles between the vectors, and the vectors can be added or subtracted. It is worth noting at this point that *voltages cannot be added to currents* although their phase differences can be shown on one diagram.

Resolution of a Vector into Two Components

23. It has been shown in Para. 14 that the resultant of two vectors OA and OB at right angles to each other is the diagonal OC of the parallelogram OACB (Fig. 19). The vector OC makes an angle θ to the reference line. This procedure can be reversed; that is, given vector OC at an angle θ to the reference line, this vector can be resolved into two components at right angles to each other.

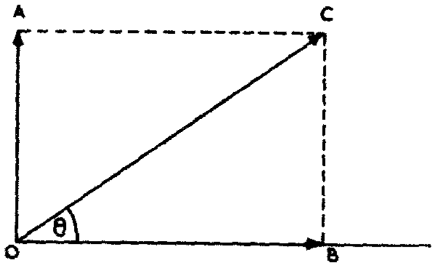


Fig. 19. QUADRATURE COMPONENTS OF A VECTOR

In Fig. 19:—

$$\sin \theta = \frac{BC}{OC} = \frac{OA}{OC}$$

$$\therefore OA = OC \sin \theta$$

$$\text{Also, } \cos \theta = \frac{OB}{OC}$$

$$\therefore OB = OC \cos \theta$$

Thus the two components at right angles into which any vector may be resolved, are given in terms of the vector and the angle which it makes to the reference line.

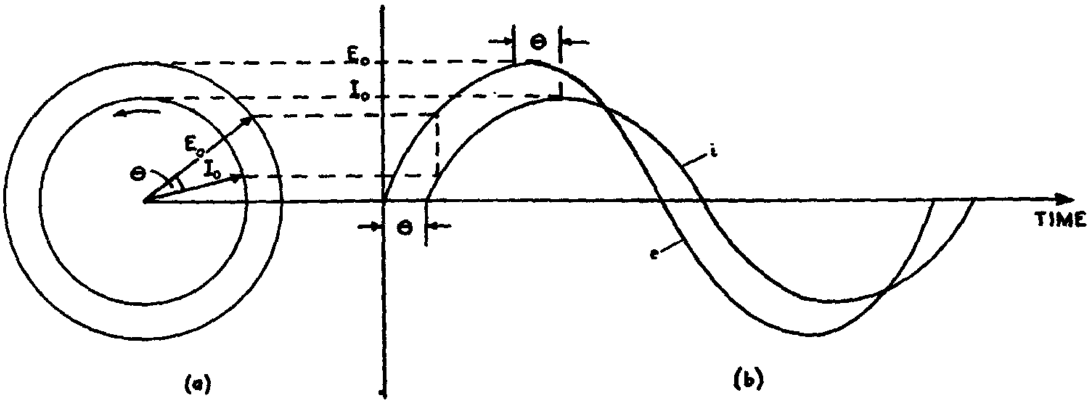
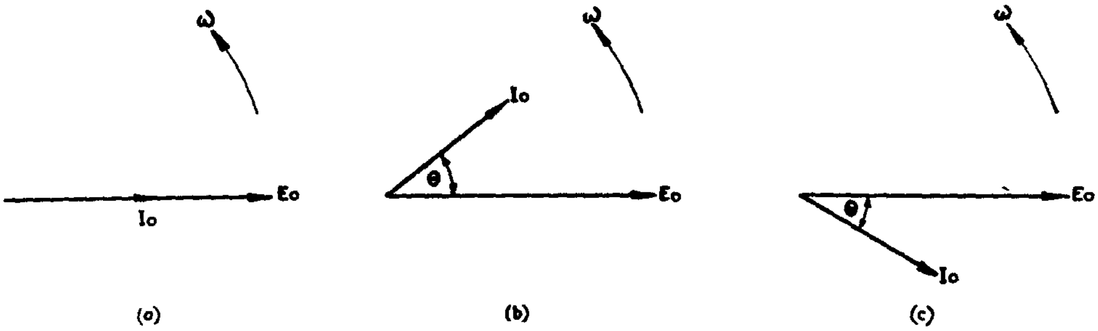


Fig. 15. PHASE DIFFERENCE BY GRAPHS AND VECTORS

required phase difference the two vectors E_0 and I_0 are displaced from each other by the constant angle θ, E_0 leading I_0. Thus, Fig. 15(*a*) and Fig. 15(*b*) both show the phase difference between the current and the voltage but the vector representation is more simple to construct and use.

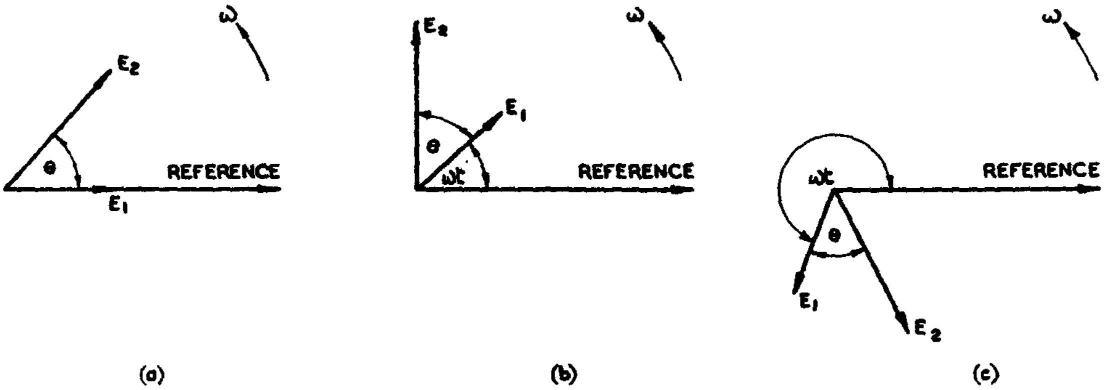
19. Let E_0 represent the peak value of an alternating voltage, and I_0 the peak value of the current in a circuit. Then, if the voltage and current are in phase, E_0 and I_0 may be represented by vectors which are coincident in direction and rotate at equal angular velocity ω radians per second. E_0 and I_0 are, therefore, represented, as in Fig. 16(*a*). In practice I_0 may be in phase with, or lead, or lag the voltage E_0. In Fig. 16(*b*) I_0 is shown as *leading* by angle θ, and in Fig. 16(*c*) as *lagging* by angle θ with reference to E_0.

20. Consider two voltages $e_1 = E_1 \sin \omega t$ and $e_2 = E_2 \sin (\omega t + \theta)$. These two voltages can be represented by vectors of length E_1 and E_2 respectively, rotating at the same angular velocity ω radians per second and displaced from each other by an angle θ. At the instant $t = 0$, E_1 is lying along the reference vector and E_2 is leading by an angle θ as shown in Fig. 17(*a*). Since both



(a) (b) (c)

Fig. 16. PHASE DIFFERENCE BY VECTOR.



(a) (b) (c)

Fig. 17. REFERENCE VECTORS

$$V_L = X_L\, I = \omega L I = \omega L \times \frac{V}{R}$$

$$\therefore V_L = \left(\frac{\omega L}{R}\right) \times V = QV$$

where $Q = \frac{\omega L}{R}$

The voltage across the inductance is Q times the applied voltage V and *voltage magnification* has taken place.

(*c*) Q as already defined is equal to $\frac{\omega L}{R}$.

However, at resonance, $\omega L = \frac{1}{\omega C}$.

Hence, Q is also equal to $\frac{1}{\omega CR}$ and the and the voltage across the capacitance at resonance is:—

$$V_c = X_c I = \frac{I}{\omega C} = \frac{1}{\omega C} \times \frac{V}{R}$$

$$\therefore V_c = \left(\frac{1}{\omega CR}\right) \times V = QV.$$

The voltage across the capacitance at resonance is equal but opposite to the voltage across the inductance (Fig. 33(*b*)).

52. When a series circuit is resonant to a given input, the voltage across either L or C at the input frequency can be many times greater than the input voltage. If $V = 0{\cdot}1V$ and $Q = 100$, the voltage across either L or C is 10V and this voltage can be applied to another circuit. A resonant series circuit is, therefore, a *voltage magnifier*.

Effect of Supply Impedance on Selectivity

53. When a series resonant circuit is used for tuning purposes, it is often intended that the voltage across the capacitor be applied to another stage. The voltage applied to the circuit itself can be considered as the supply from a generator with an internal resistance R_G (Fig. 34). If R_G is considered non-reactive, it will modify the tuned circuit characteristics as follows:—

Total circuit resistance is $R_T = R + R_G$

i.e., effective $Q = \frac{1}{R + R_G}\sqrt{\frac{L}{C}}$

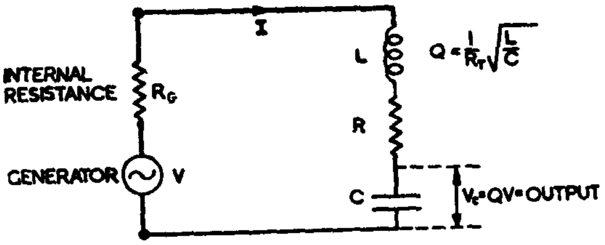


Fig. 34. EFFECT OF SUPPLY IMPEDANCE ON SELECTIVITY

If R_G is large compared with R, then Q is reduced and the voltage across the capacitor at the resonant frequency ($V_c = QV$) and the selectivity of the circuit are both relatively small. Thus, the series circuit is more selective when fed by a generator of *low* internal impedance.

The Q Factor of Components

54. It was shown in Para. 40 that the power losses associated with inductors and capacitors can be expressed in terms of the power factor $\cos\theta = \frac{R}{Z} = \frac{R}{\sqrt{R^2+X^2}}$ where R is the equivalent loss resistance of the component. In well-designed components, R is small compared with X, and the power factor approximates to $\frac{R}{X}$ (i.e., $\frac{R}{\omega L}$ for inductors and ωCR for capacitors). It is, however, more convenient to express the quality of inductors and capacitors in terms of the *reciprocal* of the power factor, namely $\frac{\omega L}{R} = \frac{1}{\omega CR} = Q$.

Thus, a coil having a high value of Q indicates a component with few losses. Q remains relatively constant with frequency since the value of loss resistance R varies with frequency in much the same way as the reactance X. At audio frequencies, Q values for coils rarely exceed 10, whilst coils normally used for radio frequencies have Q values around 50–300. Q values from 100 to 300 are common with paper dielectric capacitors and from 1,000 to 3,000 for mica capacitors.

$$\therefore Q_o = \frac{2\pi L}{R} \times \frac{1}{2\pi\sqrt{LC}} = \frac{L}{R} \times \frac{1}{\sqrt{LC}}$$

$$\therefore Q_o = \frac{1}{R}\sqrt{\frac{L}{C}} \quad \cdots \quad \cdots (15)$$

This expression supports Para. 48 as it shows that the Q or selectivity of a tuned circuit is inversely proportional to R and proportional to the ratio of L to C. Thus, a circuit with a high value of Q has high selectivity.

50. **Q and Bandwidth.** Series tuned circuits are used in radio to accept inputs at the resonant frequency and in the immediate neighbourhood of resonance. In order to present a high impedance to inputs considerably removed from resonance, the circuit must have a high Q value. In practice, values of Q vary from about 10 at audio frequencies to several hundreds at radio frequencies. The graph of current against frequency for a circuit having a high Q shows a sharp response curve (Fig. 32). An alternative way of describing this is to say that the circuit has a *narrow bandwidth*. Bandwidth is defined as the separation between two frequencies either side of the resonant frequency at which the *power* has fallen to 50% of the maximum power. This is known as the *"half-power bandwidth"*. Since power is proportional to the *square* of the current, reducing the power to one-half means reducing the current by a factor of $\sqrt{\frac{1}{2}} = 0{\cdot}707$. Thus, in terms of current the bandwidth of a series tuned circuit is the difference between two frequencies f_1 and f_2 at which the current is 70% of the resonant value (Fig. 32). The selectivity of a series circuit can therefore be defined either in terms of Q or of bandwidth. For purposes of calculation Q_o and the bandwidth as defined above are related by the expressions:—

$$Q_o = \frac{\text{Resonant frequency}}{\text{Bandwidth}} = \frac{f_o}{f_1 - f_2} \quad ..(16)$$

Thus if, the resonant frequency of a circuit is 200 kc/s and the bandwidth required is 12 kc/s, Q_o should be $\frac{200}{12} = 16{\cdot}7$.

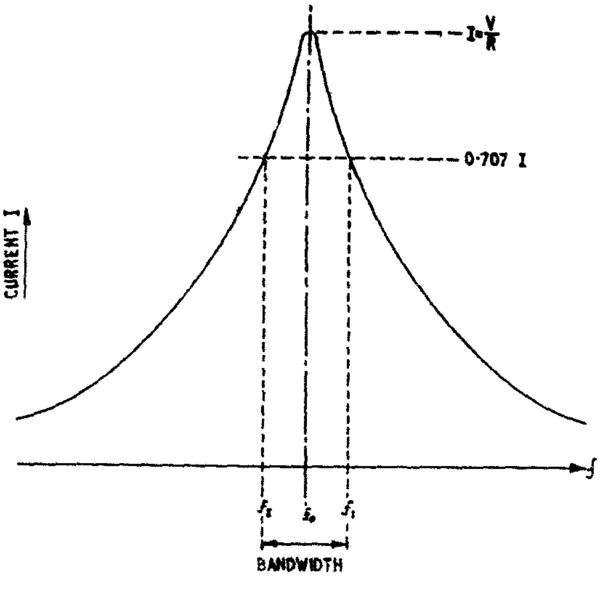


Fig. 32. THE HALF-POWER BANDWIDTH

Circuit Magnification

51. At resonance, in a series circuits, the impedance is a minimum and equal to the resistance R. The current I is a maximum and equal to $\frac{V}{R}$. P.d.s are developed across the components shown in Fig. 33(*a*) as follows:—

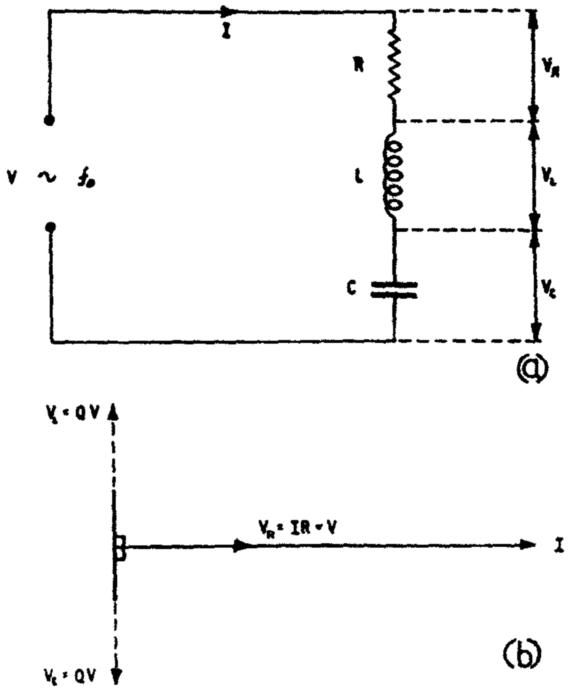


Fig. 33. CIRCUIT MAGNIFICATION

(*a*) The voltage across the resistance at resonance is:—

$$V_R = IR = \frac{V}{R} \times R = V$$

$$\therefore V_R = \text{Applied Voltage.}$$

(*b*) The voltage across the inductance at resonance is:—

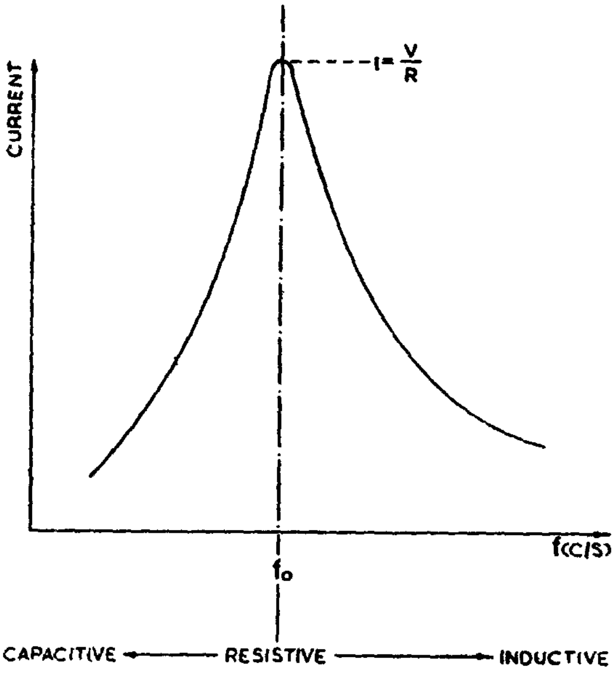


Fig. 29. VARIATION OF CURRENT WITH FREQUENCY

is to reduce the current at frequencies near resonance to a far greater extent than at other frequencies, i.e. to "flatten" the response curve as shown in Fig. 30. Selectivity is therefore reduced by an increase in resistance.

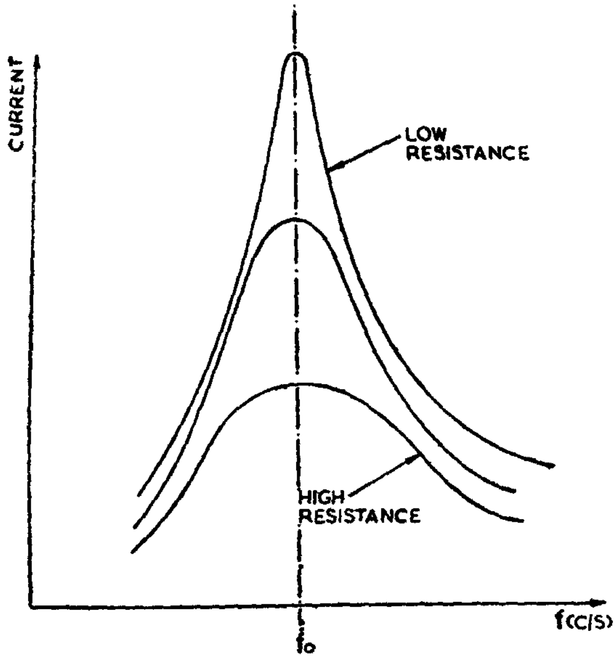


Fig. 30. EFFECT OF RESISTANCE ON SELECTIVITY

(*b*) *The ratio of L to C in the circuit.* The resonant frequency of an acceptor circuit is given by $f_o = \frac{1}{2\pi\sqrt{LC}}$. It is dependent on the *product* LC. Thus, if the inductance of a circuit is doubled and the capacitance halved the resonant frequency remains unaltered. However, the *ratio* of L to C has been increased four times and this has an effect on the selectivity of the circuit. Fig. 31 shows the response curves for two circuits having the same resonant frequency and equal values of resistance but different $\frac{L}{C}$ ratios. From this it is seen that the *greater* the $\frac{L}{C}$ ratio the *more selective* is the circuit.

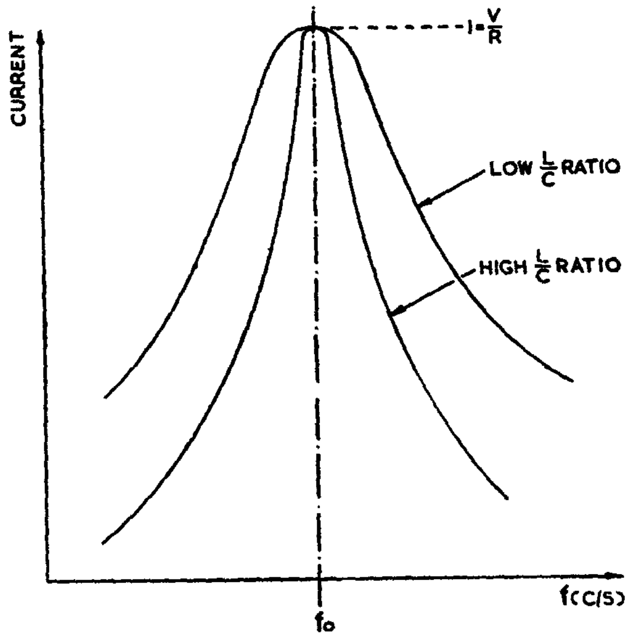


Fig. 31. EFFECT OR L/C RATIO ON SELECTIVITY

49. **Q factor.** The quantity used to represent the selectivity of a circuit is denoted by the letter Q, which is defined as the ratio of reactance to circuit resistance at the resonant frequency. It is usually considered for the coil, i.e. $Q_o = \frac{\omega_o L}{R}$.

Thus, $Q_o = \frac{2\pi f_o L}{R}$

But $f_o = \frac{1}{2\pi\sqrt{LC}}$

reactance sketches can be combined to give the total reactance variation with frequency. This is shown in Fig. 27. From this graph it is seen that at the resonant frequency f_o, $X_L = X_C$ and the total reactance is zero. The circuit is then purely resistive with the current and the applied voltage in phase.

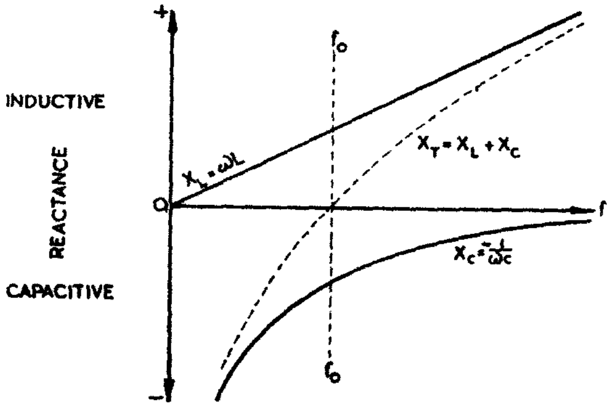


Fig. 27. REACTANCE SKETCH FOR A SERIES TUNED CIRCUIT

44. If the graph for resistance R (which for practical purposes remains constant with frequency) is combined with the graph for the total reactance X_T, a curve for impedance Z against frequency can be obtained. To plot this graph the expression $Z = \sqrt{R^2+X_T^2}$ is used, and as shown in Fig. 28, Z is all above the horizontal axis. At the resonant frequency f_o, the impedance Z is a minimum and equal to R. Either side of resonance the impedance rises, and, as seen from the reactance curve X_T, *below* resonance the circuit has *capacitive* reactance and *above* resonance the circuit has *inductive* reactance.

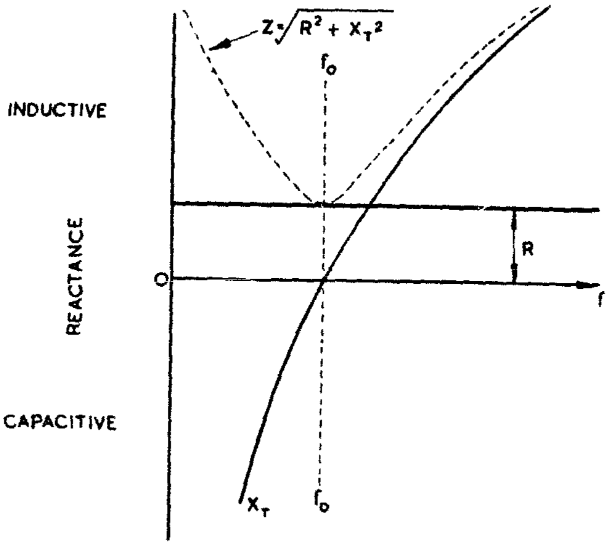


Fig. 28. VARIATION OF Z WITH FREQUENCY

45. **Resonant frequency.** At resonance:—

$$\omega L = \frac{1}{\omega C}$$

$$\therefore \omega^2 = \frac{1}{LC}$$

$$\therefore \omega = \frac{1}{\sqrt{LC}} = 2\pi f_o$$

$$\therefore f_o = \frac{1}{2\pi\sqrt{LC}} \quad \cdots \quad \cdots \quad \cdots \quad (14)$$

This gives the resonant frequency of a series tuned circuit. By altering either L or C the frequency at which resonance occurs is changed. It should be noted that R has no effect on this.

Selectivity of a Series Circuit

46. Fig. 28 shows that the impedance Z of a series tuned circuit falls as resonance is approached. Such a circuit is often termed an "acceptor" circuit, because it accepts frequencies to which it is resonant since the impedance is then a minimum.

47. The sharpness of response over a range of frequencies near resonance gives an indication of the *selectivity* of the circuit. Selectivity is the property of a tuned circuit which enables it to respond strongly to a particular signal and to disregard others of different frequencies. It is a natural product of the phenomenon of resonance, for the ability to respond most strongly to the resonant frequency implies a power to select it from among others. For the series tuned circuit, selectivity is a measure of the ease with which the circuit can accept inputs at the resonant frequency in relation to inputs off resonance. A graph showing the variation of *current* with frequency is shown in Fig. 29.

48. The selectivity of a series circuit depends on two factors:—

(*a*) *The resistance R of the circuit.* At resonance $I = \frac{V}{R}$, so that if R is doubled the current at resonance is halved. In a properly designed acceptor circuit, above resonance X_L is much greater than R and below resonance X_C is greater than R. Thus the effect of R is progressively less important as the frequency is moved away from resonance. The effect of resistance

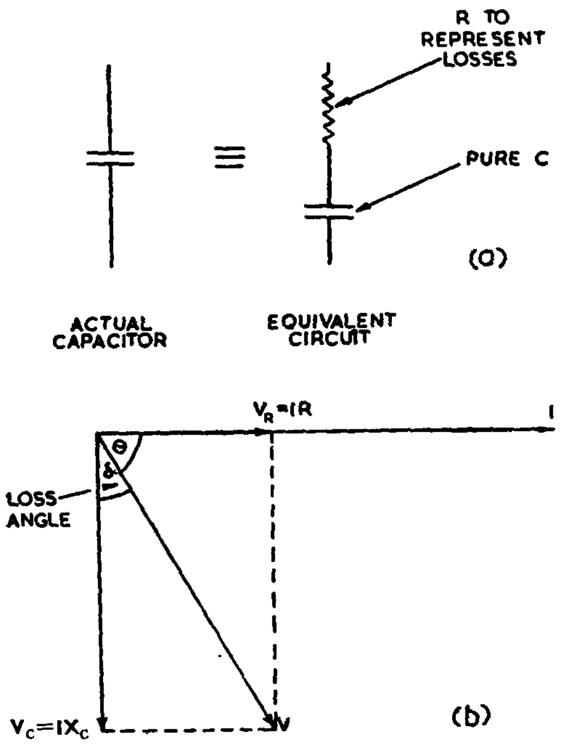


Fig. 24. POWER FACTOR OF A CAPACITOR

capacitor has, therefore, a power factor cos θ. In practice it is nearer zero than that of an inductor and in normal circuits of capacitor and inductor in series, the major part of the losses are in the inductor.

Resonance in the Series Circuit

41. Consider the circuit shown in Fig. 25 where R is the combined loss resistance of L and C. When the frequency of the supply is such that the capacitive reactance X_C is greater than the inductive reactance X_L, the impedance is capacitive and the phase angle negative indicating that the applied voltage is lagging the current. When the frequency of the supply is such that X_L is greater than X_C, the impedance is inductive and the phase angle positive, indicating that the applied voltage is leading the current. In between these conditions, at some particular frequency X_L will equal X_C and V_L will equal V_C. This is shown vectorially in Fig. 26. The circuit is then said to be *at resonance.* Where the frequency of the supply is fixed, either L or C can be varied to give the condition $X_L = X_C$. The circuit has then been *tuned to resonance* with the supply frequency.

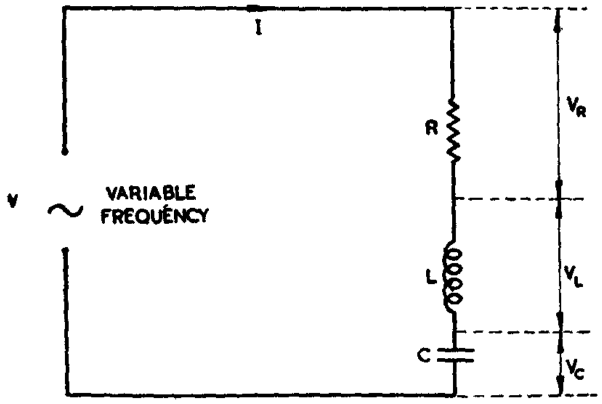


Fig. 25. SERIES RESONANT CIRCUIT

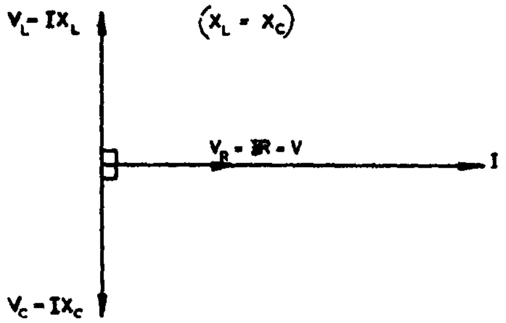


Fig. 26. SERIES RESONANCE

42. **Conditions at resonance.**

(*a*) At resonance, $\omega L = \frac{1}{\omega C}$ and the impedance $Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} = R$.

Thus at resonance in a series tuned circuit the impedance Z is a *minimum* and equal to R. In a well-designed circuit R will be small, being only the loss resistance of L and C.

(*b*) The current $I = \frac{V}{Z} = \frac{V}{R}$ has a *maximum* value at resonance.

(*c*) The phase angle θ, given by $\tan\theta = \frac{(\omega L - {}^1/\omega C)}{R}$ is zero. Thus, at resonance in a series circuit the current and the applied voltage are *in phase.*

43. **Reactance sketches.** A reactance sketch is a graph relating reactance and frequency. The reactance sketch for an inductance is given in Fig. 5 and that for a capacitance in Fig. 8. In a series circuit where both capacitance and inductance are present, the

37. Referring again to Fig. 22(*b*) it is seen that the voltage $V = I\sqrt{R^2 + X^2} = IZ$.

$$\therefore \cos\theta = \frac{IR}{IZ} = \frac{R}{Z} = \text{Power factor} \quad (13)$$

This gives an alternative expression for the power factor in terms of the components in a circuit.

38. **Adjustment of power factor.** In a pure resistance the power factor, $\cos\theta = \frac{R}{Z} = 1$. In a pure reactance it is zero. In a practical circuit it lies between these extremes. The value of power factor required by a circuit depends on what the circuit is to be used for. Where the power developed must be kept to a minimum, the power factor should be as nearly zero as possible (i.e., R should be small). Where a large power is required to be developed, e.g., electric fires, the power factor should be as nearly unity as possible (i.e., X should be small).

39. Where a power factor of unity is required, the out-of-phase component of current must be zero (i.e., the reactive component must be zero.) When the circuit is inductive the current will lag the voltage by an angle θ and the power factor will be less than unity. This can be adjusted by inserting a capacitive component of such value that it balances the inductive component. The current and voltage will then be in phase and the power factor will be unity.

40. **Power losses in components.** The power factor of an inductor or a capacitor should, in theory, be zero. That is, these components should be pure reactances. This cannot apply in practice however, since power losses are developed in the components. Most of the losses listed below increase with the frequency of the supply. They occur mainly in high frequency circuits and because of this are more important in radio engineering than in power engineering.

(*a*) *Inductors.* Power losses occur in inductors because of:—

(i) Eddy current in the cores.

(ii) Hysteresis in the cores.

(iii) Skin effect in the coils.

(iv) Proximity effect of conductors in the near vicinity.

(v) Ohmic resistance of the windings.

All these losses can be lumped together by supposing that the inductor has in series with it a resistance R, such that the power loss is I^2R, where I is the r.m.s. value of the current. An inductor can thus be represented as L and R in series, where L is a pure inductance and R represents the losses (Fig. 23(*a*)). The vector diagram is then as shown in Fig. 23(*b*) from which it is seen that I no longer lags V by 90° but by an angle θ, this being less than 90° by δ (the loss angle). The inductor has, therefore, a power factor cos θ.

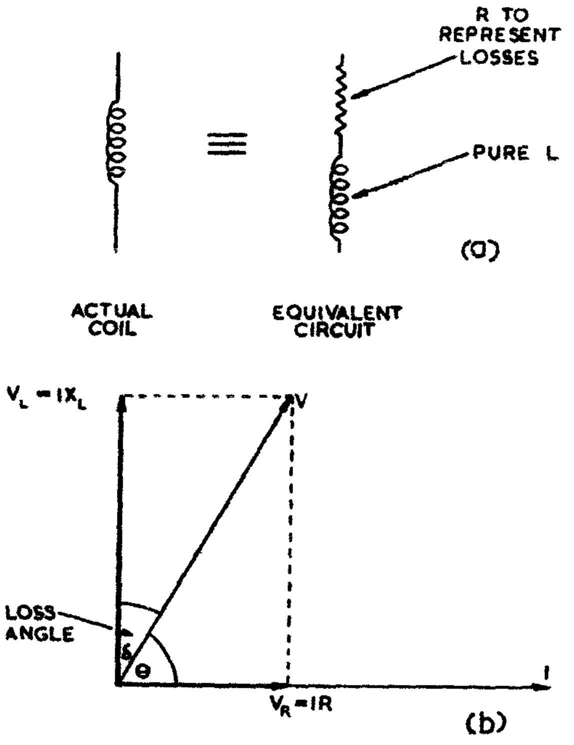


Fig. 23. POWER FACTOR OF A COIL

(*b*) *Capacitors.* Power losses occur in capacitors because of:—

(i) Leakage through the dielectric.

(ii) Dielectric hysteresis or absorption.

(iii) Brush discharge.

(iv) Ohmic resistance of the connecting leads.

(v) Skin effect in the plates and connecting leads.

Remarks similar to those for the inductor apply to the capacitor, and a capacitor can be represented as C and R in series, where C is a pure capacitance and R represents the losses (Fig. 24(*a*)). The vector diagram is then as shown in Fig. 24(*b*) from which it is seen that I leads V by an angle θ, this being less than 90° by the angle δ (the loss angle). A positive or negative depending on the relative values of ωL and $\frac{1}{\omega C}$.

(*c*) The *impedance* of the circuit is $Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$.

Power in A.C. Circuits

33. When an alternating voltage is applied across a pure resistance R, current and voltage are always in phase. The power developed in R, is VI watts where V and I are r.m.s. values.

34. In a coil which is considered to be a pure inductance, there is no dissipation of energy as heat when the coil is connected across an alternating supply of voltage. The current and voltage are 90° out of phase, and the energy ($\frac{1}{2}LI^2$) supplied during that part of the cycle when the current is growing to its peak value, is stored in the magnetic field and restored to the source as the current decays to zero. Similarly, in a pure capacitance the current and voltage are 90° out of phase, and the energy ($\frac{1}{2}CV^2$) supplied to the capacitance during that part of the cycle when the voltage is building up to its peak value, is stored in the electric field and restored to the source as the voltage falls to zero. Thus, when an alternating voltage is applied across a pure reactance, either inductive or capacitive, no power is developed at all, and the current is said to be "*wattless*".

35. In an actual circuit containing both resistance and reactance there is a phase difference θ between the current and the voltage. The vector diagram for the circuit of Fig. 22(*a*) is given in Fig. 22(*b*), from which it is seen that the current I lags the applied voltage V by an angle θ. It was shown in Chap. 1 that a vector I at an angle θ to the reference, can be resolved into two components at right angles to each other in terms of I and θ. This is shown in Fig. 22(*c*). The component of current I which is in phase with the applied voltage (termed the *in-phase component* of I) is I cos θ ; the *out-of-phase component* of I at right angles to the applied voltage is I sin θ. The former develops a power of VI cos θ watts. The latter develops no power at all since it is 90° out of phase with the applied voltage; for this reason it is termed the *wattless component* of current.

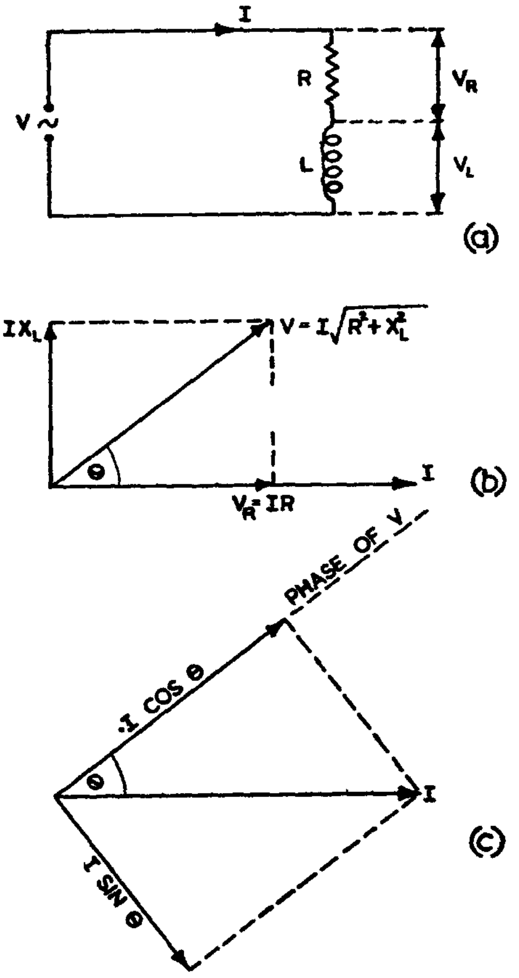


Fig. 22. WORKING AND WATTLESS COMPONENTS OF CURRENT

36. **Power factor.** The factor cos θ is known as the *power factor* of a circuit. The product VI is that of the r.m.s. values of voltage and current and is termed the volt-amperes (VA) or the *apparent power* in a circuit. The *true power* in a circuit where V and I are out of phase is given by the product of the applied voltage and the *in-phase component* of current. Thus:—

$$P = VI \cos \theta$$

$$\cos \theta = \frac{P}{VI}$$

$$\therefore \text{ Power factor} = \frac{\text{True power}}{\text{Apparent power}} \quad .. \quad (12)$$

The power factor may be found by measuring the true power with a wattmeter, and the apparent power with a voltmeter and ammeter (see Sect. 6.).

$$= I^2R^2 + (I\ X_L + IX_c)^2$$

$$= I^2[\ R^2 + (X_L + X_c\ )^2]$$

$$\therefore\ V = I\sqrt{R^2 + (X_L + X_c)^2}$$

$$\therefore\ V = I\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} \quad .. \quad (9)$$

In this case, V *leads* I by an angle θ where:—

$$\tan\ \theta = \frac{(V_L + V_c)}{V} = \frac{(X_L + X_c)}{R}$$

$$\therefore\ \tan\ \theta = \frac{(\omega L - {}^1/\omega C)}{R}$$

$$\therefore\ \theta = \tan^{-1}\frac{\text{Reactance}}{\text{Resistance}} \quad .. \quad .. \quad (10)$$

The factor tan θ is positive in this instance since X_L is greater than X_c.

29. The impedance of this circuit is:—

$$Z = \frac{V}{I} = \sqrt{R^2 + (X_L + X_c)^2} \quad .. \quad (11)$$

The corresponding impedance triangle is shown in Fig. 20. It should be noted that X_L is a positive reactance and X_c is a negative reactance so that it is, in fact, their *numerical difference* which gives the magnitude of the total reactance in a circuit.

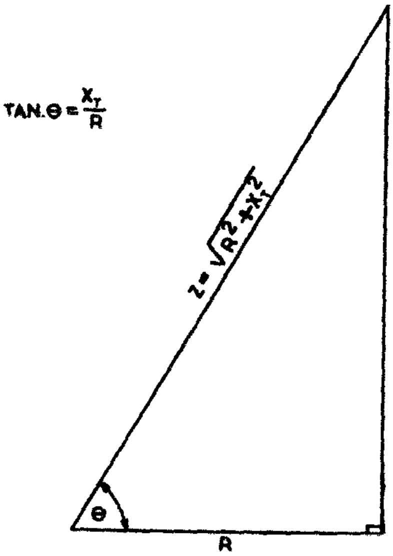

$\text{TAN.}\theta = \frac{X_T}{R}$
$Z = \sqrt{R^2 + X_T^2}$
$X_T = (\omega L - \frac{1}{\omega C})$
θ
R

Fig. 20. IMPEDANCE TRIANGLE, R, L, AND C IN SERIES

30. A variation in the frequency of the supply has an effect on the result. The vector diagrams of Fig. 18 and 19 apply when the frequency of the supply voltage is such that X_L is greater than X_c. As the frequency is reduced, X_L is reduced and X_c is increased. Thus, the vector $V_L = IX_L$ is correspondingly reduced and the vector $V_c = IX_c$ increased. The condition where the frequency of the supply voltage is such that X_c is greater than X_L is shown in Fig. 21.

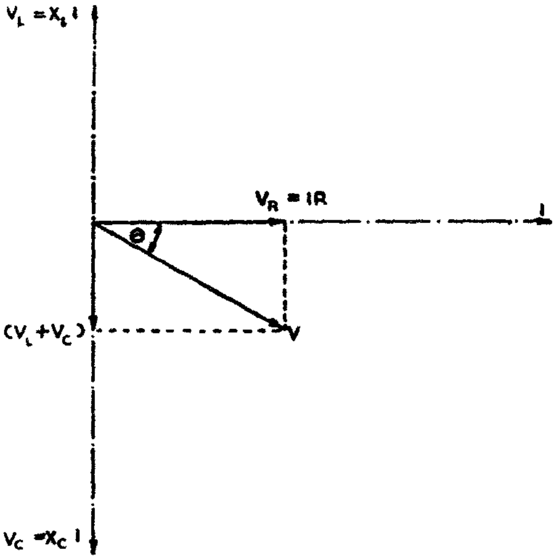


Fig. 21. EFFECT OF VARIATION OF FREQUENCY

From this vector diagram it is seen that the resultant voltage V now *lags* the current I. Thus, the applied voltage V either leads or lags the current I depending on whether V_L is greater than V_c, i.e. $X_L > X_c$, or V_c is greater than V_L, i.e. $X_c > X_L$. If the voltage leads the current (tan θ positive) the circuit is "inductive"; if the voltage lags the current (tan θ negative) the circuit is "capacitive".

31. Similar results are obtained if the frequency of the supply is fixed and the value of L or C is altered. With a constant value of ω the inductive reactance ωL can be altered by varying the value of L. Similarly, a variation in C will alter the capacitive reactance $-\frac{1}{\omega C}$. In this way, X_L or X_c can be varied to give resultant vector diagrams similar to Figs. 19 and 21.

32. **Summary.** In a circuit containing resistance, inductance and capacitance in series.:—

(*a*) The *magnitude* of the applied voltage is $V = I\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$.

(*b*) The *phase* of the applied voltage relative to the current is found from $\tan\ \theta = \frac{(\omega L - {}^1/\omega C)}{R}$. The angle is either

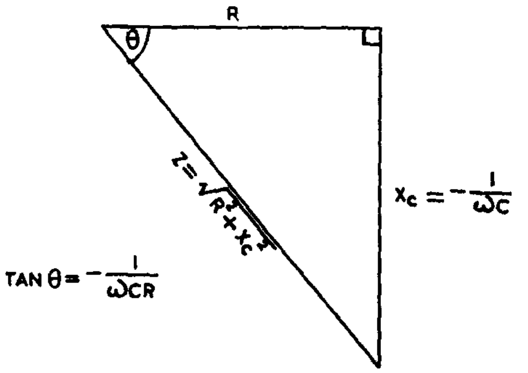


Fig. 16. IMPEDANCE TRIANGLE, R AND C IN SERIES

(*b*) The *phase* of the current relative to the applied voltage is found from $\tan\theta = \frac{X_c}{R}$. The current *leads* the voltage in a capacitive circuit.

(*c*) The *impedance* of the circuit is $Z = \sqrt{R^2 + X_c^2}$.

Resistance, Inductance and Capacitance in Series

26. Fig. 17 shows a circuit consisting of R, L, and C connected in series across an alternating supply of frequency f. With a current of r.m.s value I established in the circuit, p.d.s. are developed across each component.

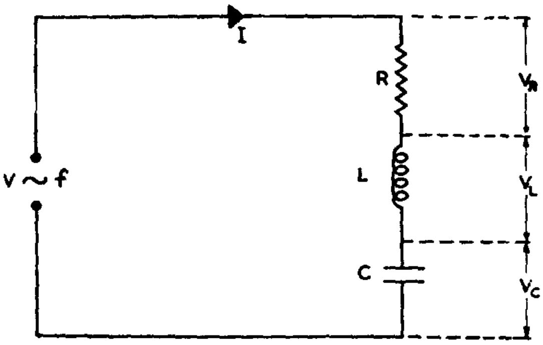


Fig. 17 R, L, AND C IN SERIES

(*a*) The p.d. across R is $V_R = I\,R$ and is *in phase* with I.

(*b*) The p.d. across L is $V_L = IX_L$ and it *leads* I by 90°. Its magnitude increases with frequency.

(*c*) The p.d. across C is $V_C = IX_C$ and it *lags* I by 90°. Its magnitude decreases as the frequency increases.

27. The resultant vector diagram is constructed as shown in Fig. 18, the current I, which is common to all components, being the reference vector. By Kirchhoff's second law the resultant of V_R, V_L, and V_C must equal the applied voltage V. To obtain this:—

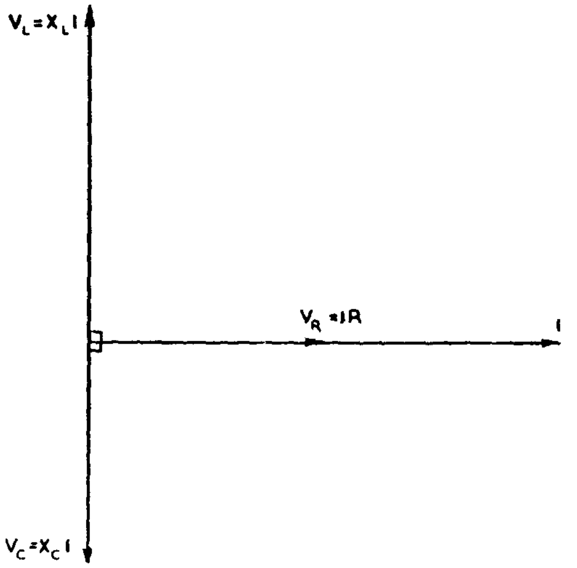


Fig. 18. VECTORS FOR R, L, AND C IN SERIES

(*a*) The two vectors $V_L = IX_L$ and $V_C = IX_C$ are added to give a single resultant vector $I\,(X_L + X_C)$. Since the capacitive reactance is negative with respect to the inductive reactance and is given by $X_C = -\frac{1}{\omega C}$, the resultant vector is $I(\omega L - \frac{1}{\omega C})$.

(*b*) Apply the rule of the parallelogram of forces to vector V_R and the combined vector $(V_L + V_C)$ to obtain the resultant applied voltage vector V. This is shown in Fig. 19.

28. Apply Pythagoras' theorem to Fig. 19:—

$$V^2 = V_R{}^2 + (V_L + V_C)^2$$

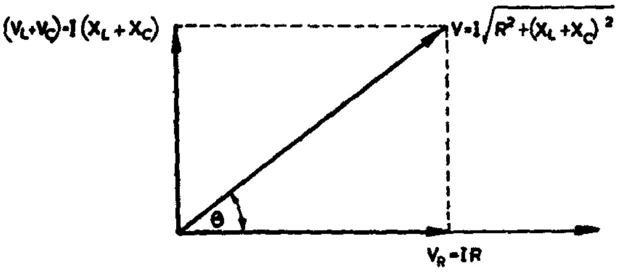


Fig. 19. RESULTANT APPLIED VOLTAGE

SECTION 5

CHAPTER 3

PARALLEL A.C. CIRCUITS

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PART 1, SECTION 5, CHAPTER 3

PARALLEL A.C. CIRCUITS

Introduction

1. In this Chapter, circuits consisting of components connected *in parallel* across an a.c. supply are considered. The phase and magnitude relationships for currents and voltages are discussed, as they were for the series circuits of Chap. 2. There are however, important differences in the construction of vector diagrams for series and for parallel a.c. circuits, and these are summarized in Table 1.

Series Circuit	**Parallel Circuit**
(*a*) All components carry the *same current* in the same phase.	(*a*) All branches have have the *same voltage* across them in the same phase.
(*b*) The resultant applied voltage is the vector sum of the individual voltages across the separate components.	(*b*) The resultant supply current is the vector resultant of the individual currents in the separate branches.
(*c*) Vector diagrams are constructed by drawing the voltage vectors relative to the *current*, which is the reference vector.	(*c*) Vector diagrams are constructed by drawing the current vectors relative to the *voltage*, which is the reference vector.

TABLE 1.
COMPARISON OF SERIES AND PARALLEL A.C. CIRCUITS

Admittance, Conductance, and Susceptance

2. When discussing parallel circuits it is often more convenient to consider the *reciprocals* of impedance, resistance, and reactance. For instance, for resistors in parallel, the reciprocal of the total resistance equals the sum of the reciprocals of the individual resistances:—

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ .. \quad \text{(ohms).}$$

This can be written in the reciprocal form:—

$$G_T = G_1 + G_2 + G_3 + \ .. \quad ..\text{(mhos),}$$

$$\text{where } G = \frac{1}{R}\ .$$

3. The reciprocal of impedance Z is termed *admittance* (symbol Y). Thus:—

$$Z = \frac{V}{I} \text{ (ohms)}$$

$$\therefore\ Y = \frac{1}{Z} = \frac{I}{V} \text{ (mhos).}$$

In general, the impedance Z is a combination of resistance R and reactance X and is given by:—

$$Z = \sqrt{R^2 + X^2} \quad \text{(ohms).}$$

Similarly, the admittance Y has two components termed *conductance* (symbol G) and *susceptance* (symbol B) and it is given by:—

$$Y = \sqrt{G^2 + B^2} \quad \text{(mhos).}$$

$$\text{where, } G = \frac{R}{R^2 + X^2}$$

$$B = \frac{X}{R^2 + X^2}$$

Note. Conductance $G = \frac{1}{R}$ only when Z is purely resistive ($X = O$).

Susceptance $B = \frac{1}{X}$ only when Z is purely reactive ($R = O$).

Resistance and Inductance in Parallel

4. The vector diagram for the circuit of Fig. 1(*a*) is given in Fig. 1(*b*). This vector diagram is constructed as follows:—

(*a*) The applied voltage V is common to both R and L and is the reference vector.

(*b*) The current through the resistor is $I_R = \frac{V}{R}$ and is *in phase* with V. The vector I_R is drawn to any convenient scale in line with V.

(*c*) The current through the inductor is $I_L = \frac{V}{X_L}$ and *lags* 90° on V. The vector I_L is drawn to the same scale as I_R, lagging V by 90°.

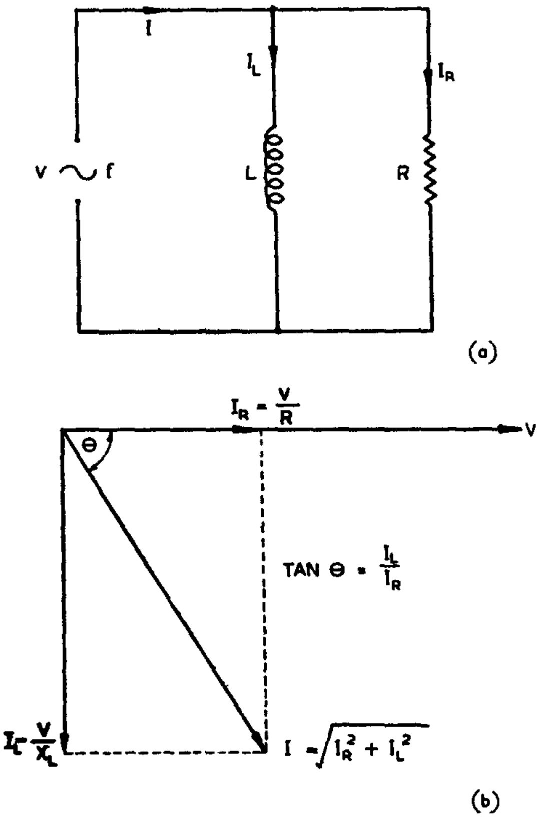


Fig. 1. R AND L IN PARALLEL

(*d*) The resultant supply current I is the vector sum of I_R and I_L and is found by completing the parallelogram and applying Pythagoras' theorem to Fig. 1(*b*). Thus:—

$$I = \sqrt{I_R^2 + I_L^2}$$

$$I = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L}\right)^2}$$

$$I = V\sqrt{\frac{1}{R^2} + \frac{1}{X_L^2}} \quad \cdots \quad \cdots \quad (1)$$

(*e*) I *lags* V by an angle θ, where

$$\tan\theta = \frac{I_L}{I_R} = \frac{V/X_L}{V/R}$$

$$\tan\theta = \frac{R}{X_L} = \frac{\text{Resistance}}{\text{Reactance}} \quad \cdots \quad \cdots \quad (2)$$

(*f*) From equation (1) :—

$$\frac{V}{I} = \frac{1}{\sqrt{\frac{1}{R^2} + \frac{1}{X_L^2}}} = \text{Impedance Z} \; (3)$$

5. The same results can be obtained by considering the admittances of the separate branches, and adding vectorially:—

(*a*) In the resistive branch there is no reactance X so that $G = \frac{1}{R}$ and $B = O$; hence $Y_R = \frac{1}{R}$.

(*b*) In the inductive branch there is no resistance R so that $G = O$ and $B = \frac{1}{X_L}$; hence $Y_L = \frac{1}{X_L}$.

(*c*) The total admittance is:—

$$Y = \sqrt{Y_R^2 + Y_L^2}$$

$$Y = \sqrt{\frac{1}{R^2} + \frac{1}{X_L^2}}$$

(*d*) The impedance is:—

$$Z = \frac{1}{Y} = \frac{1}{\sqrt{\frac{1}{R^2} + \frac{1}{X_L^2}}}.$$

Resistance and Capacitance in Parallel

6. The vector diagram for the circuit of Fig. 2(*a*) is given in Fig. 2(*b*). This vector diagram is constructed as follows:—

(*a*) The applied voltage V is the reference vector.

(*b*) The current through the resistor is $I_R = \frac{V}{R}$ and is *in phase* with V.

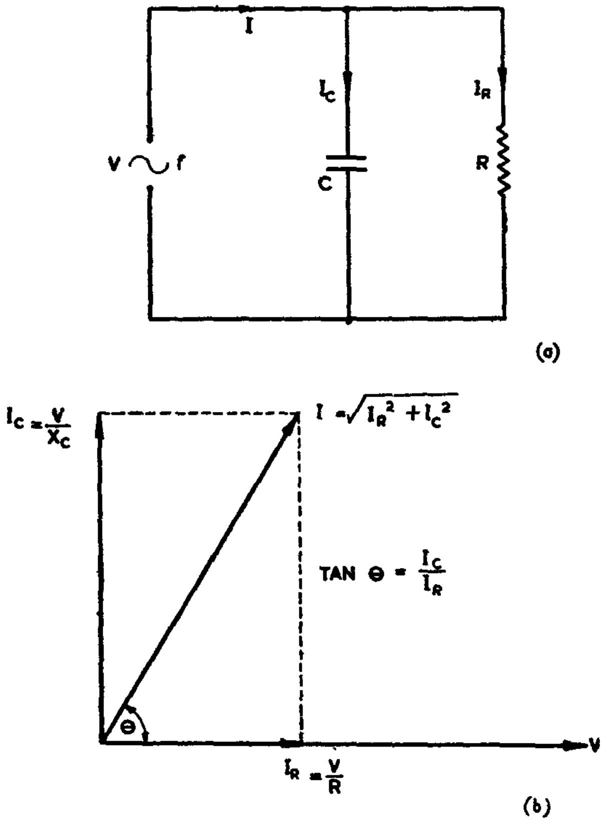


Fig. 2. R AND C IN PARALLEL

(*c*) The current through the capacitor is $I_c = \frac{V}{X_c}$ and *leads* 90° on V.

(*d*) The resultant supply current I is the vector sum of I_R and I_c and is found by completing the parallelogram and applying Pythagoras' theorem to Fig. 2(*b*). Thus:—

$$I = \sqrt{I_R^2 + I_c^2}$$

$$I = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_c}\right)^2}$$

$$I = V\sqrt{\frac{1}{R^2} + \frac{1}{X_c^2}} \quad \ldots \quad \ldots \quad (4)$$

(*e*) I *leads* V by an angle θ, where:—

$$\tan\theta = \frac{I_c}{I_R} = \frac{V/X_c}{V/R}$$

$$\tan\theta = \frac{R}{X_c} = \frac{\text{Resistance}}{\text{Reactance}} \quad \ldots \quad (5)$$

(*f*) From equation (4):—

$$\frac{V}{I} = \frac{1}{\sqrt{\frac{1}{R^2} + \frac{1}{X_c^2}}} = \text{Impedance } Z \quad (6)$$

Parallel Tuned Circuit with Resistance

7. Consider the circuit of Fig. 3. This shows a practical arrangement of coil and capacitor in a parallel tuned circuit. The coil has certain power losses and these are represented by a resistance in series with the inductance of the coil. The capacitor losses are small in comparison and are neglected. This approximation is satisfactory for most purposes and is normally assumed when considering low power r.f. circuits.

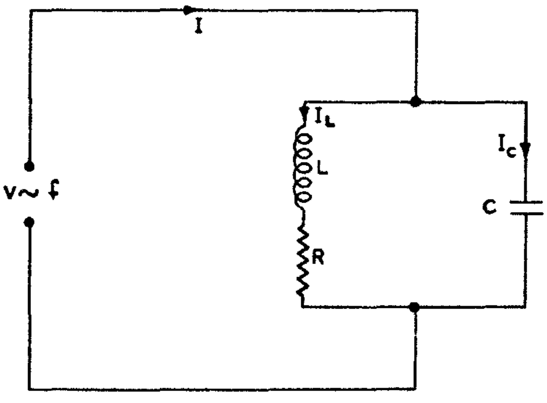


Fig. 3. PRACTICAL PARALLEL TUNED CIRCUIT

8. The vector diagram for the circuit is constructed as shown in Fig. 4:—

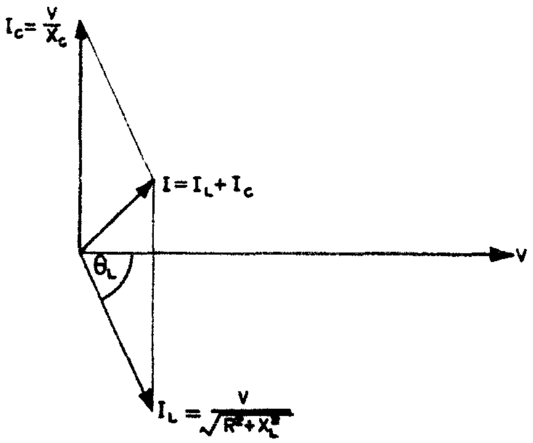


Fig. 4. VECTOR RELATIONSHIPS IN A PARALLEL TUNED CIRCUIT

(*a*) The applied voltage V is the reference vector.

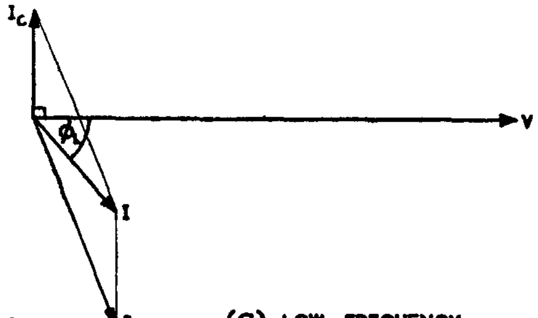
(*b*) The capacitive current is $I_c = \frac{V}{X_c}$ and *leads* V by 90°. I_c *increases* with frequency.

(*c*) The inductive current is $I_L = \frac{V}{\sqrt{R^2 + X_L^2}}$ and *lags* V by an angle θ_L which is less than 90° by virtue of the series resistance. I_L *decreases* as the frequency increases.

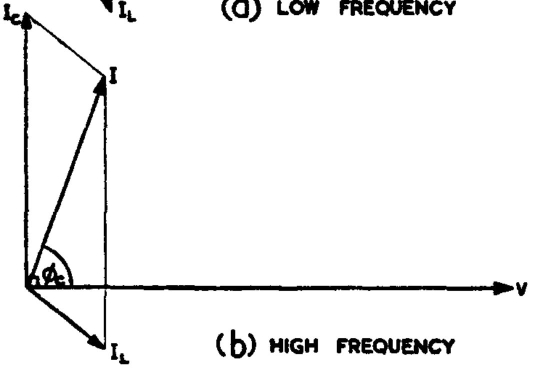
(*d*) The resultant supply current I is the vector sum of I_c and I_L.

9. The *phase* of the supply current I relative to the applied voltage V depends on the supply frequency, as well as on the values of L and C. At a low frequency, $I_c = \frac{V}{X_c} = V\omega C$ will be small and $I_L = \frac{V}{\sqrt{R^2 + \omega^2 L^2}}$ will be large so that I *lags* V by an angle $\varnothing_L$ (Fig. 5(*a*)). At a high frequency, I_c will be large and I_L small so that I *leads* V by an angle $\varnothing_c$ (Fig. 5(*b*)). At a particular frequency (the resonant frequency) I will be *in phase* with V (Fig. 5(*c*)).

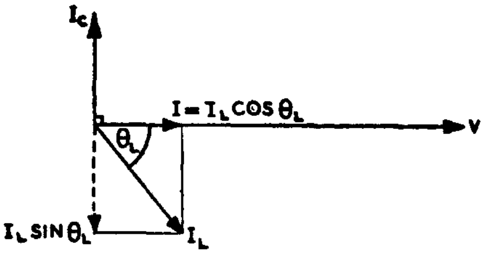
10. Resonance occurs in a parallel tuned circuit when the supply current I is in phase with the applied voltage V. For this to be so, *the out-of-phase* (*or reactive*) *component* of I_L must equal I_c. There is then no reactance offered by the circuit and its impedance is a pure resistance. Thus, the



(a) LOW FREQUENCY



(b) HIGH FREQUENCY



(c) RESONANT FREQUENCY

Fig. 5. EFFECT OF VARIATION OF FREQUENCY

condition for resonance in a parallel tuned circuit is that of *zero reactance.*

11. **Resonant frequency.** The out-of-phase component of a current I_L at an angle θ_L to the reference is $I_L \sin \theta_L$ as shown in Fig. 5(*c*). Thus for resonance to occur:—

$$I_L \sin \theta_L = I_C$$

Now, $I_C = \frac{V}{X_C}$

$$I_L = \frac{V}{\sqrt{R^2 + X_L^2}} = \frac{V}{Z_L}$$

$$\sin \theta_L = \frac{X_L}{Z_L} \text{ (from Fig. 6)}$$

$$\therefore I_L \sin \theta_L = \frac{V}{Z_L} \times \frac{X_L}{Z_L}$$

$$\therefore I_L \sin \theta_L = \frac{VX_L}{Z_L^2}$$

Hence, at resonance:—

$$I_L \sin \theta_L = I_C$$

$$\therefore \frac{VX_L}{Z_L^2} = \frac{V}{X_C}$$

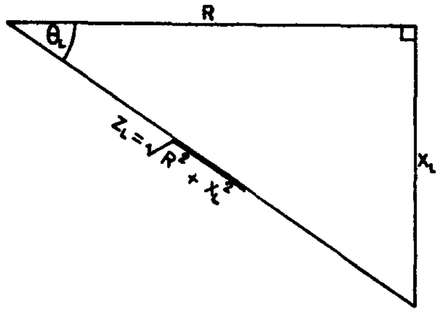


Fig. 6. IMPEDANCE TRIANGLE, R AND L IN SERIES

$$\frac{X_L}{Z_L^2} = \frac{1}{X_C}$$

$$\frac{\omega L}{R^2 + \omega^2 L^2} = \omega C$$

$$\frac{L}{C} = R^2 + \omega^2 L^2$$

$$\therefore \omega^2 L^2 = \frac{L}{C} - R^2 \quad \ldots \quad \ldots \quad (7)$$

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\therefore f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad \ldots \quad (8)$$

This gives the frequency at which resonance occurs, and unlike the series tuned circuit R has an effect on the resonant frequency of a parallel tuned circuit. In most cases, the resistance is very small and $\frac{R^2}{L^2}$ can be neglected, so that for practical purposes the resonant frequency can be taken as $f_o = \frac{1}{2\pi\sqrt{LC}}$.

12. **Dynamic impedance.** At resonance in a parallel tuned circuit there is zero reactance and the supply current I is in phase with the applied voltage V. From Fig. 5(*c*) it is seen that at resonance, I is equal to *the in-phase* (*or resistive*) *component* of I_L. Thus at resonance:—

$$I = I_L \cos \theta_L$$

But $I_L = \frac{V}{\sqrt{R^2 + X_L^2}}$

And $\cos \theta_L = \frac{R}{\sqrt{R^2 + X_L^2}}$ (From Fig. 6)

$$I = I_L \cos \theta_L$$

$$\therefore I = \frac{VR}{R^2 + X_L^2}$$

$$I = \frac{VR}{R^2 + \omega^2 L^2}$$

$$\therefore Z = \frac{V}{I} = \frac{R^2 + \omega^2 L^2}{R}$$

From equation (7):—

$$\omega^2 L^2 = \frac{L}{C} - R^2$$

$$\therefore Z = \frac{R^2 + (L/C - R^2)}{R} = \frac{L/C}{R}$$

$$\therefore Z = \frac{L}{CR} \text{ (ohms)} \quad .. \quad .. \quad (9)$$

The expressions $Z = \frac{L}{CR}$ is known as the *dynamic impedance* and is the purely resistive impedance of a parallel tuned circuit *at resonance.*

13. **Conditions at resonance.**

(*a*) The frequency at which resonance occurs is:—

$$f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}.$$

The resonant frequency is partly dependent on the loss resistance R.

(*b*) The impedance at resonance is a *maximum* and equal to $\frac{L}{CR}$. An interesting feature about this expression is that the dynamic resistance is *inversely* proportional to R.

(*c*) The supply current is a *minimum* at resonance and is given by:—

$$I = \frac{V}{Z} = \frac{V}{L/CR}$$

$$I = \frac{VCR}{L} \quad .. \quad .. \quad .. \quad (10)$$

The supply current at resonance is, therefore, increased by an *increase* either in R or in the $\frac{C}{L}$ ratio.

(*d*) At resonance in the parallel tuned circuit the supply current is *in phase* with the applied voltage, the reactance in the circuit being zero.

Note. A parallel tuned circuit is often termed a *rejector* circuit since it presents maximum impedance to an input at its resonant frequency.

14. **Conditions of f resonance.**

(*a*) The impedance falls and the supply current rises either side of the resonant frequency. The amount by which these factors varies depends on:—

(i) The departure from resonance.
(ii) The ratio of C to L.
(iii) The value of R.

(*b*) If the applied frequency is *greater* than the resonant frequency of the circuit, the current $I_c = V\omega C$ through the capacitor will increase and the current $I_L = \frac{V}{\sqrt{R^2 + \omega^2 L^2}}$ through the inductive branch will decrease. The resultant supply current therefore *leads* the applied voltage and the circuit is *capacitive* (see Fig. 5(*b*)).

(*c*) If the applied frequency is *less* than the resonant frequency the reverse occurs. The supply current then *lags* the applied voltage and the circuit is *inductive* (see Fig. 5(*a*)).

15. **Resistance neglected.** If the coil losses are so small that R can be neglected, the parallel tuned circuit has the "ideal" form of Fig. 7. The vector diagrams for this circuit will be as shown in Fig. 8. If the applied frequency, or the values of L and C are such that X_c is greater than X_L, I *leads* V by 90° and the circuit behaves as a pure capacitance (Fig. 8(*a*)). If X_L is greater than X_c, I *lags* V by 90°, and the circuit behaves as a pure inductance (Fig. 8(*b*)). If X_L equals X_c, the supply current into the circuit is *zero* and the circuit behaves as an *infinite resistance.* This is the resonant condition (Fig. 8(*c*)), which occurs at a frequency $f_o = \frac{1}{2\pi\sqrt{LC}}$.

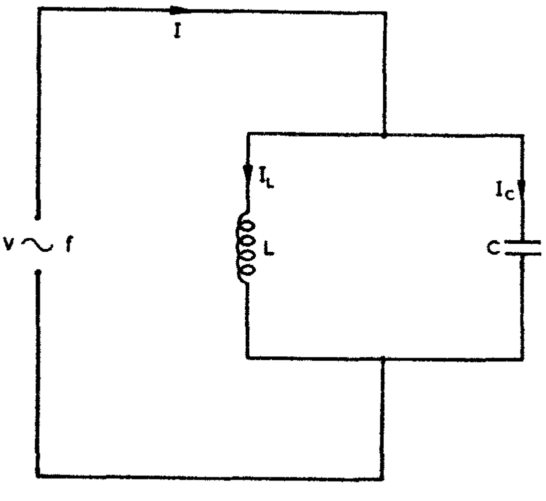


Fig. 7. IDEAL PARALLEL TUNED CIRCUIT

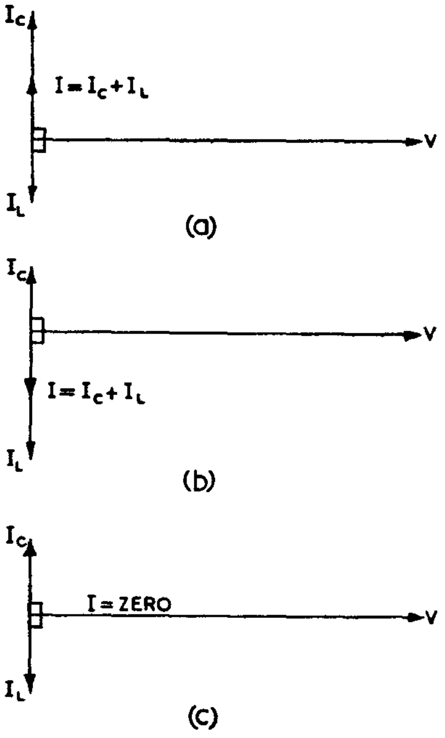


Fig. 8. EFFECT OF VARYING X_C AND X_L

Circuit Magnification

16. The *supply current* is the current into a parallel tuned circuit from the supply. At resonance, this current is a minimum and is given by equation (10). Either side of resonance, the supply current increases and the phase angle varies as described in Para. 14.

17. The current *within* the closed LCR circuit is known as the *circulating current* and is equal to I_L or I_C. It has a maximum value at *resonance* and thus varies inversely with the supply current. In a practical tuned circuit, R is small in comparison with X_L and for most purposes can be neglected. On this assumption, at resonance:—

$$\text{Circulating current} = I_C = I_L$$

$$\therefore I_L^2 = I_L I_C$$

$$I_L^2 = \frac{V}{\omega L} \cdot V\omega C.$$

$$I_L^2 = \frac{V^2 C}{L}$$

$$\therefore I_L = V\sqrt{\frac{C}{L}} = \text{Circulating current (11)}$$

Alternatively:—

$$\text{Circulating current} = I_L \simeq \frac{V}{\omega L}$$

$$\text{But } V = IZ,$$

$$\text{where } Z = \frac{L}{CR} \text{ at resonance}$$

$$\therefore V = \frac{IL}{CR}$$

$$\therefore I_L = \frac{IL}{CR} \times \frac{1}{\omega L}$$

$$I_L = \frac{I}{\omega CR}$$

$$\therefore I_L = QI \quad .. \quad .. \quad .. \quad (12)$$

$$\text{where } Q = \frac{1}{\omega CR} = \frac{\omega L}{R} \text{ (by definition)}$$

$$\text{and } I = \text{Supply current.}$$

Thus, at resonance in a parallel tuned circuit, the circulating current is a maximum and equal to Q times the supply current. *Current magnification* has taken place. It will be remembered that a series tuned circuit is a *voltage magnifier.*

Selectivity in a Parallel Circuit

18. **Selectivity.** This is the property of a tuned circuit which enables it to discriminate between the desired signal and signals at other frequencies. The variation of supply current with frequency is shown in Fig. 9(*a*). The sharpness of the response curve denotes the selectivity of the circuit. For parallel circuits, selectivity is often defined in terms of impedance rather than of current, and the higher the impedance at resonance in relation to the impedance at frequencies off resonance, the greater is the selectivity of the circuit. Fig. 9(*b*) shows the variation of impedance with frequency.

19. An *increase* in resistance R will *reduce* the impedance at resonance ($Z = \frac{L}{CR}$) to a greater extent than at frequencies removed from resonance. Since $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$ is reduced, the response curve is "flattened" and the circuit is made less selective. A variation in the ratio of L to C will also affect the impedance at resonance and, hence, the selectivity of the circuit. Thus, a circuit with a high value of Q has a high selectivity, and *vice versa* (Fig. 10).

20. **Q and bandwidth.** Parallel tuned circuits are used in radio equipments to "reject" inputs at, and near, the resonant frequency by offering a high impedance at those frequencies. To inputs at frequencies considerably removed from the resonant frequency, the parallel tuned circuit should

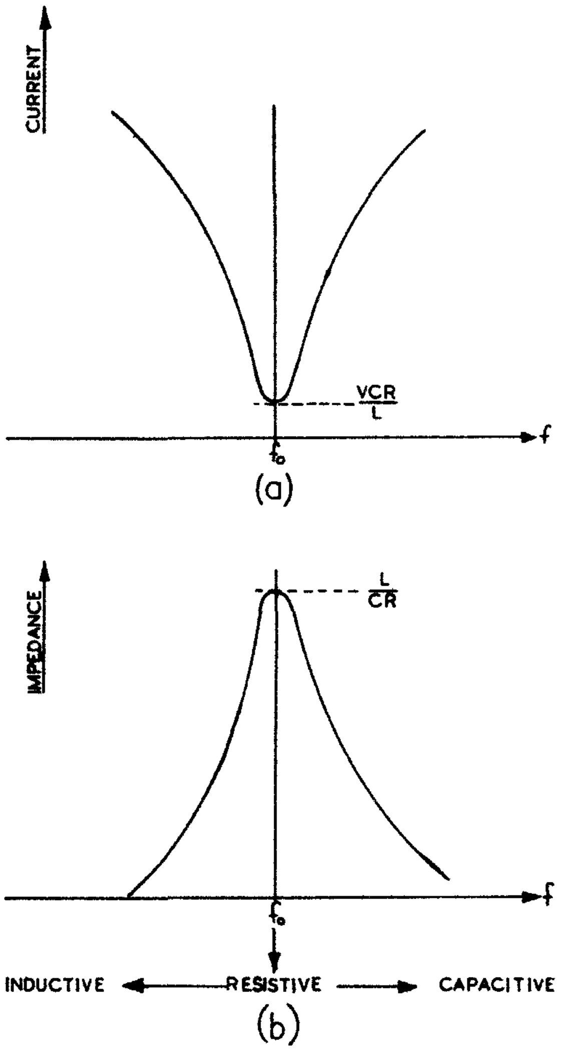


Fig. 9. VARIATION OF I AND Z WITH FREQUENCY ABOUT RESONANCE

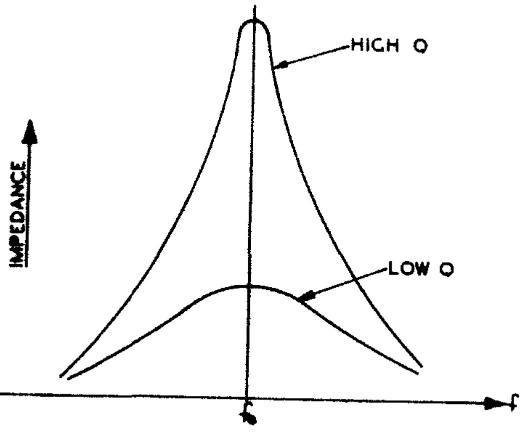


Fig. 10. Q AND SELECTIVITY

offer a low impedance. To do this satisfactorily, the circuit must have a high Q or a *narrow bandwidth.* The bandwidth of a parallel tuned circuit is defined as the difference between two frequencies f_1 and f_2 at which the impedance has fallen to 70% of the resonant value (Fig. 11). For purposes of calculations, Q and the bandwidth are related by the expression:—

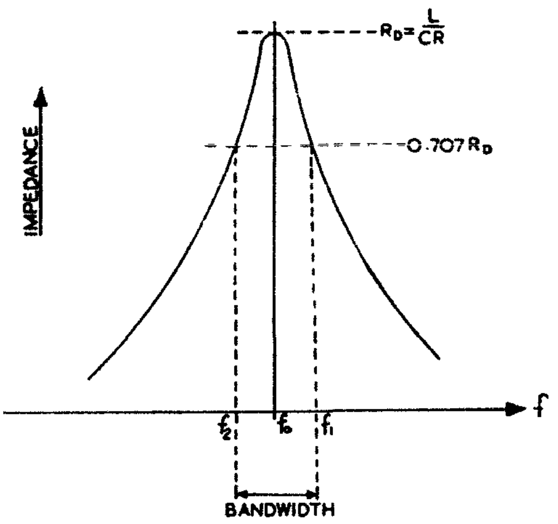
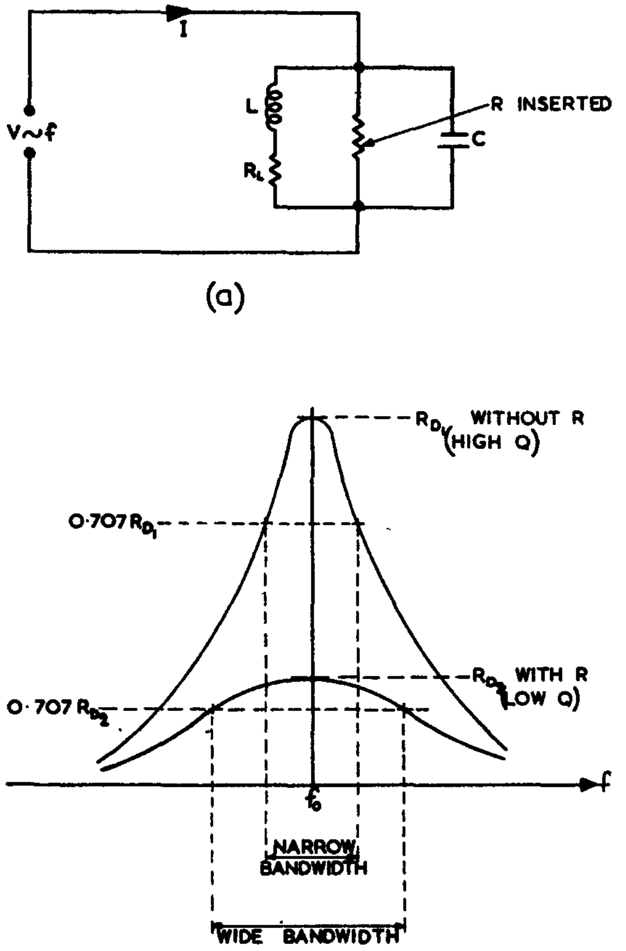


Fig. 11. THE HALF-POWER BANDWIDTH

$$Q = \frac{\text{Resonant frequency}}{\text{Bandwidth}}$$

$$Q = \frac{f_o}{f_1 - f_2} \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

21. **"Damping" of parallel circuits.** In some circuits in radio equipments it is desired to "pass" a wide band of frequencies in the neighbourhood of resonance and for this, a circuit with a wide bandwidth is required. From Para. 20 it is seen that the bandwidth of a parallel tuned circuit is inversely proportional to Q (i.e., Bandwidth $= \frac{f_o}{Q}$). Thus, a circuit with a low value of Q will give a wide bandwidth. To reduce Q, an actual resistor is inserted in parallel with the rejector circuit to "damp" the response (Fig. 12(*a*)). The impedance at resonance is then the dynamic impedance R_D ($= \frac{L}{CR_L}$) in parallel with R. Consequently the reduction of Q results in reduced impedance at resonance and a flattened response curve with a corresponding increase in bandwidth (Fig. 12(*b*)).


I
L
R_L
V~f
R INSERTED
C
R_D1 WITHOUT R (HIGH Q)
0·707 R_D1
R_D2 WITH R (LOW Q)
0·707 R_D2
f
f_0
NARROW BANDWIDTH
WIDE BANDWIDTH

(a)

(b)

Fig. 12. EFFECT OF DAMPING ON SELECTIVITY

22. **Effect of supply impedance on selectivity.** When a parallel circuit is used for tuning purposes, the idea is that the voltage across the capacitor shall be applied to a further stage. The voltage applied to the circuit itself can be considered to be applied from a generator, and the behaviour of the circuit then depends on the impedance of the generator as well as on the characteristics of the circuit itself. If the internal impedance of the generator R_G is small compared with the impedance of the parallel tuned circuit then V_C will be approximately equal to V at all frequencies (Fig. 13). If however, R_G is large compared with the impedance of the circuit, then the current will be approximately equal to $\frac{V}{R_G}$. Thus, V_C will be proportional to the impedance of the parallel circuit and will vary with frequency, being a maximum at resonance. Hence the parallel circuit is selective only if supplied from a generator having a *high* internal impedance.

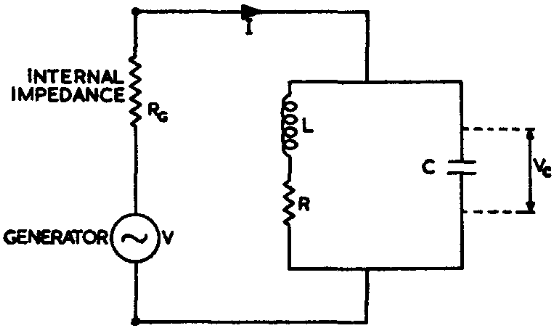

I
INTERNAL IMPEDANCE
R_G
L
C
V_c
R
GENERATOR
V

Fig. 13. EFFECT OF SUPPLY IMPEDANCE ON SELECTIVITY

SECTION 5

CHAPTER 4

POLYPHASE A.C. SYSTEMS

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PART 1, SECTION 5, CHAPTER 4

POLYPHASE A.C. SYSTEMS

Introduction

1. The simple a.c. generator discussed in Sect. 3, Chap. 1 produced a *single-phase* a.c. output at its slip rings. In other a.c. generators the armature winding consists of two or more groups of series-connected coils with their outer ends connected to separate slip rings. Two or more alternating voltages are then produced at the slip rings, these voltages being in general out of phase with each other. Such machines are known as *polyphase* or *multiphase* a.c. generators.

Effect of Load on Supply

2. In polyphase systems, loads can be made to draw power from the generator at a uniform rate, so that the machine runs steadily under a uniform torque. Fig. 1 shows the relationship between current and voltage when a *single-phase* a.c. generator is supplying an *inductive* load. The instantaneous power is the product VI at each instant and is indicated by the shaded portion of the graph. When the power graph is above the time axis, it indicates that energy is being drawn from the generator; below this axis, energy is being returned from the inductive load. Since the load on the generator not only fluctuates during each half cycle but also changes sign, the torque changes in a similar manner. Though this may happen at the same stage in the *individual* cycles of each phase in a multiphase system, the power peak and zero values will not normally coincide because of the phase differences. Power is, therefore, drawn from the generator at a much more uniform rate and the torque remains more steady.

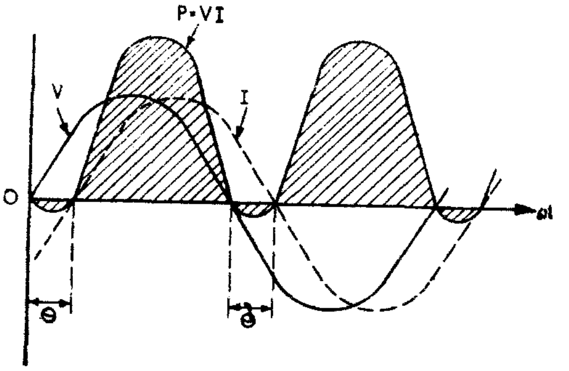


Fig. 1. POWER VARIATION WITH INDUCTIVE LOAD IN A SINGLE PHASE SUPPLY

Generation of Polyphase Voltages

3. The output of a simple a.c. generator consisting of a single group of coils rotating uniformly in a uniform magnetic field is a sine wave V_1 as shown in Fig. 2(*a*) which represents the voltage available at the slip rings, A and A_1.

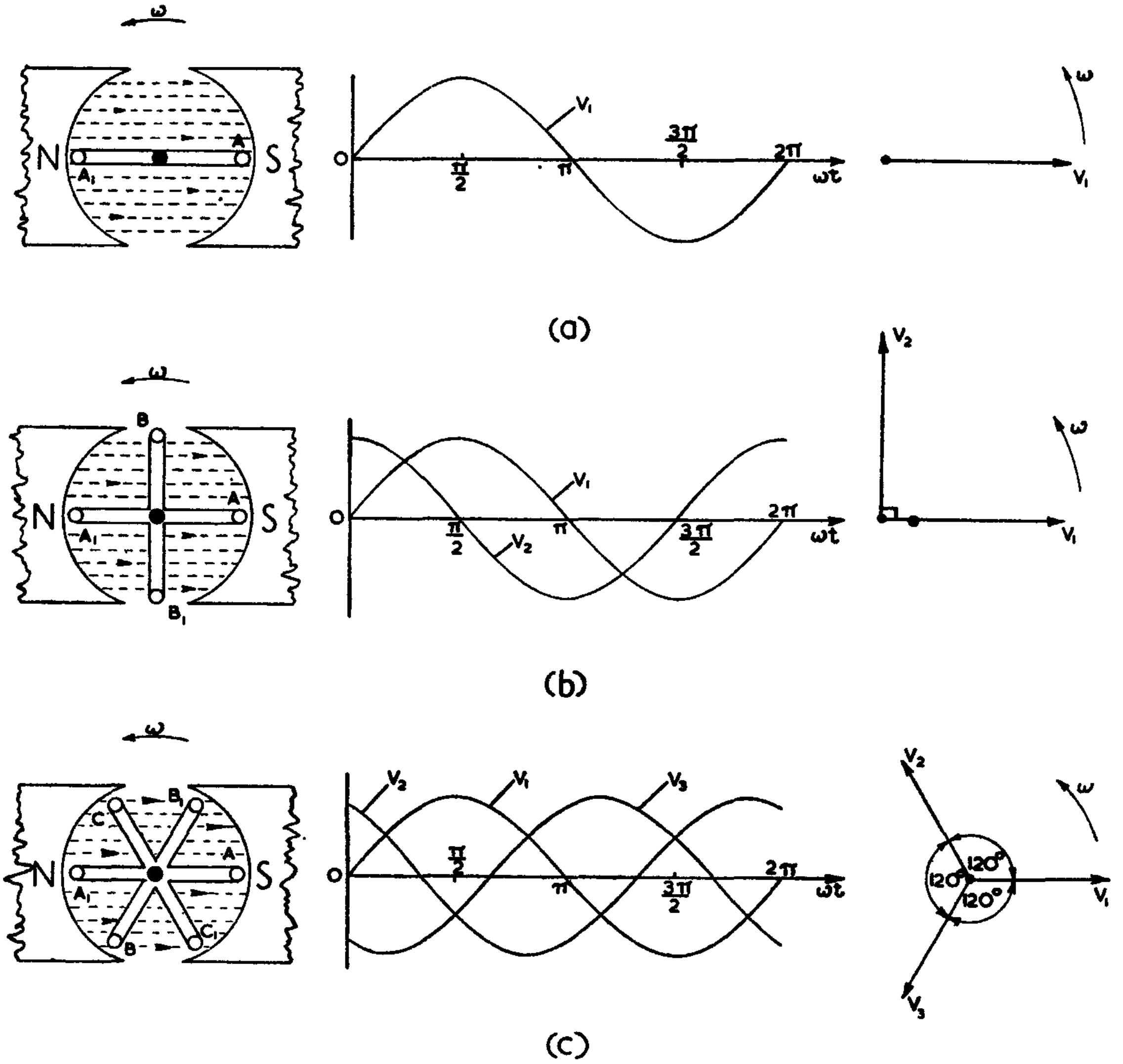
4. If, on the same armature core, a second group of coils is mounted at right angles to the first and connected to a second pair of slip rings B and B_1, two independent voltages are available, differing in phase by 90°. This arrangement, shown in Fig. 2(*b*), represents a *two-phase* generator.

5. With three groups of coils wound independently on the same armature and connected to three separate pairs of slip rings (AA_1, BB_1, and CC_1) three independent voltages are generated as indicated in Fig. 2(*c*). The *start* of one coil is at an angle of 120° to the start of the next (e.g., A is at an angle of 120° to B) and the three voltages differ in phase by 120°, the arrangement representing a *three-phase* generator. It should be noted that when the voltage in one phase is at peak value, the voltage in each of the other phases is at *half* peak value in the *opposite* direction. The sum of the voltages in the three phases is thus zero; this holds good for all points in the cycle.

6. The armature can be wound with any number of symmetrically spaced groups of coils and independent pairs of slip rings; six-phase and twelve-phase a.c. supplies are sometimes used. However, since many of the advantages of polyphase voltages are available with three phases, the three-phase system is that in most general use for the generation and transmission of power.

Advantages of Polyphase Systems

7. The advantages of polyphase systems can be summarized as follows:—



(a)

(b)

(c)

Fig. 2. GENERATION OF POLYPHASE VOLTAGES

(*a*) For a given size, the power rating of a polyphase machine increases with the number of phases.

(*b*) The heating loss for a given power transmitted and the line voltage drop are both less than they would have been if the whole power had been transmitted by a single phase only.

(*c*) Loads can be made to draw power from the generator at a uniform rate (see Para. 2).

Three-phase Generator

8. A simple form of a.c. generator consisting of a coil caused to rotate in a magnetic field by a prime mover is described in Sect. 3, Chap. 1. Most actual machines are designed the other way round. That is, the rotating part (rotor) consists of electromagnets energized from a d.c. supply, while the coils in which the generated e.m.f. is induced are wounds on a fixed frame (stator). The two arrangements are electrically equivalent since the *relative* motion of flux and coils that gives rise to an induced e.m.f. is the same in both cases. However, fixed connections to stationary windings are easier to insulate at high voltages than slip rings would be, and whether the output is single-phase or polyphase, only one pair of slip rings is needed—that for the relatively low voltage d.c. energising supply.

9. Fig. 3 shows the arrangement of a two-pole, *single-phase* a.c. generator of the rotating field type. The stator winding consists of a number of coils connected

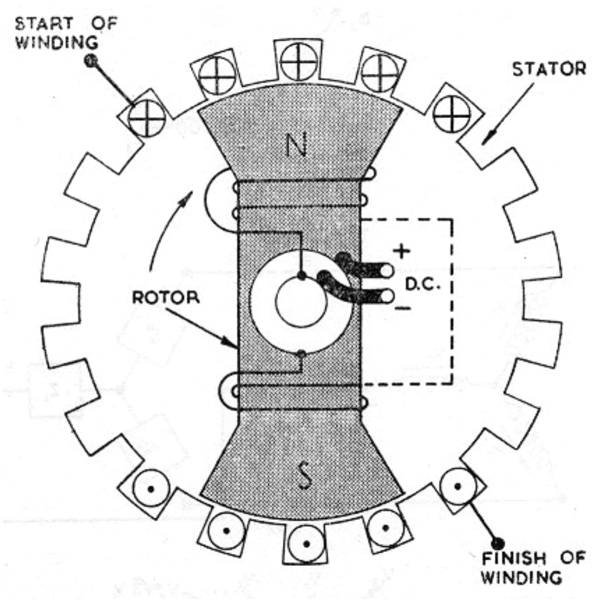


Fig. 3. TWO-POLE, SINGLE-PHASE A.C. GENERATOR

in series and inserted in slots cut in the inner surface of the laminated frame. The rotor is driven by a prime mover and carries the field windings which are energised from a d.c. source via slip rings as shown.

10. The stator of a *three-phase* a.c. generator has three separate windings equally spaced round the interior of the frame and occupying all the slots in the laminations. The poles of the rotor sweep past each winding in turn so that three alternating voltages are produced, differing in phase by 120°. This is shown in Fig. 4.

11. The e.m.f. generated in each conductor completes one cycle as it is passed by a *pair* of rotor poles. The machine shown in Fig. 4 has two poles, or one pair of poles on the rotor. However, some machines have many pairs of poles on the rotor with the stator windings spaced accordingly, in a manner similar to that for the multi-pole d.c. generator discussed in Sect. 3, Chap. 1. Thus, in one revolution of the rotor, the e.m.f. will complete a number of cycles corresponding to the number of pairs of poles in the generator. The frequency of the supply given by the machine will be:—

$$\text{Frequency } f = \frac{pN}{60} \text{ (c/s)} \quad \ldots \quad (1)$$

where, p = Number of pairs of poles
N = Speed in r.p.m.

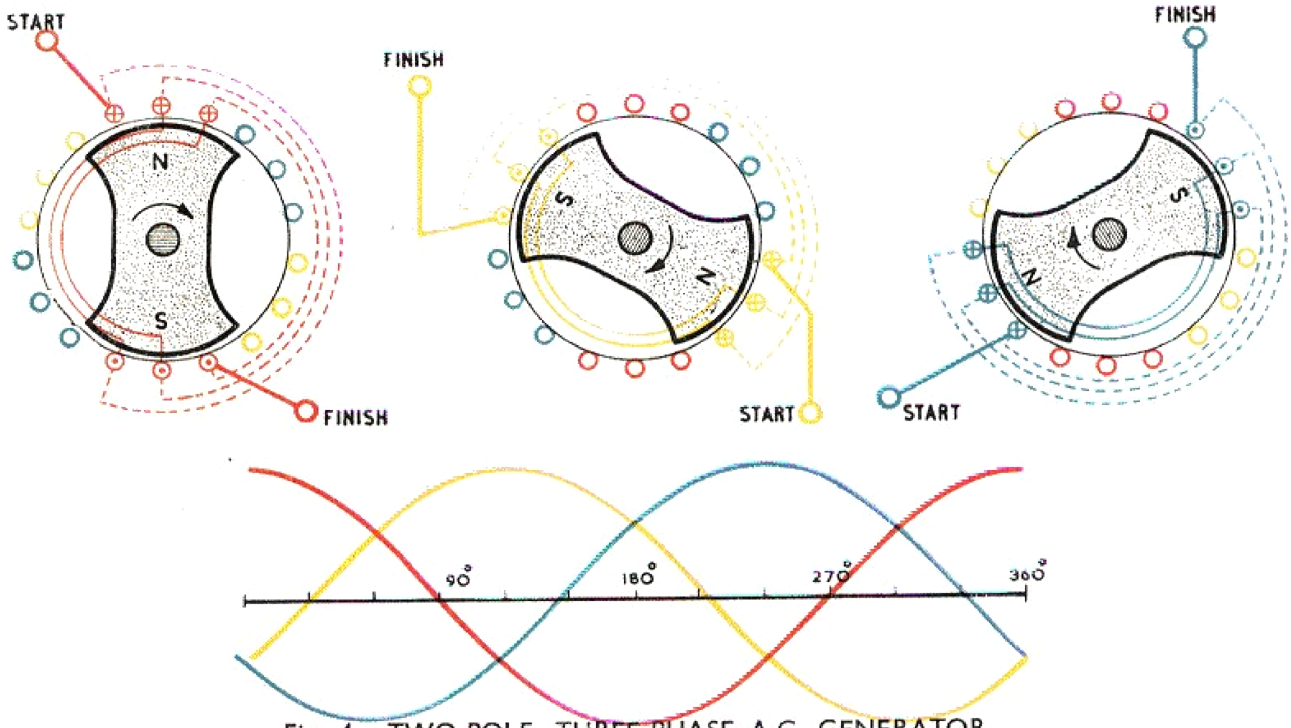


Fig. 4. TWO-POLE, THREE-PHASE A.C. GENERATOR

"This leaf issued with A.L. 3"

The factor N is called the *synchronous speed;* it is the speed at which the machine must run in order to generate the required frequency. Thus:—

$$N = \frac{f}{p} \times 60 \text{ (r.p.m.)} \quad .. \quad .. \quad (2)$$

Interconnection of Phases

12. Each phase of a three-phase generator may be brought out to separate terminals and used to supply separate loads independently of one another. This method requires a pair of lines for each phase. The number of wires may be reduced, with a consequent saving in cable, if the phases are inter-connected. There are two methods of connecting the generator windings and the loads in three-phase systems:—

(*a*) The "star" or "Y" connection.

(*b*) The "delta" or "mesh" connection.

13. **Star connection.** It is standard practice to identify each phase in a three-phase system by the colours *red, yellow,* and *blue* as shown in Fig. 4. Each phase is then referred to its colour. In Fig. 5(*a*), the starting ends of the three phase windings A_1, B_1, and C_1, are joined together at a common junction and the finishing ends A_2, B_2, and C_2 taken to the terminals T_1, T_2, and T_3. Loads Z_1, Z_2, and Z_3 may also be connected in this way (Fig. 5(*b*)) so that conditions are reproduced at the far end of the line, with the phase voltage applied across the corresponding load. The voltage between lines 1 and 2 is the vector *difference* between the individual phase voltages V_1 and V_2. Similarly for voltages between other pairs of lines. In the vector diagram of Fig. 5(*c*), the phase e.m.f.s are represented by the vectors V_1, V_2 and V_3 mutually displaced by 120°. Hence, the voltage between lines 1 and 2 is obtained by the vector addition of V_1 and V_2 reversed and is represented by V_{12}. Similarly, the voltages between lines 1 and 3, and between 2 and 3, are represented by V_{13} and V_{23} respectively. By calculation, the line voltage is found to be $\sqrt{3}$ *times* the phase voltage and leads it by 30°. Since each line is in series with its individual phase winding, the line current must always equal the phase current. For balanced loads, the three line currents will be equal in magnitude and spaced 120° in phase (Fig. 5(*d*)). For purely resistive loads, each *line current* will be in phase with its corresponding *phase voltage.*

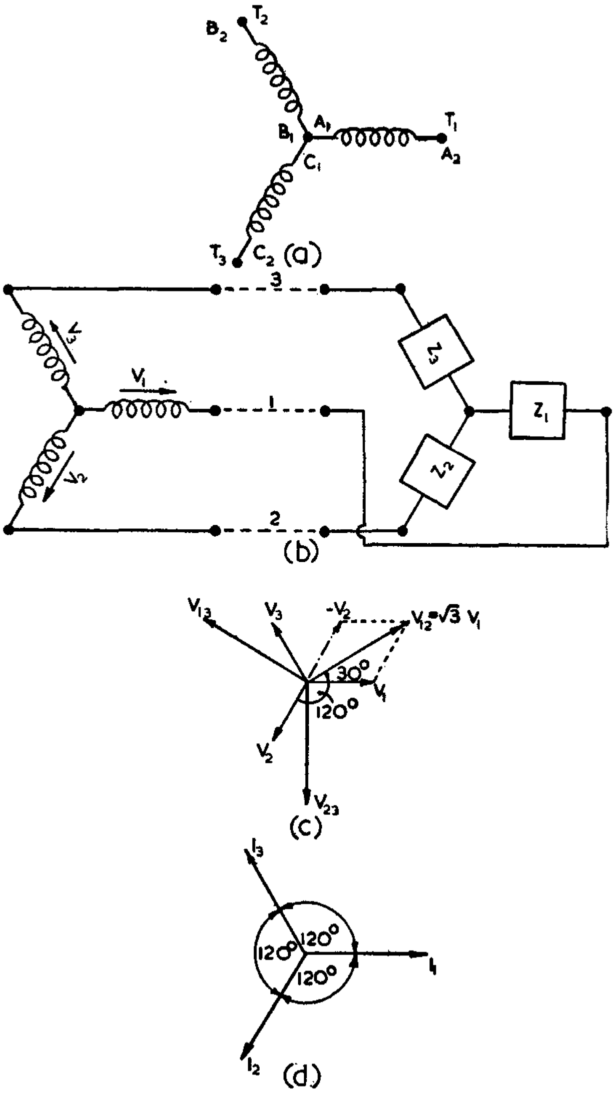


Fig. 5. THREE-PHASE STAR CONNECTION

14. **Delta (Δ) connection.** The three phase windings are connected in series to form a closed mesh, with the loads connected in a similar manner as shown in Fig. 6(*a*). Only one phase winding is connected between each pair of lines, so that the voltage between any two lines must equal the phase voltage; that is, V_{12} is equal to and in phase with V_2, and so on, as shown in Fig. 6(*b*). For balanced loads, the phase currents are equal to one another and differ in phase by 120°. The current in line 1, from the junction marked B, is the vector *difference* between that flowing to the junction from the red phase 1 and that flowing away down

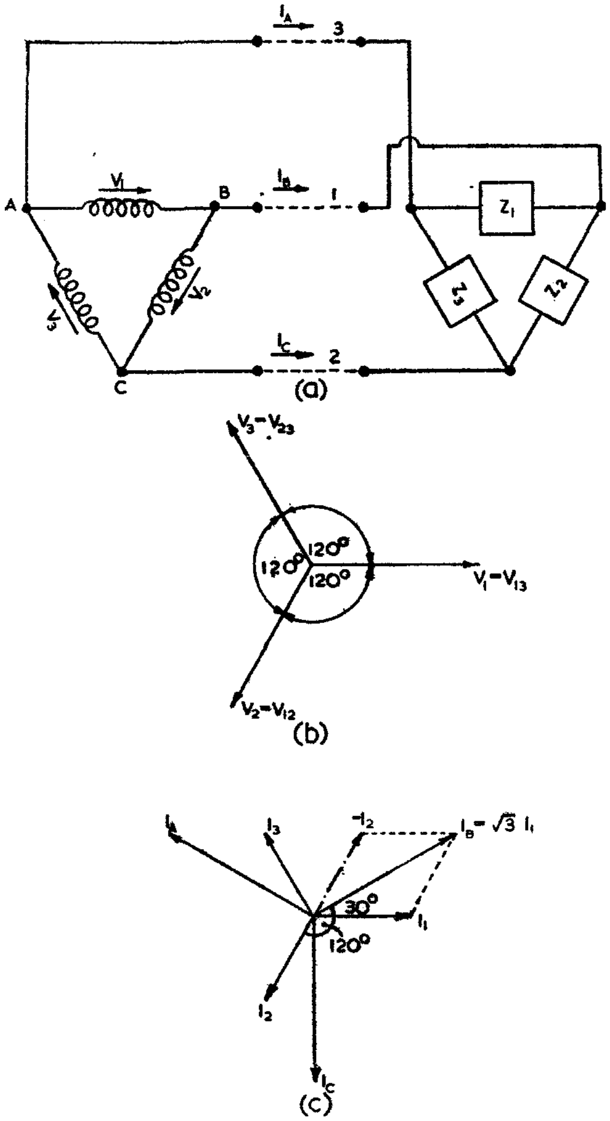


Fig. 6. THREE-PHASE DELTA CONNECTION

the blue phase 2. Similarly for the other line currents. In the vector diagram of Fig. 6(*c*) the phase currents are represented by I_1, I_2 and I_3 mutually displaced by 120°. Hence the current in line 1 is obtained by the vector addition of I_1 and I_2 reversed, and is represented by I_B. Similarly, the currents in line 2 and line 3 are represented by I_C and I_A respectively. By calculation, the line current is found to be $\sqrt{3}$ *times* the phase current and leads it by 30°. For purely resistive loads, each *line voltage* will be in phase with its corresponding *phase current*.

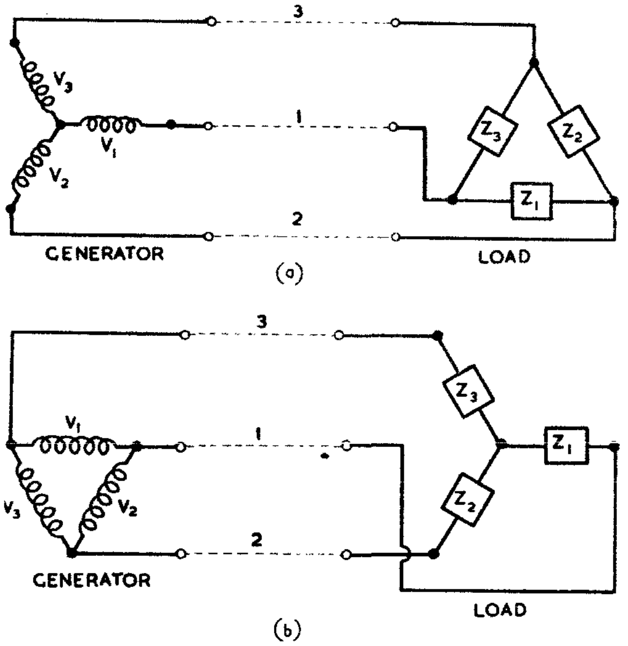


Fig. 7. STAR-DELTA TRANSFORMATION

15. **Star-delta transformation.** In some circumstances a delta-connected load to a star-connected generator (Fig. 7(*a*)), or a star-connected load to a delta-connected generator (Fig. 7(*b*)) may be required. With a star-delta connection, as in Fig. 7(*a*), the load voltage equals the line voltage which, for a star-connected generator, is $\sqrt{3}$ times the phase voltage. For the power to remain the same, the load current must be $\frac{1}{\sqrt{3}}$ times the line current. A voltage and a current transformation has taken place, with the power developed at each end remaining constant. With a delta-star connection between generator and load, as in Fig. 7(*b*), a current step-up and a voltage step-down occurs at the load. The power transmitted in each case must be the same.

16. **Power in three-phase circuits.** In a balanced three-phase circuit, the total power equals three times the power per phase:—

$$P = 3VI\cos\theta \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

Expressing this in line quantities:—

(*a*) *Star.*

$$P = 3\,\frac{V_L}{\sqrt{3}}\,I_L\cos\theta$$

$$P = \sqrt{3}\,V_L\,I_L\cos\theta$$

(*b*) *Delta.*

$$P = 3\,V_L\frac{I_L}{\sqrt{3}}\cos\theta$$

$$P = \sqrt{3}\,V_L\,I_L\cos\theta \quad \ldots \quad \ldots \quad (4)$$

where θ is the angle between phase voltage and phase current.

Production of a Rotating Field

17. Certain types of a.c. motors depend for their operation on the fact that current is induced in the rotor of the machine by a magnetic field which is *rotating* about the axis of the rotor. A rotating magnetic field can be produced by applying a three-phase supply to a *stationary* group of coils, provided the latter are suitably wound, spaced and connected. The field produced is of unvarying strength and its speed of rotation is directly related to the frequency of the supply.

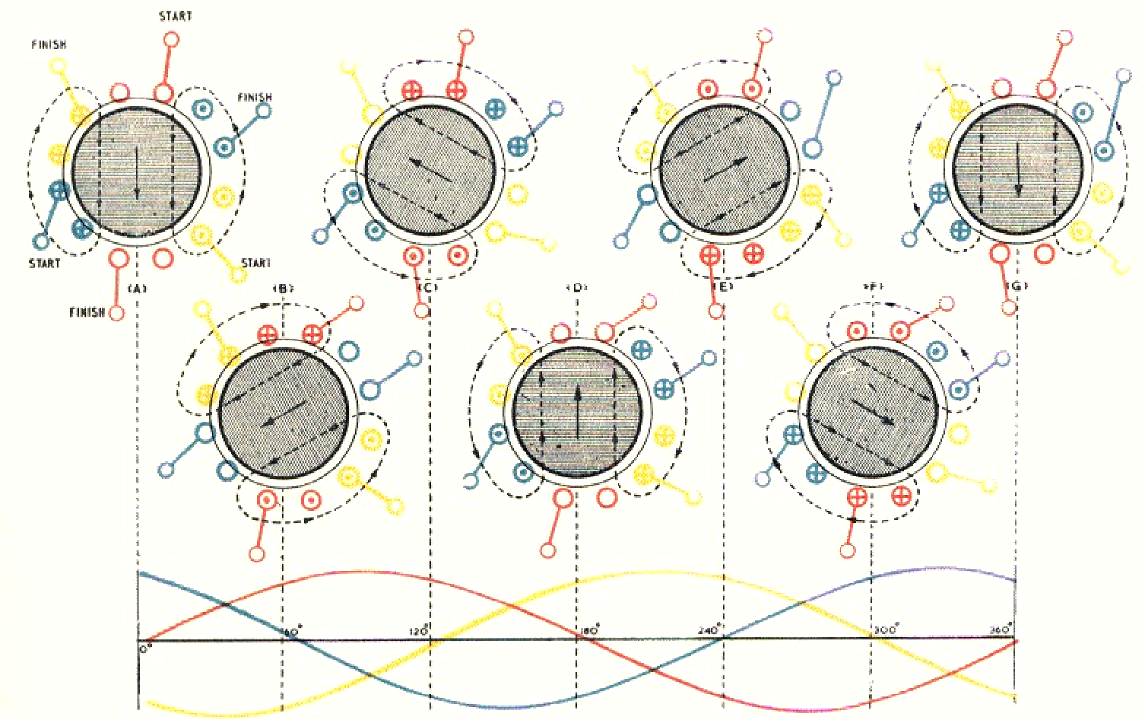


Fig. 8. PRODUCTION OF ROTATING FIELD WITH TWO-POLE, THREE-PHASE STATOR

18. Fig. 8 shows at intervals of 60° during one complete cycle, the resultant magnetic field produced by a three-phase supply when applied to the stator windings of a three-phase generator. Fig. 8(A) shows the magnetic state when the current in the red phase winding is zero and about to increase in the positive direction. At that instant, the resultant field is produced by the currents in the coils of the blue and yellow phases, the combined effect of which is to produce a field acting vertically downwards through the rotor as shown by the arrow.

19. Fig. 8(B) shows the condition when the supply cycle has advanced 60°. The blue phase winding carries no current, but the currents in the red and yellow phases produce a resultant field which has advanced 60° *clockwise* from the former position. Figs. 8(C), (D), (E), (F) and (G) show the conditions of the currents in the windings, together with the resultant magnetic fields, when the supply cycle passes through 120°, 180°, 240°, 300°, and 360° respectively. From these diagrams it is seen that the magnetic field rotates through 360° (a full revolution) during one complete cycle of the three-phase supply. It is thus in time with (i.e., synchronous with) the a.c. input. A frequency of 50 c/s will therefore produce a field rotating at 50 revolutions per second or at a synchronous speed of 3,000 r.p.m. In a similar manner it may be shown that for all numbers of phases and for all numbers of poles, when polyphase windings are traversed by corresponding polyphase currents, the magnetic field established rotates at a synchronous speed of $\frac{60f}{p}$ revolutions per minute.

Note. In determining the direction of the currents in Fig. 8, the winding direction of the coils must be considered. For example in Fig. 8(A) the currents in the yellow and blue phases are equal in magnitude but opposite in direction. The opposing directions are not however apparent in the stator view unless the directions shown are considered in relation to the "start" and "finish" of the phase windings.

Synchronous Motors

20. The a.c. generator, like the d.c. generator, is a reversible machine; if *supplied* with electrical energy it will run as a motor. An a.c. generator used in this manner operates as a *synchronous motor*, so called because the speed of the machine is dependent upon the synchronous speed of the rotating field produced by applying an input to the stator windings.

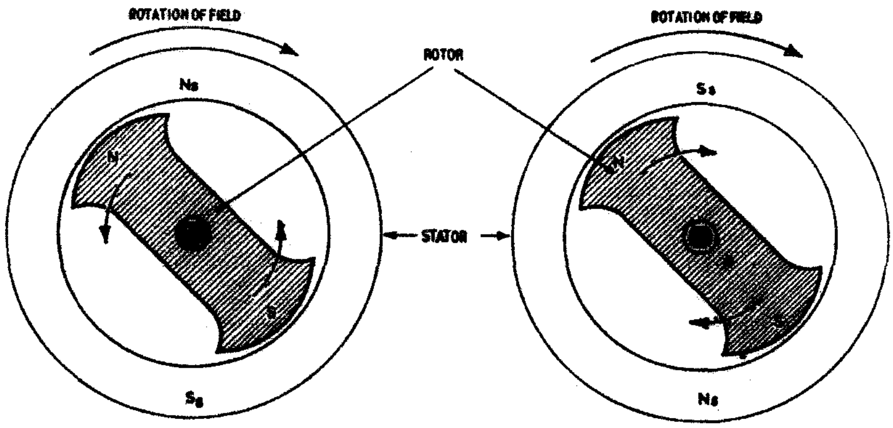


FIG. a FIG. b

Fig. 9. PRINCIPLE OF OPERATION OF SYNCHRONOUS MOTOR

21. **Principle.** When a three-phase stator winding is supplied with three-phase a.c. a magnetic field of constant magnitude and rotating at synchronous speed is produced. In a two-pole stator supplied at 50 c/s, this is equivalent to two poles, N_s and S_s, rotating at 3,000 r.p.m. The rotor carries the field windings which are supplied with d.c. to produce two poles, N and S. With the rotor stationary in the position shown in Fig. 9(*a*), there will be repulsion between N and N_s, and between S and S_s. A torque will therefore be exerted on the rotor in an *anti-clockwise* direction.

22. Half a cycle (i.e., 0·01 sec) later, the poles of the rotating stator field will have changed position as shown in Fig. 9(*b*). There is now attraction between N and S_s and between S and N_s, so that a *clockwise* torque is applied to the rotor. Because of its inertia the rotor cannot respond to this rapidly alternating torque and so remains at rest. The synchronous motor cannot therefore be started, simply by applying a.c. to the stator winding, even if the rotor is already supplied with d.c.

23. Suppose now that the rotor, driven by an external force, is already turning in a *clockwise* direction at just below synchronous speed. The *relative* speed between the rotating field and the rotor will be low, and eventually the N pole of the rotor will become adjacent to the S_s pole of the rotating field as the latter overtakes the rotor. At this point, the two magnetic fields will "lock together" and the rotor will maintain its position relative to the rotating field; that is, it will rotate at synchronous speed. The synchronous motor is usually started and run up towards the synchronous speed with the help of a small *induction motor* (see Para. 26).

24. **Characteristics.**

(*a*) *Effect of load.* When a mechanical load is applied to a synchronous motor, the electrical input to the motor increases. If too great a load is applied the machine is pulled out of synchronism and comes to rest. The torque which causes this is called "the pull-out torque".

(*b*) *Effect of d.c. excitation.* If the mechanical load is kept constant and the d.c. excitation varied, the back e.m.f. varies and, hence, the supply current to the

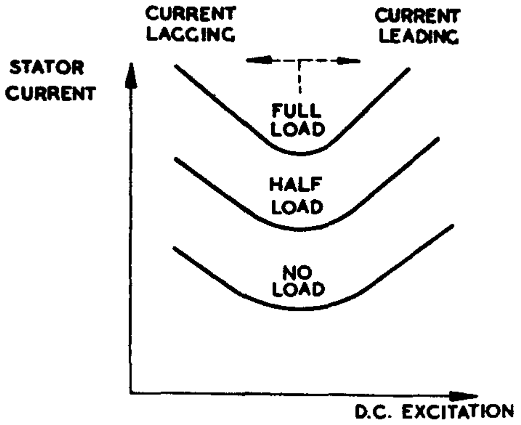


Fig. 10. EFFECT OF VARYING THE D.C. EXCITATION TO A SYNCHRONOUS MOTOR

stator. A graph showing the variation of stator current as the excitation is varied is shown in Fig. 10. The greater the load on the machine, the greater is the current and the higher is the curve on the graph. At a low value of d.c. excitation the stator current is large and *lags* the applied voltage. At normal excitation the current is a minimum and the phase angle zero. At a high value of excitation the current again increases but it now *leads* the applied voltage. This is an important property of the synchronous motor since, by over-exciting the field, the machine is made to take a leading current which compensates for any lagging current taken by any other apparatus. The power factor is, thereby, improved.

25. **Summary.** The synchronous motor:—

(*a*) requires to be started by an external prime mover:

(*b*) runs only at the synchronous speed; this is an advantage where constant speed is required, but a disadvantage where variable speed is required:

(*c*) can be used to adjust the power factor of a system at the same time as it is driving a mechanical load.

Induction Motors

26. **Construction.** Fig. 11 shows the basic construction of a simple type of three-phase induction motor. In consists of a three-phase stator winding supplied with three-phase a.c. to produce a rotating magnetic field. The rotor consists of a set of stout copper conductors laid in slots in a soft-iron armature and welded to copper end rings, thus forming a closed circuit. This is termed a *squirrel-cage rotor* and it will be seen that there is no electrical connection to the rotor.

27. **Principle.** If a conductor is set at right angles to a magnetic field as in Fig. 12(*a*) and moved across the flux from left to right, the direction of the induced e.m.f. will be into the paper (Fleming's right hand rule). If this conductor is part of a complete circuit, a current will be established in the direction of this voltage and there will be a force on the conductor tending to urge it from right to left (Fleming's left hand rule). The same *relative* motion of field and conductor obtains if the conductor is stationary and

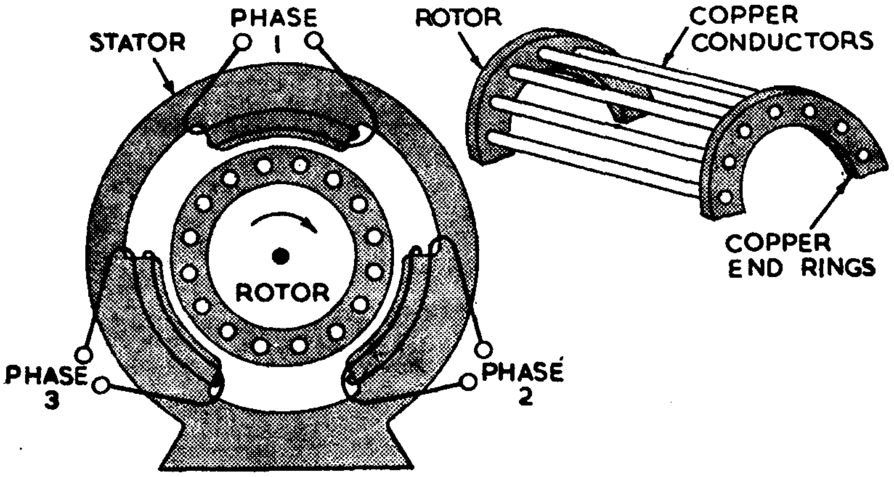


Fig. 11. THREE-PHASE INDUCTION MOTOR

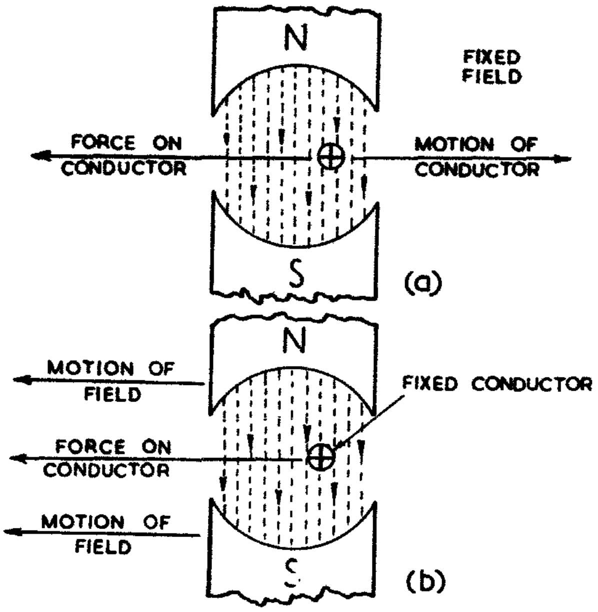


Fig. 12. PRINCIPLE OF INDUCTION MOTOR

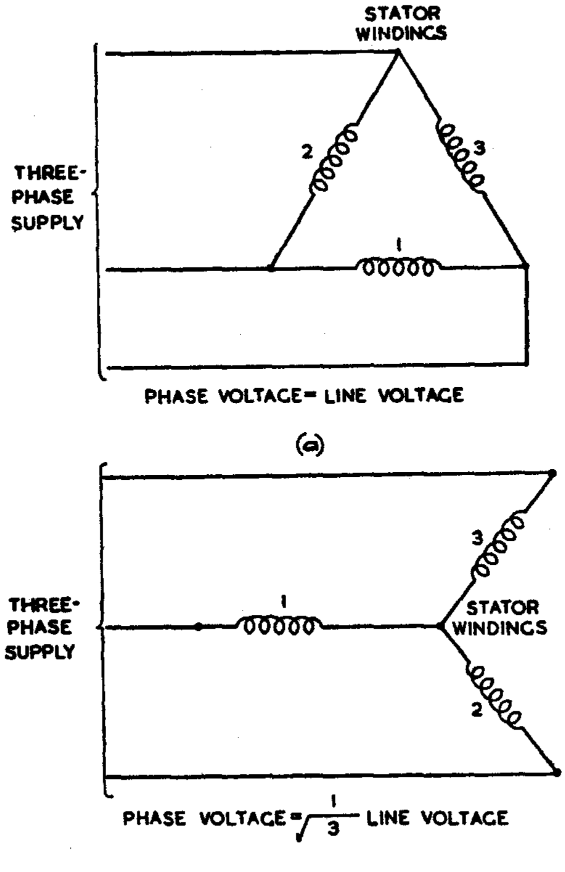
the field moves from right to left as in Fig. 12(*b*). Also in this case, if the circuit is completed so that current can be established in the conductor, the conductor tends to move from right to left. The conductor experiences a force moving it in the *same direction* as the field's motion. As the conductor follows the field, the relative motion is reduced, thereby reducing the conductor current and the force on the conductor. Thus, the conductor speed is limited to something less than that of the field, otherwise there will be no relative motion, no current, and no torque.

28. The squirrel-cage rotor of the induction motor set in the rotating field of the stator should accelerate until it is running steadily at a speed which is *slightly less* than the synchronous speed at which the magnetic field rotates. The greater the difference between the synchronous speed and the rotor speed, the greater the relative speed of conductors and field, and the greater the force on each conductor and the torque exerted by the whole.

29. **Slip.** The difference between the synchronous speed and the rotor speed, measured in r.p.m., is termed the *slip speed.* The ratio of slip speed to synchronous speed, expressed as a percentage, is termed *slip.* For example, a six-pole motor supplied at 50 c/s will have a synchronous speed of $N = \frac{60f}{p} = \frac{60 \times 50}{3}$ = 1,000 r.p.m. With a rotor speed of 960 r.p.m., the slip speed will be 1,000 − 960 = 40 r.p.m. and the slip will be $\frac{40}{1{,}000} \times 100$ = 4%. In practice the value of slip varies from 2% for large machines to 6% for small machines.

30. **Starting current.** When the stator winding is energised with the rotor stopped, the slip is 100% and maximum e.m.f. is induced in the rotor. A heavy current is thus established in the rotor and this produces a flux which opposes and weakens the stator flux. The self-induced e.m.f. in the stator is therefore reduced, and a heavy current is taken by the *stator* winding on starting. To reduce this heavy starting current, the voltage applied to the stator windings should be reduced until the rotor is turning at such a speed that its effect on the stator current is negligible. The normal way of doing this is to use a "star-delta" starting switch. For normal running, the motor is designed to operate with the stator phases mesh or *delta* connected to the supply via the switch (Fig. 13(*a*)) so that the phase voltage is equal to the line voltage. For starting, the stator windings are connected up in *star* via the switch to the supply (Fig. 13(*b*)) so that the phase voltage is $\frac{1}{\sqrt{3}}$ of the normal voltage. This reduced voltage limits the starting current to a safe level.

31. **Torque.** The frequency of the current induced in the rotor is the frequency with which the stator field rotates relative to each conductor. When the rotor is at rest, this equals the supply frequency. When the motor is running lightly loaded the slip is small, and the frequency of the induced rotor current may be only a few cycles per second. Now the resistance of a squirrel-cage rotor is small and its inductance high. Its reactance ($2\pi fL$) will therefore be large at the frequency of the supply when the rotor is stationary, and much less when it is running. Thus on starting, the rotor current and the rotor e.m.f. are nearly 90° out of phase. The flux produced by this lagging rotor current is such that there is little interaction between it and the stator flux, and the starting torque is poor. As the rotor


STATOR WINDINGS
THREE-PHASE SUPPLY
1
2
3
PHASE VOLTAGE = LINE VOLTAGE
THREE-PHASE SUPPLY
1
2
3
STATOR WINDINGS
PHASE VOLTAGE = $\frac{1}{\sqrt{3}}$ LINE VOLTAGE

(a)

(b)

Fig. 13. STAR-DELTA STARTER

current comes into phase with the rotor e.m.f. with increased rotor speed (decreased slip and inductive reactance) the rotor and stator fluxes come more into phase and the torque increases. The torque-speed characteristic is shown in Fig. 14.

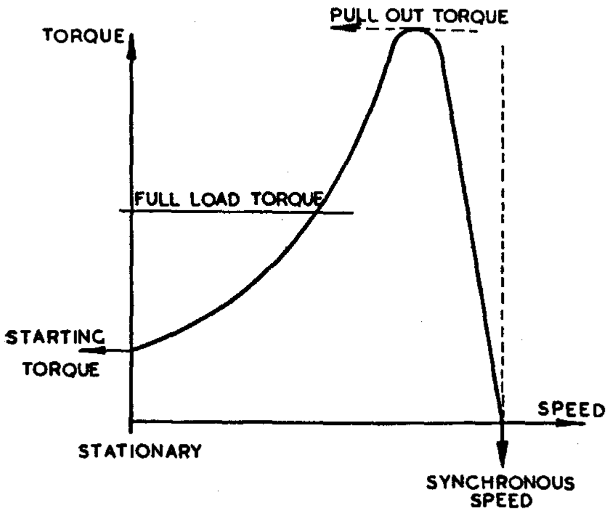


Fig. 14. TORQUE-SPEED CHARACTERISTIC OF INDUCTION MOTOR

32. **Load-speed characteristic.** Off load, the torque developed by the rotor is only that required to overcome friction and wind resistance and in this condition the rotor speed is almost synchronous. When a load is applied the rotor slows down to the point at which the resultant increased driving torque balances the load torque. The fall in speed from no-load to full load is small as shown in Fig. 15. The speed of a squirrel-cage motor is not easily controlled, since it is related to synchronous speed, and its main use is on devices where a constant speed is required. One typical use is as the prime mover for generators used in control system where ratings of 2 to 30 h.p. at speeds of up to 3,000 r.p.m. may be required. The squirrel-cage motor can be readily adapted for frequent reversing. To do this it is necessary to reverse the direction of rotation of the stator field, and this is achieved by changing over any two of the three connections at the stator terminals.

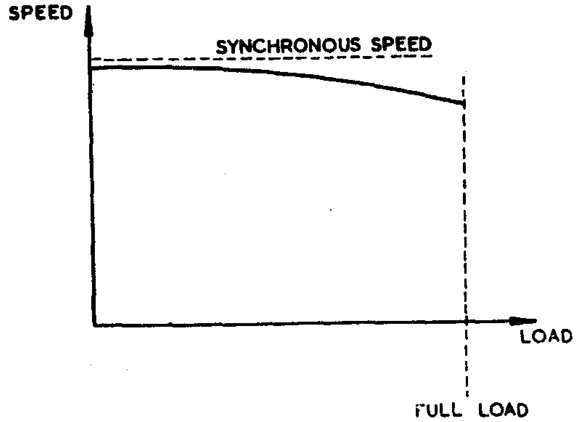


Fig. 15. SPEED-LOAD CHARACTERISTIC OF INDUCTION MOTOR

33. **Summary.** The three-phase squirrel-cage motor:—

(*a*) has a high starting current (reduced by a star-delta starter);

(*b*) has a poor starting torque;

(*c*) runs at almost synchronous speed, and the speed cannot easily be varied;

(*d*) can be adapted for frequent reversing.

Single-phase Induction Motors

34. Fig. 16 represents a single-phase induction motor with one pair of stator poles and a squirrel-cage rotor. Such a motor is not capable of producing a rotating field in

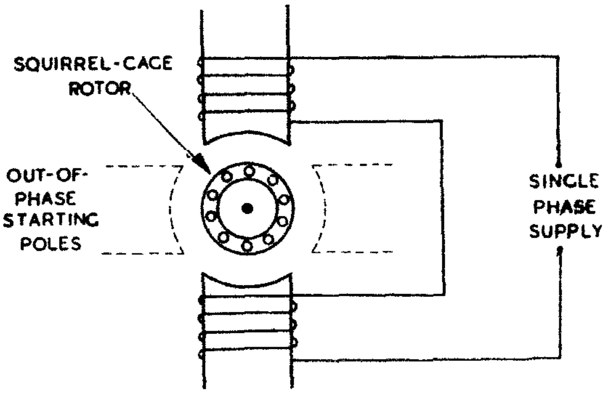


Fig. 16. SINGLE-PHASE INDUCTION MOTOR

the manner previously described and it is not self-starting. Suppose that a field of magnitude H is rotating *anti-clockwise* with angular velocity ω radians per second. At time t, when the vector H is at an angle ωt to the horizontal, the horizontal and vertical components of H are $H \cos \omega t$ and $H \sin \omega t$ (Fig. 17(*a*)). Another field of the same magnitude H, starting at the same instant as the first but rotating *clockwise* will have as its components at time t, $H \cos \omega t$ and $-H \sin \omega t$ (Fig. 17(*b*)). If the two are superposed, the resultant horizontal component is $H \cos \omega t + H \cos \omega t = 2H \cos \omega t$, and the vertical component $H \sin \omega t - H \sin \omega t = O$ (Fig. 17(*c*)). That is, two rotating fields of the same amplitude and in phase, but rotating in opposite directions, are together equivalent to a single alternating field of twice the individual amplitudes. So a single-phase alternating field as given by the windings of Fig. 16 can be regarded as producing two equal fields of *half* the magnitude of the peak value, which rotate at the same speed in opposite directions. Thus, if a single-phase induction motor is started and brought up to speed by some auxiliary device, the rotor will be acted upon by *one* of the rotating components of the alternating field (depending on the initial direction of rotation) and it then continues to run as a single-phase motor.

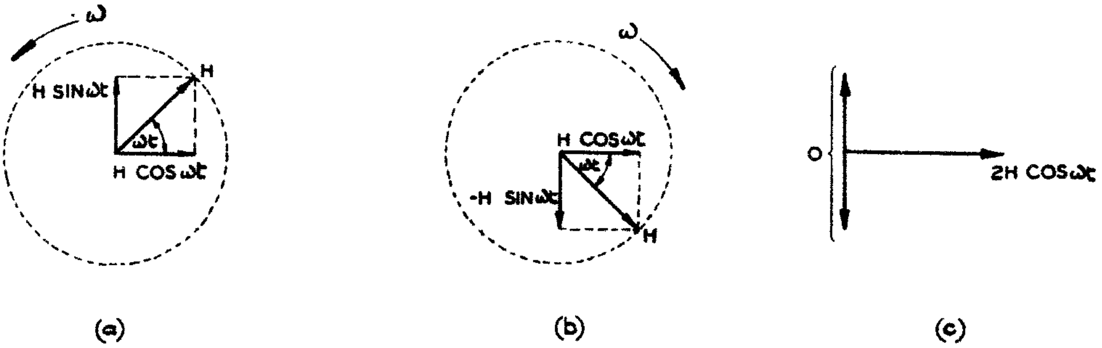


Fig. 17. ROTATING COMPONENTS OF SINGLE-PHASE A.C.

35. The starting device takes the form of an auxiliary stator winding spaced 90° from the main winding (Fig. 16) and connected in series with an impedance to the main supply. This impedance is chosen to produce as great a phase displacement as possible between the currents in the main and auxiliary windings so that the machine starts up virtually as a two-phase motor (Fig. 18). A switch, usually operated by centrifugal action, cuts out the auxiliary winding when a fair speed has been attained and the machine continues to run on the main stator winding. Single-phase induction motors operating on 230 V 50 c/s supply and rated up to 5 h.p. are used in the Service to provide a constant-speed drive to d.c generators.

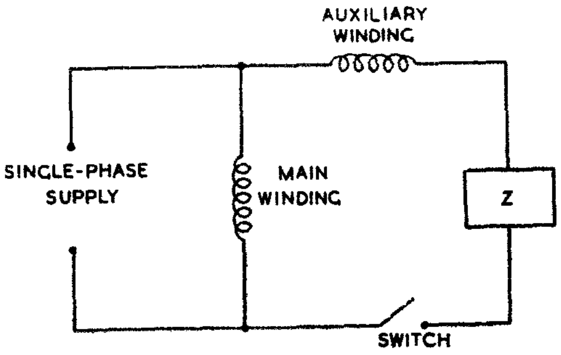


Fig. 18. STARTING DEVICE FOR SINGLE-PHASE INDUCTION MOTOR

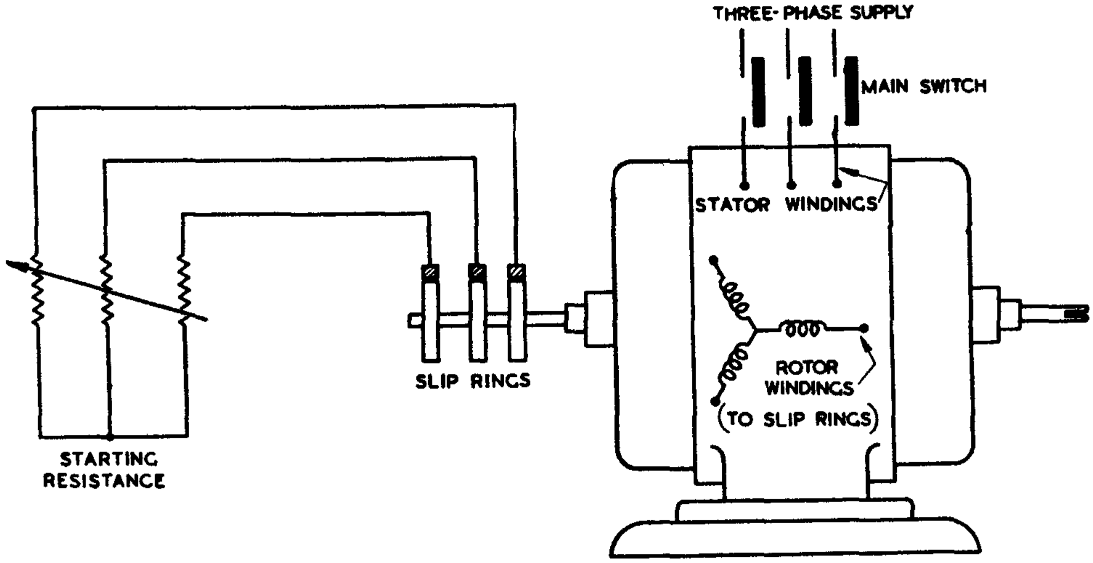


Fig. 19. SLIP-RING INDUCTION MOTOR

Wound Induction Motors

36. The squirrel-cage motor takes a large starting current and has a poor starting torque (see Para. 33) both due to the low-resistance rotor. These features are improved in the "wound" or "slip-ring" type of induction motor. The stators of wound induction motors are identical with those of squirrel-cage motors, but the rotor conductors are insulated and form a three-phase, star-connected winding, the three ends of which are connected to three insulated slip-rings mounted on the motor shaft (Fig. 19). When the motor is running normally these slip-rings are short-circuited to give a low-resistance rotor equivalent to the squirrel-cage. For starting, the slip-rings are connected to a three-phase, star-connected resistance as shown in Fig. 19 and maximum resistance is inserted in the rotor circuit to give a low starting current. The resistance is gradually cut out as the machine speeds up until finally the three slip-rings are short-circuited and the motor runs as for a squirrel-cage machine.

Synchronous Induction Motors

37. The features of constant speed and capacitor action of the synchronous motor are often combined with the self-starting feature of the induction motor by using a machine which is virtually a hybrid of the two types. One type of synchronous induction motor in wide use consists of a slip-ring induction motor coupled to a d.c. exciter; the rotor windings are connected via the slip rings to the exciter and a three-phase starting resistance as shown in Fig. 20. The exciter is driven by the motor, which is started up as a slip-ring induction motor. There is no appreciable d.c. from the exciter until the motor has attained some 90% of full speed. As synchronous speed is approached the machine automatically pulls itself into synchronism and continues to run as a synchronous motor with d.c. applied to the rotor windings from the exciter. If the machine is overloaded, it pulls out of synchronism like an ordinary synchronous motor, but continues to run at reduced speed as an induction motor.

Single-phase Commutator Motors

38. The induction and synchronous types of a.c. motor, although possessing many excellent features, are not variable-speed machines. Furthermore, the starting torque of squirrel-cage motors is poor, it takes a large starting current, and the power factor is low. For these reasons a.c. commutator motors have been developed in an attempt to obtain improved speed control, starting torque, and power factor.

39. An ordinary d.c. series motor, when connected to an a.c. supply, will rotate and exert a *undirectional* torque. The ordinary d.c. series motor would not however,

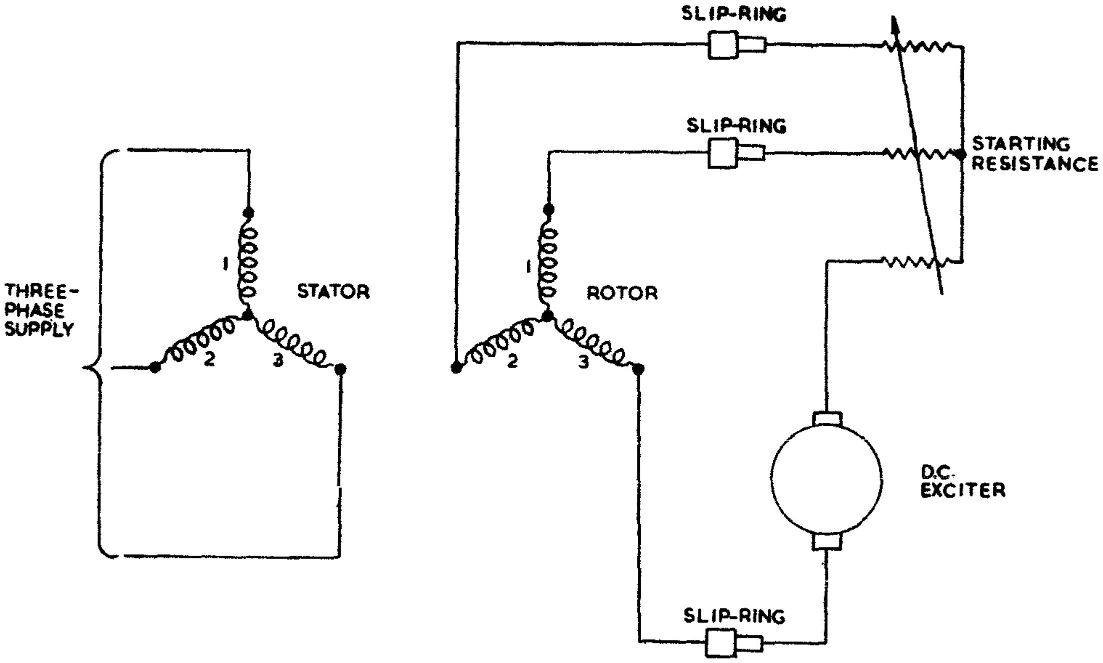


Fig. 20. SYNCHRONOUS INDUCTION MOTOR

be capable of giving an efficient and satisfactory performance for the following reasons:—

(*a*) Large eddy currents in the field magnets would cause undue heating.

(*b*) Each coil of the armature winding when short-circuited by the brushes would give rise to destructive sparking at the brushes.

(*c*) The power factor would be very low because of the highly inductive nature of the field and armature windings.

40. To overcome these drawbacks, a series motor designed for use on a.c. supply (or for use on either a.c. or d.c., in which case it is known as a "universal" motor), is modified in the following manner:—

(*a*) The entire magnetic circuit is laminated.

(*b*) The field winding is distributed in core slots like the stator winding of an induction motor.

(*c*) The armature winding is sub-divided to a greater extent than in d.c. machines. This necessitates a relatively large number of commutator segments and gives a large commutator for the size of the motor.

(*d*) The brushes are of high resistance carbon and each brush is restricted in width so that it bridges only two commutator segments.

(*e*) Connection between armature coils and commutator segments is often made through resistors, instead of by direct soldering. This reduces the circulating current when a coil is short-circuited by a brush.

41. The main characteristics of the a.c. series motor are similar to those of d.c. series motors (see Sect. 3, Chap. 2), and the variable series resistor method for starting and speed control of the d.c. motor is used for the a.c. motor. A.c. commutator motors are used to provide a variable-speed drive to generators which are used, for example, on aerial turning gear assemblies for ground radar equipments.

SECTION 5

CHAPTER 5

THE OPERATOR "j"

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PART 1, SECTION 5, CHAPTER 5

THE OPERATOR "j"

Introduction

1. If the vector P of Fig. 1(*a*) is multiplied by (-1) it becomes $(-P)$. Thus, multiplication of a vector by (-1) rotates it through 180° as shown in Fig. 1(*b*). If the vector is multiplied *twice* by (-1), i.e. by $(-1)^2$, it will be rotated through $(2 \times 180°) = 360°$. Multiplication by $(-1)^n$ rotates the vector through $(n \times 180°)$. If this rule is to apply for all values of n, then on putting $n = \frac{1}{2}$, multiplication by $(-1)^{\frac{1}{2}}$ must be considered to rotate the vector through $(\frac{1}{2} \times 180°) = 90°$. The factor $(-1)^{\frac{1}{2}} = \sqrt{-1}$ is denoted by the letter "j". Multiplication of a vector P by j is equivalent to rotating the vector in an *anti-clockwise* direction through 90° as shown in Fig. 1(*c*).

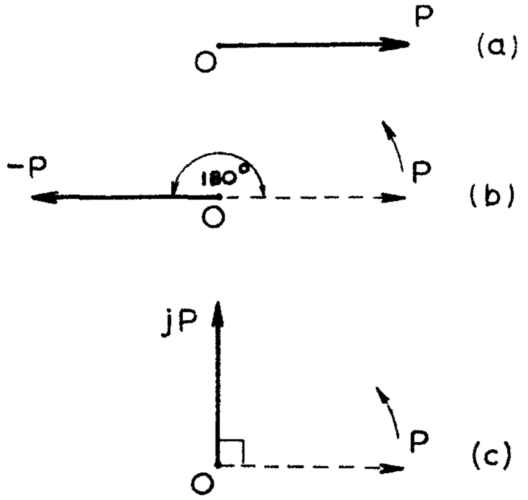

Fig. 1. THE OPERATOR "j"

2. The operator j obeys all the ordinary rules of algebra. Thus:—

(*a*) $j = \sqrt{-1}$
Multiplication by j rotates a vector anti-clockwise through 90° (Fig. 2(*b*)).

(*b*) $j^2 = (\sqrt{-1})^2 = (-1)$.
Multiplication by j^2 rotates a vector through 180° (Fig. 2(*c*)).

(*c*) $j^3 = j^2 \times j = (-1) \times j = -j$.
If multiplication by j rotates a vector through + 90°, then multiplication by −j rotates it through −90° (or 270°) as shown in Fig. 2(*d*).

(*d*) $j^4 = j^2 \times j^2 = (-1) \times (-1) = +1$.
Multiplication by j^4 rotates a vector through 360° (Fig. 2(*e*)).

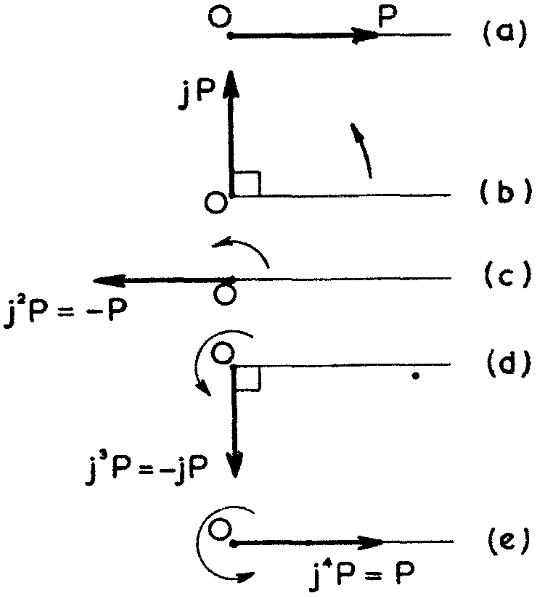

Fig. 2. POWERS OF "j"

3. The results of Para. 2 are summarised in Table 1.

Symbol	Algebraic Equivalent	Rotation (anti-clockwise)
j	$\sqrt{-1}$	90°
j^2	-1	180°
j^3	$-j$	270°
j^4	$+1$	360°

TABLE 1. POWERS OF j

Representation of Vectors

4. Consider the vector OP of Fig. 3. Draw perpendiculars PN and PM from P to the axes and let ON = x, OM = y. Then

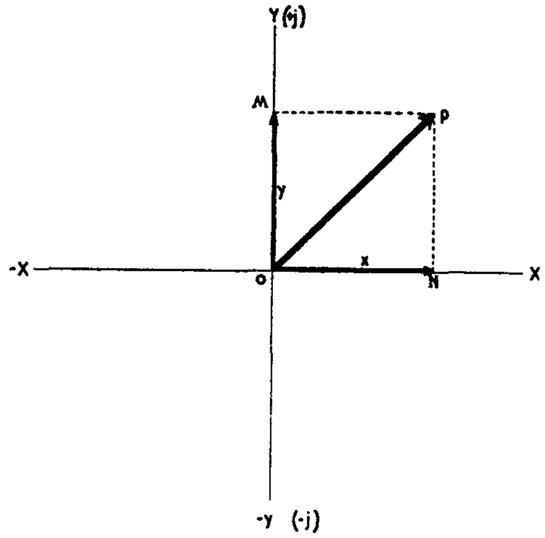

Fig. 3. REPRESENTATION OF VECTORS

(x,y) are termed the *co-ordinates* of P. The vector OP is the sum of the vectors ON and OM (since ONPM is a parallelogram). But ON has length x and lies along the X axis (or "*real*" axis); therefore it is equal to the vector x. OM has length y and lies along the Y axis (or "*imaginary*" axis); it is consequently equal to the vector jy, the operator j indicating a rotation through 90° anti-clockwise. Therefore:—

Vector OP = Vector sum of ON and OM
$\therefore$ OP $= x + jy$.

5. In this way *any* vector can be written in the form $(x + jy)$ where x is the distance projected along the real axis and y is the distance projected along the imaginary (j) axis. This defines the *rectangular notation* of a vector. For example, Fig. 4 shows two vectors OA and OB which in the rectangular notation are denoted as vectors $(1 + j3)$ and $(-3 - j2)$ respectively.

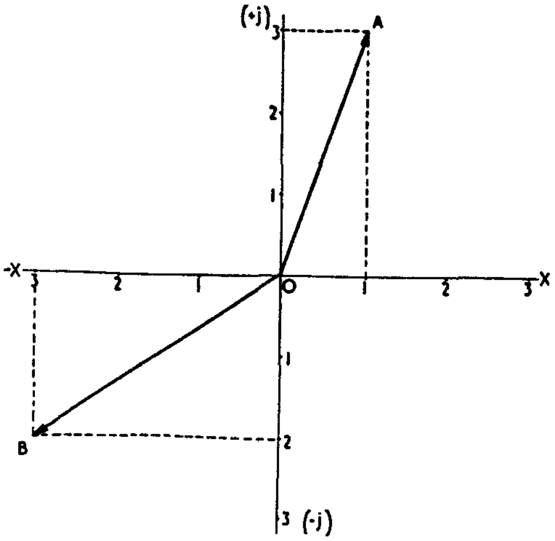

Fig. 4. THE RECTANGULAR NOTATION

Addition and Subtraction of Vectors

6. If two vectors OA and OB are to be added, the resultant can be obtained by sketching the vector diagram for OA and OB, and then completing the parallelogram to find the resultant OC (Fig. 5). However, the addition of vectors will be simplified if the vectors are in the form $(x + jy)$. The resultant of adding in this notation is found by adding all the real terms and all the imaginary terms separately. Thus, in Fig. 5, the sum of OA = $(1 + j3)$ and OB = $(2 - j2)$ is:—

$$OC = OA + OB$$
$$OC = (1 + j3) + (2 - j2)$$
$$\therefore OC = (3 + j1).$$

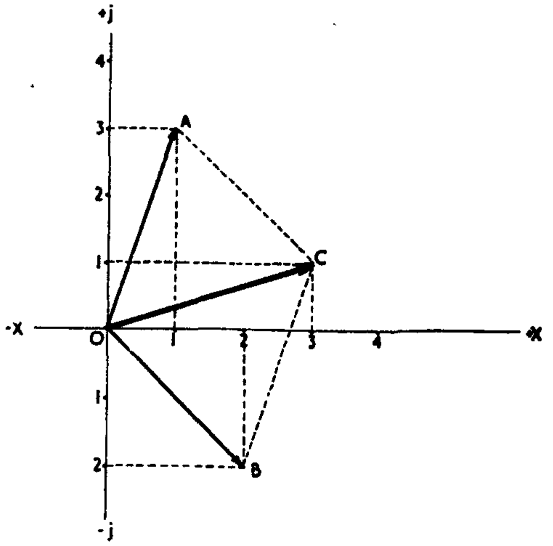

Fig. 5. ADDITION OF VECTORS

7. The resultant of subtracting one vector from another is found by subtracting all the real terms and all the imaginary terms separately. Thus, if OA = $(3 + j2)$ and OB = $(2 - j4)$, the resultant of subtracting OB from OA is:—

$$OC = OA - OB$$
$$OC = (3 + j2) - (2 - j4)$$
$$OC = 3 + j2 - 2 + j4$$
$$\therefore OC = (1 + j6).$$

Multiplication of Vectors

8. To multiply two vectors of the form $(x + jy)$ the ordinary rules of algebra apply.

Thus, if OA = (7 + j3) and OB = (4 + j5) then:—

$$
\begin{aligned}
OC &= OA \times OB\\
OC &= (7 + j3) \times (4 + j5)\\
OC &= 28 + j12 + j35 + j^2 15\\
OC &= 28 + j47 - 15 \quad (\text{since } j^2 = -1)\\
\therefore\ OC &= 13 + j47.
\end{aligned}
$$

Division of Vectors

9. When two vectors are given in the form (x +jy) and one vector is to be divided by the other it is necessary to "*rationalize*". For instant, if a vector (2 + j2) is to be divided by a vector (4 + j3) the quotient is written as $\frac{(2 + j2)}{(4 + j3)}$. Rationalize to remove the j in the denominator. To do this multiply the numerator and the denominator by the "*conjugate*" of (4 + j3), which is (4 − j3). Thus:— $\frac{(2 + j2)}{(4 + j3)} \times \frac{(4 - j3)}{(4 - j3)} = \frac{8 + j8 - j6 - j^2 6}{16 + j12 - j12 - j^2 9}$

$$
\begin{aligned}
&= \frac{8 + j2 + 6}{16 + 9} \quad (\text{since } j^2 = -1)\\
&= \frac{14 + j2}{25}
\end{aligned}
$$

$$\therefore \frac{(2 + j2)}{(4 + j3)} = \frac{14}{25} + j\frac{2}{25}.$$

Use of j in A.C. Theory

10. **Pure inductive circuit.** When an alternating voltage is applied across a pure inductance:—

(*a*) The voltage V leads the current I by 90°.

(*b*) The ratio $\frac{V}{I}$ equals the inductive reactance $X_L = \omega L$.

Therefore, $V = \omega LI$ and leads I by 90° (Fig. 6). This lead of 90° can be denoted by j, the magnitude and phase of V relative to I then being given by the single expression:—

$$V = j\omega LI.$$

Also, $\frac{V}{I} = j\omega L =$ Inductive reactance X_L.

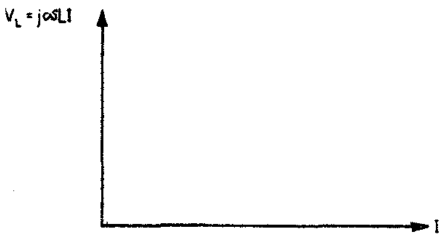

Fig. 6. PHASE ANGLE IN AN INDUCTANCE SHOWN BY 'j'

11. **Pure capacitive circuit.** When an alternating voltage is applied across a pure capacitance:—

(*a*) The voltage V lags the current I by 90°.

(*b*) The ratio $\frac{V}{I}$ equals the capacitive reactance $X_c = \frac{1}{\omega C}$.

Therefore, $V = \frac{I}{\omega C}$ and lags I by 90° (Fig. 7). This lag of 90° can be denoted by (−j), the magnitude and phase of V relative to I then being given by the single expression:—

$$V = \frac{-jI}{\omega C}$$

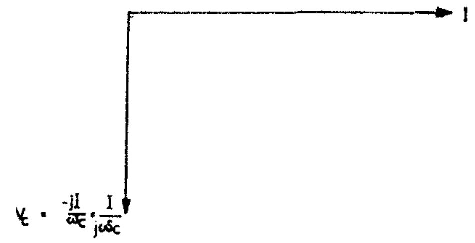

Fig. 7. PHASE ANGLE IN A CAPACITANCE SHOWN BY 'j'

Also, $\frac{V}{I} = \frac{-j}{\omega C} =$ Capacitive reactance X_c.

Note. The factor (—j) can be written as $\frac{1}{j}$ since:—

$$
\begin{aligned}
-j \times \frac{j}{j} &= \frac{-j^2}{j}\\
&= \frac{-(-1)}{j}\\
\therefore -j &= \frac{1}{j}
\end{aligned}
$$

Thus, $X_c = \frac{-j}{\omega C} = \frac{1}{j\omega C}$.

12. **R and L in series.** The vector diagram for the circuit of Fig. 8(*a*) is given in Fig. 8(*b*). From this vector diagram it is seen that:—

$$
\begin{aligned}
V &= V_R + V_L\\
V &= IR + jIX_L\\
V &= I(R + j\omega L)\\
\therefore \frac{V}{I} &= (R + j\omega L) = \text{Impedance } Z.
\end{aligned}
$$

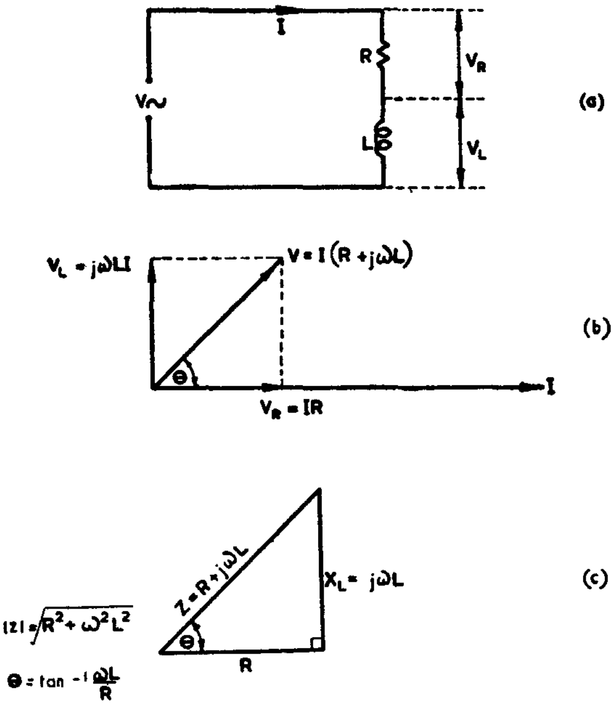
I
R
V~
L
V_R
V_L
(a)
$V_L = j\omega LI$
$V = I(R + j\omega L)$
θ
$V_R = IR$
I
(b)
$Z = R + j\omega L$
$X_L = j\omega L$
$|Z| = \sqrt{R^2 + \omega^2 L^2}$
$\theta = \tan^{-1}\frac{\omega L}{R}$
R
(c)

Fig. 8. R AND L IN SERIES

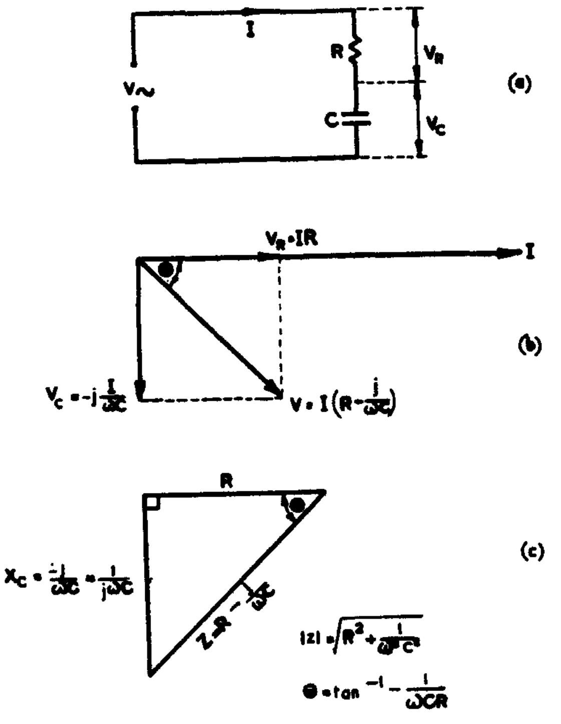
I
R
V~
C
V_R
V_C
(a)
$V_R = IR$
I
θ
$V_C = -j\frac{I}{\omega C}$
$V = I(R - \frac{j}{\omega C})$
(b)
R
θ
$X_C = \frac{-j}{\omega C} = \frac{1}{j\omega C}$
$Z = R - \frac{j}{\omega C}$
$|Z| = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$
$\theta = \tan^{-1} - \frac{1}{\omega CR}$
(c)

Fig. 9. R AND C IN SERIES

This represents an impedance vector of the form (x + jy) as shown in Fig. 8(*c*). R is the real component and jωL the imaginary component of the impedance vector. The magnitude of a vector, without reference to its direction, is termed its "*modulus*". The modulus of Z, written |Z|, equals $\sqrt{R^2 + \omega^2 L^2}$.

13. **R and C in series.** The vector diagram for the circuit of Fig. 9(*a*) is given in Fig. 9(*b*). From this vector diagram it is seen that:—

$$V = V_R + V_C$$
$$V = IR - jIX_C$$
$$V = I(R - \frac{j}{\omega C})$$
$$\therefore \frac{V}{I} = (R - \frac{j}{\omega C}) = \text{Impedance } Z.$$

This represents an impedance vector of the form (x + jy) as shown in Fig. 9(*c*). R is the real component and $\frac{-j}{\omega C}$ (or $\frac{1}{j\omega C}$) the imaginary component of the impedance vector. The modulus of Z is:—

$$|Z| = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

14. **R, L and C in series.** The vector diagram for the circuit of Fig. 10(*a*) is given in Fig. 10(*b*). The condition assumed is that where X_L is greater than X_C. From this vector diagram it is seen that:—

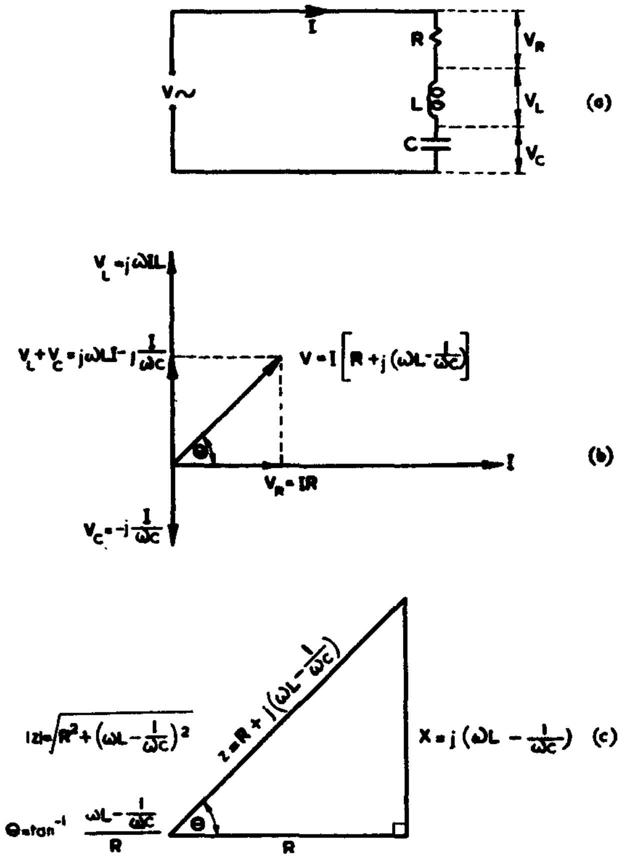
I
R
V~
L
C
V_R
V_L
V_C
(a)
$V_L = j\omega LI$
$V_L + V_C = j\omega LI - I\frac{1}{j\omega C}$
$V = I[R + j(\omega L - \frac{1}{\omega C})]$
θ
$V_R = IR$
I
$V_C = -j\frac{I}{\omega C}$
(b)
$|Z| = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$
$Z = R + j(\omega L - \frac{1}{\omega C})$
$X = j(\omega L - \frac{1}{\omega C})$
$\theta = \tan^{-1}\frac{\omega L - \frac{1}{\omega C}}{R}$
θ
R
(c)

Fig. 10. R, L, AND C IN SERIES

$$V = V_R + (V_L + V_C)$$

$$V = IR + (j\omega LI - \frac{jI}{\omega C})$$

$$V = I\left[R + j(\omega L - \frac{1}{\omega C})\right]$$

$$\therefore \frac{V}{I} = R + j(\omega L - \frac{1}{\omega C}) = \text{Impedance } Z.$$

This represents an impedance vector of the form (x + jy) as shown in Fig. 10(*c*).

R is the real component and $j(\omega L - \frac{1}{\omega C})$ the imaginary component of the impedance vector. The modulus of Z is:—

$$|Z| = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}.$$

Series Resonant Circuit

15. The impedance of a circuit containing R, L and C in series is:—

$$Z = R + j(\omega L - \frac{1}{\omega C}).$$

Resonance occurs when the reactive (*j*) term is zero; that is, when:—

$$\omega L - \frac{1}{\omega C} = O$$

$$\text{And, } \omega L = \frac{1}{\omega C}.$$

The resonant frequency is $f_o = \frac{1}{2\pi\sqrt{LC}}$ and the value of Z at resonance is R. Since the circuit is purely resistive, the current and voltage are then in phase.

16. At any frequency other than the resonant frequency, Z is a complex impedance containing resistance and reactance, the voltage being given by:—

$$V = IZ$$

$$V = I\left[R + j(\omega L - \frac{1}{\omega C})\right].$$

The phase angle is given by:—

$$\theta = \tan^{-1}\frac{\omega L - 1/\omega C}{R}.$$

Consider such a series circuit where, at a certain frequency, $R = 4\Omega$, $X_L = 85\Omega$, and $X_C = 82\Omega$. If the current established is 2A, the voltage is:—

$$V = 2[4 + j(85 - 82)].$$

$$V = 2(4 + j3)$$

$$\therefore V = 8 + j6.$$

This is a vector of the form (x + jy) and is shown in Fig. 11. From this vector diagram the modulus of V is seen to be:—

$$|V| = \sqrt{8^2 + 6^2}$$

$$|V| = \sqrt{100}$$

$$\therefore |V| = 10 \text{ volts.}$$

Its phase angle relative to the current is:—

$$\theta = \tan^{-1}\frac{3}{4}$$

$$\therefore \theta \triangleq 37° \textit{ leading.}$$

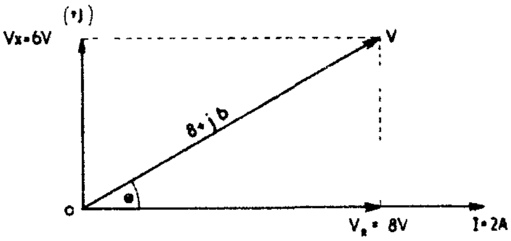

Fig. 11. USE OF j IN A.C. CIRCUITS

Admittance, Conductance and Susceptance

17. It was stated in Chap. 3 that the admittance Y of a circuit is the reciprocal of the impedance Z and is given by:—

$$Y = \sqrt{G^2 + B^2},$$

$$\text{where } G = \frac{R}{R^2 + X^2}$$

$$B = \frac{X}{R^2 + X^2}$$

These relationships can now be shown using the operator j.

18. In a circuit containing resistance and reactance in series:—

$$Z = (R + jX).$$

$$\text{But, } Y = \frac{1}{Z}$$

$$\therefore Y = \frac{1}{(R + jX)}.$$

Rationalize to remove the j from the denominator.

$$Y = \frac{1}{(R + jX)} \times \frac{(R - jX)}{(R - jX)}$$

$$Y = \frac{(R - jX)}{R^2 + jXR - jXR - j^2X^2}$$

$$Y = \frac{R - jX}{R^2 + X^2} \text{ (since } -j^2 = +1)$$

$$Y = \frac{R}{R^2 + X^2} - j\frac{X}{R^2 + X^2}$$

$$\therefore Y = G - jB,$$

$$\text{where } G = \frac{R}{R^2 + X^2}$$

$$B = \frac{X}{R^2 + X^2}$$

The modulus of Y is $\sqrt{G^2 + B^2}$.

Example. An a.c. at 48V is applied to a circuit consisting of a pure resistance in parallel with a pure inductance. At the frequency of the supply, $R = 3\,\Omega$ and $X_L = 4\,\Omega$. The resultant current is:—

$$I = \frac{V}{Z} = VY.$$

(*a*) In the resistive branch:—

$$Y_R = G - jB$$

$$Y_R = \frac{R}{R^2 + X^2} - j\frac{X}{R^2 + X^2}$$

$$Y_R = \frac{1}{R} \text{ (since } X = O).$$

$$\therefore Y_R = \frac{1}{3}.$$

(*b*) In the inductive branch:—

$$Y_L = \frac{R}{R^2 + X_L^2} - j\frac{X_L}{R^2 + X_L^2}$$

$$Y_L = \frac{-j}{X_L} \text{ (since } R = O).$$

$$\therefore Y_L = \frac{-j}{4}.$$

(*c*) The total admittance is:—

$$Y = Y_R + Y_L$$

$$\therefore Y = \frac{1}{3} - \frac{j}{4}.$$

Now, $I = VY$

$$\therefore I = 48\left(\frac{1}{3} - \frac{j}{4}\right)$$

$$I = \frac{48}{3} - j\frac{48}{4}$$

$$\therefore I = 16 - j12.$$

The vector is shown in Fig. 12.

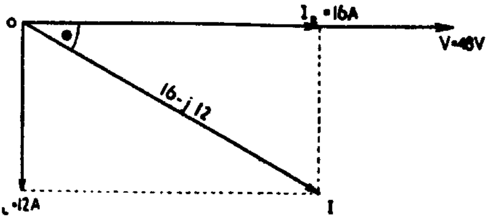

Fig. 12. R AND L IN PARALLEL, EXAMPLE

From this, the modulus of I is:—

$$|I| = \sqrt{(16)^2 + (12)^2}$$

$$|I| = \sqrt{400}$$

$$\therefore |I| = 20A.$$

The phase angle of the current relative to the voltage is:—

$$\theta = \tan^{-1} - \frac{12}{16}$$

$$\theta = \tan^{-1} - 0{\cdot}75$$

$$\therefore \theta \simeq 37° \textit{ lagging}.$$

Parallel Tuned Circuit

19. In Chap. 3, the resonant frequency and the dynamic resistance was calculated for a circuit containing a resistive inductance in parallel with a pure capacitance. The use of the operator j simplifies these calculations. The circuit to be considered is shown in Fig. 13.

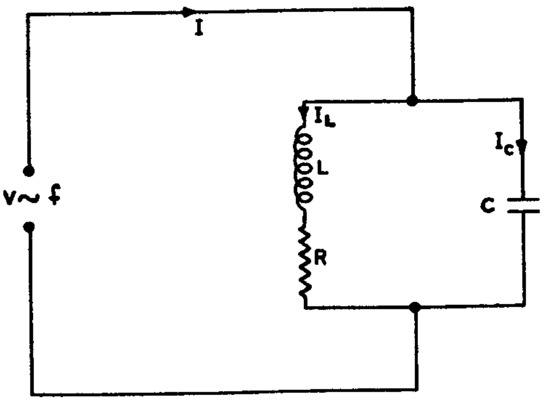

Fig. 13. PARALLEL TUNED CIRCUIT

Now:—

$$Y = \frac{R}{R^2 + X^2} - j\frac{X}{R^2 + X^2}.$$

(*a*) In the capacitive branch:—

$$Y_C = \frac{-j}{X_C} \text{ (since } R = O).$$

$$Y_C = \frac{-j}{-1/\omega C}$$

$$\therefore Y_C = j\omega C.$$

(*b*) In the inductive branch:—

$$Y_L = \frac{R}{R^2 + X_L^2} - j\frac{X_L}{R^2 + X_L^2}$$

$$\therefore Y_L = \frac{R}{R^2 + \omega^2L^2} - j\frac{\omega L}{R^2 + \omega^2L^2}$$

(*c*) The total admittance is:—

$$Y = Y_c + Y_L$$

$$Y = j\omega C + \frac{R}{R^2 + \omega^2 L^2} - j\frac{\omega L}{R^2 + \omega^2 L^2}$$

$$\therefore Y = \frac{R}{R^2 + \omega^2 L^2} + j\omega\left(C - \frac{L}{R^2 + \omega^2 L^2}\right) \quad \text{(i)}$$

This gives the admittance at any frequency in the form (x + jy). For resonance to occur, the reactance in a parallel tuned circuit must be zero (i.e., the j term must be zero). Thus:—

$$\left(C - \frac{L}{R^2 + \omega^2 L^2}\right) = O$$

$$C = \frac{L}{R^2 + \omega^2 L^2}$$

$$R^2 + \omega^2 L^2 = \frac{L}{C} \quad \text{(ii)}$$

$$\omega^2 L^2 = \frac{L}{C} - R^2$$

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\therefore f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

This gives the frequency at which resonance occurs.

From equation (i):—

$$Y = \frac{R}{R^2 + \omega^2 L^2} + j\omega\left(C - \frac{L}{R^2 + \omega^2 L^2}\right).$$

But at resonance:—

$$\left(C - \frac{L}{R^2 + \omega^2 L^2}\right) = O$$

$$\therefore Y = \frac{R}{R^2 + \omega^2 L^2} \text{ (at resonance).}$$

From equation (ii):—

$$R^2 + \omega^2 L^2 = \frac{L}{C}$$

$$\therefore Y = \frac{R}{L/C}$$

$$Y = \frac{CR}{L}$$

$$Z = \frac{1}{Y}$$

$$\therefore Z = \frac{L}{CR} = \text{Dynamic resistance.}$$

SECTION 6

MEASURING INSTRUMENTS

Chapter 1 .. .. **Ammeters and Voltmeters**
Chapter 2 .. .. **Test Instruments**
Chapter 3 .. .. **Measurement of Power**

SECTION 6

CHAPTER 1

AMMETERS AND VOLTMETERS

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PART 1, SECTION 6, CHAPTER 1

AMMETERS AND VOLTMETERS

Introduction

1. Reference has been made in previous chapters to the *values* of various electrical quantities without any explanation of how such quantities are measured in practice. The quantities which commonly require measurement are the currents established in, and the p.d.s developed across, various parts of a circuit. The measuring instruments used for these purposes are termed *ammeters* and *voltmeters* respectively. In this Chapter the operation of certain types of these "meters" is discussed.

Essentials of a Measuring Instrument

2. Measuring instruments have a moving system (usually mounted in jewelled bearings and free to rotate about a fixed axis) which carries a pointer over a graduated scale. When discussing the principle of operation of such instruments three forces have to be considered:—

(*a*) *Deflecting force.* This is the force which moves the pointer over the scale and it exists only when the meter is connected to a "live" circuit. The rotating system is deflected from its zero position by a torque that is some function of the quantity to be measured.

(*b*) *Controlling force.* This force acts in opposition to the deflecting force and brings the pointer to rest at a position on the scale where the deflecting and controlling forces are equal. In addition, the controlling force returns the pointer to zero when the deflecting force is removed, and for this reason it is sometimes termed the *restoring force.* The majority of meters have a "zero adjustment" on the controlling system so that the instrument can be set to zero when there is no deflecting force. Most instruments use a hairspring system to supply the controlling force, but control by a system of weights is sometimes used.

(*c*) *Damping force.* It is important that the movement should take up its final position quickly and without oscillation. Mechanical damping by a small plunger moving in a cylinder, or electromagnetic damping by induced eddy currents, are the means employed to secure this.

Differences Between Ammeters and Voltmeters

3. Most meters are current-operated; that is, the deflecting force results from a *current* passing through the instrument, although the meter may be scaled to read either amperes or volts. Thus in general, ammeters and voltmeters both work on the same principles. (An important exception is the electrostatic voltmeter described in Para. 26). The differences that occur in design are due only to their different functions:—

(*a*) *Ammeter.* This instrument is inserted *in series* with the circuit so that the current to be measured passes through it. The ammeter must be of *low resistance*, otherwise the circuit current will be considerably reduced when the meter is inserted and test conditions upset.

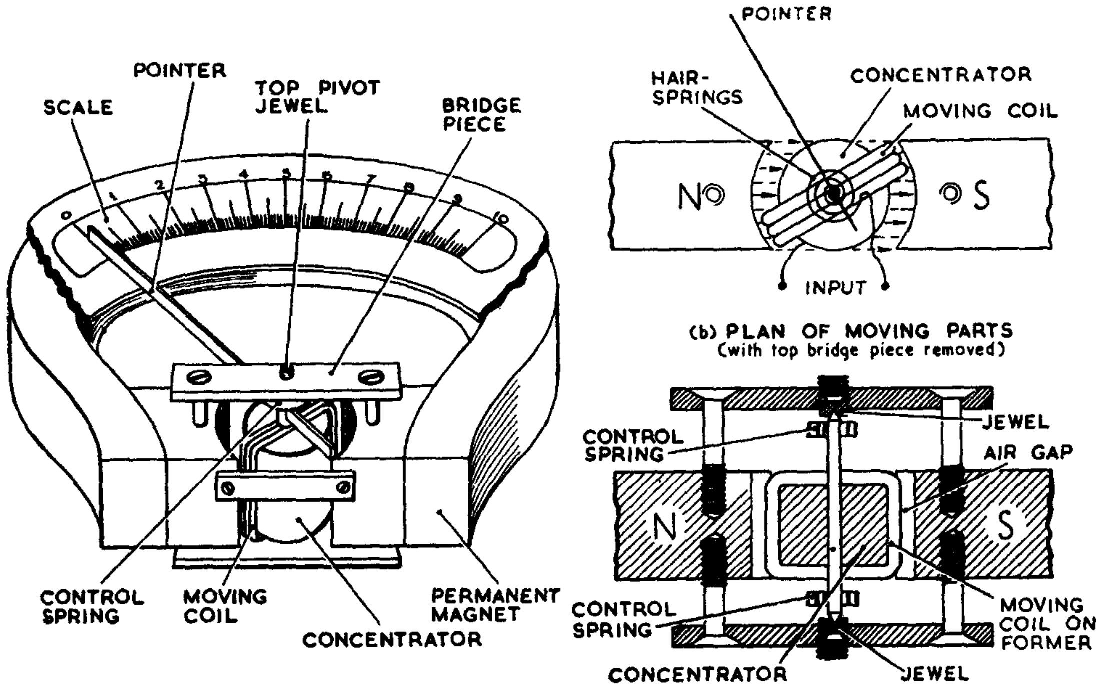
(*b*) *Voltmeter.* This instrument measures p.d. and so must be connected *across* the two points in a circuit between which the voltage is to be found. It must therefore have a *high resistance*, otherwise it will by-pass considerable current from the circuit being examined and test conditions will be upset.

4. The same instrument can be used either as an ammeter or as a voltmeter by suitably adjusting its resistance value (see Para. 9). The various types of ammeters and voltmeters in general use are classified as follows:—

(*a*) Moving coil.
(*b*) Rectifier.
(*c*) Moving Iron.
(*d*) Hot-wire.
(*e*) Thermo-junction.
(*f*) Electrostatic (voltmeter only).

Moving Coil Instruments

5. The operation of the moving coil meter is based on "the motor principle"; that is a current-carrying coil in a magnetic field experiences a torque (see Sect. 3, Chap. 2). The construction of a moving coil meter is illustrated in Fig. 1. The magnetic field is provided by a horse-shoe permanent magnet. Between the poles is fixed a cylindrical soft-iron core (concentrator) which serves to concentrate the flux in the air gap and give



(b) PLAN OF MOVING PARTS (with top bridge piece removed)

(a) GENERAL VIEW

(c) SECTION THROUGH MOVING PARTS

Fig. 1. MOVING COIL METER, CONSTRUCTION

a uniform *radial* field. A moving coil of fine copper wire is mounted on a light, rectangular, aluminium former placed over the iron core. The coil is pivoted to move freely in the air gap between the pole faces and the cylindrical core. The electrical connections to the coil are made by two hairsprings, one at either end of the coil former. The hairsprings are of phosphor-bronze or beryllium-copper, both of which have very stable elastic properties. The springs also provide the controlling force and are coiled in opposite directions to reduce the effect of temperature variation. The tension of the springs can be varied slightly to give zero adjustment. The pointer is attached to the same spindle as the coil former.

6. When the instrument is inserted in a circuit, current flows through the hairsprings to the coil. The resultant force tends to turn the coil on its pivots in a direction dependent upon that of the current (Fleming's left hand rule). Since the magnetic field is of the same intensity at all coil positions, the coil is subjected to a *constant* torque directly proportional to the current in it. As the coil rotates, the hairsprings become twisted and set up an opposing torque proportional to the angle through which the coil has turned. The coil therefore takes up a position in which the controlling force due to the springs just balances the deflecting force due to the current. Since these forces are proportional to the angle of rotation of the coil and to the current in the coil respectively, it follows that the deflection of the pointer attached to the coil is directly proportional to the current in the coil; the scale is therefore evenly divided. When the aluminium former is in motion, circulating eddy currents are established in it. From Lenz's law, the eddy currents produce a force tending to oppose the motion producing them, thereby acting as a damping device to bring the pointer to rest without oscillation.

7. The main *attributes* of moving coil meters are:—

(*a*) High sensitivity.

(*b*) Low power consumption.

(*c*) Suitability for adaptation as voltmeters or ammeters.

(*d*) Freedom from the effects of external magnetic fields.

(*e*) Linear scale.

(*f*) Excellent damping.

8. A *disadvantage* of moving coil meters is that, unless suitably modified, they are useful only for d.c. measurements. Since the direction in which the coil rotates is depend-

ent on the direction of the current, the meter must be connected in the correct polarity. For this reason the two test terminals are marked with + and − signs. Some moving coil meters have a centre-zero scale and can be read in either direction, in which case polarity is not important. If an alternating current of *very low frequency* is applied to such a moving coil meter, the meter will be able to follow the alternations and the pointer will move from side to side about the zero mark. At frequencies in excess of a few cycles per second however the pointer will be unable to follow and will read the *average* value of the current — i.e., zero. Hence, alternating current will not register on a moving coil meter. On the other hand, if the current consists of d.c. *plus* an a.c. component, a moving coil meter in the circuit will read the d.c. and be unaffected by the a.c.

Extension of Meter Range

9. It was stated in Para. 4 that a meter can be adapted as either an ammeter or voltmeter by suitably adjusting its resistance value. Consider a moving coil meter, having a coil resistance of 100 Ω and a full scale deflection (f.s.d.) current of 1 mA, as shown in Fig. 2.

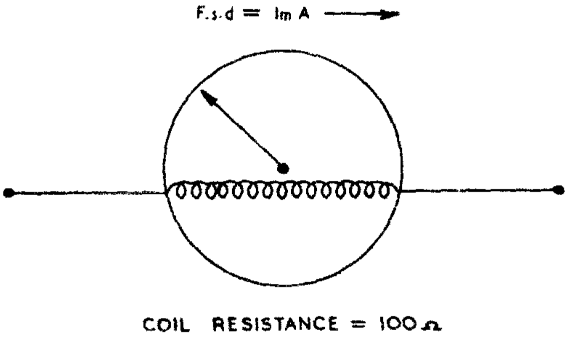


Fig. 2. BASIC MOVING COIL METER CIRCUIT

The voltage necessary for f.s.d. is:—

$$V = IR \text{ (volts)}$$

$$V = \frac{1}{1{,}000} \times 100$$

$$\therefore V = 0{\cdot}1\text{V or } 100 \text{ mV.}$$

Thus, the meter scale could be calibrated to read either *current* or *voltage*, but only to a maximum of 1mA or 100mV respectively. To extend these to more practical ranges, suitable meter "shunts" or "multipliers" must be incorporated to change the meter resistance.

10. **Adapting as a Voltmeter.** The basic instrument is adapted by inserting a suitable resistor (known as a *multiplier*) *in series* with the coil. These resistors are usually made of a material which has a low temperature coefficient (e.g., manganin) to minimize inaccuracies due to temperature changes. The value of the necessary multiplier can be calculated by Ohm's law. Fig. 3 shows a moving coil meter which has a coil resistance of 100 ohms and a f.s.d. current of 1 milliampere. To adapt the instrument for use as a voltmeter reading 0—10 volts, a resistor must be inserted in series with the coil. To find the value of this series multiplier:—

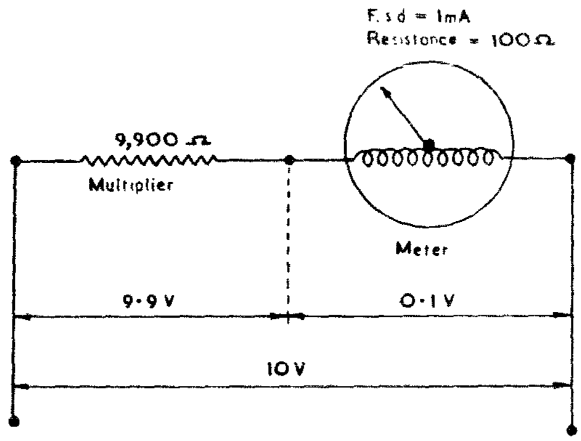


Fig. 3. ADAPTATION AS A VOLTMETER

Maximum voltage to be measured = 10V

Maximum (f.s.d.) current = 1mA = 10^{-3}A

Total resistance of coil and multiplier = R_T

$$\therefore R_T = \frac{V}{I}$$

$$R_T = \frac{10}{10^{-3}}$$

$$\therefore R_T = 10{,}000\ \Omega$$

Resistance of coil = 100 Ω

$\therefore$ Resistance of multiplier = 10,000—100
= 9,900 Ω

11. **Ohms Per Volt Rating.** A basic requirement of a meter is that it has minimum effect on the circuit under test. Thus, the less the current taken by a voltmeter, the more independent the circuit conditions will be of the meter loading. In practice the "resistance per volt" is made high to permit a low power consumption, 1,000 ohms per volt being a useful standard.

12. **Adapting as an Ammeter.** Currents larger than the normal f.s.d. current can be measured by connecting a resistor (known as a *shunt*) *in parallel* with the coil of a moving coil meter. The resistance of the shunt is adjusted so that a definite fraction of the total current passes through the coil of the instrument, the remainder passing through the shunt. The scale of the meter is graduated to read the *total* current. Fig. 4 shows a moving coil meter, which has a coil resistance of 100 ohms and a f.s.d. current of 1 milliampere. To adapt the meter for use as a milliammeter reading 0—100 milliamperes, a shunt resistor must be connected in parallel with the coil. With 100 milliamperes total current, the division of current must be:—

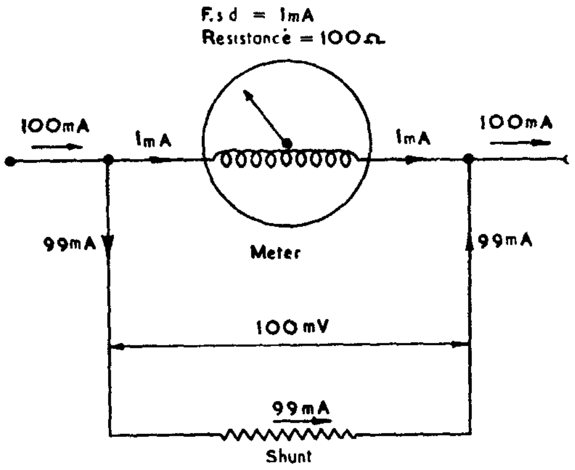


Fig. 4. ADAPTATION AS AN AMMETER

Coil current = 1mA

Shunt current = 100 — 1 = 99 mA

P.d. across coil = Coil current × coil resistance

$$\therefore V = \frac{1}{1{,}000} \times 100$$

$$\therefore V = 100\text{mV}$$

Since the coil and shunt are in parallel, the p.d. across the shunt is also 100mV.

$$\therefore \text{Resistance of shunt} = \frac{\text{Shunt p.d.}}{\text{Shunt current}}$$

$$R = \frac{100 \times 10^{-3}}{99 \times 10^{-3}}$$

$$\therefore R \simeq 1\,\Omega$$

13. **Swamping resistors.** The shunt of an ammeter is made of manganin or similar alloy, having a low temperature coefficient, while the coil of the instrument is of fine copper wire which has a relatively high temperature coefficient. The passage of current through the instrument increases the temperature of the coil and results in an increase in coil resistance; but the resistance of the shunt will remain virtually constant. The ratio of coil resistance to shunt resistance is thus varied, and the meter tends to become inaccurate. The inaccuracy can be reduced by inserting a "swamping" resistor of manganin in series with the coil (Fig. 5).

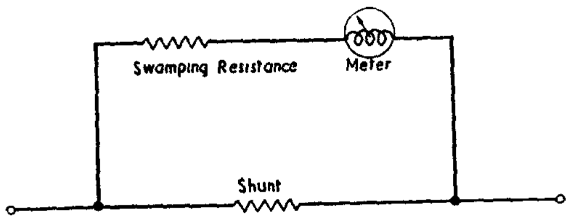


Fig. 5. SWAMPING RESISTOR

If the added resistance is high in relation to the coil resistance, any small changes in coil resistance due to heating are "swamped" by the constancy of the remainder of the resistance in the coil circuit.

Rectifier Instruments

14. A rectifier is a component which, while permitting the free passage of current in one direction, virtually stops the flow in the opposite direction, (see Sect. 9). Thus, on applying a.c. to a rectifier, current flows in one direction only. A rectifier fitted in series with the coil of a moving coil meter will ensure that only current in the correct direction is established in the meter, thereby

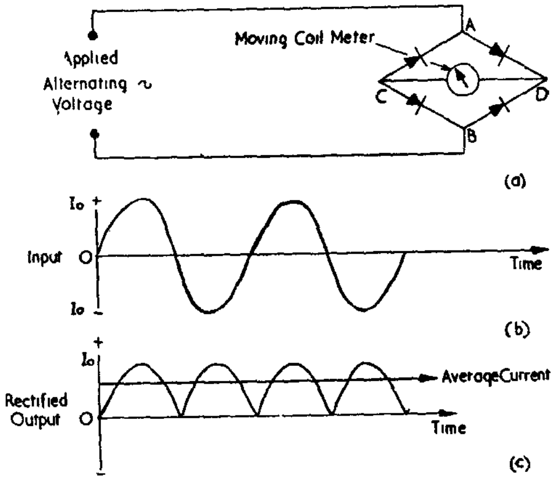


Fig. 6. PRINCIPLE OF THE RECTIFIER INSTRUMENT

preventing reversal of the deflecting force on the coil. By this means, a moving coil meter *with a rectifier* can be used to measure alternating currents and voltages. A typical arrangement is shown in Fig. 6(*a*). When the alternating voltage of Fig. 6(*b*) is applied between terminals A and B, the current through the moving coil meter connected between C and D is a "rectified" current as shown in Fig. 6(*c*). The meter reads the *average* value of the rectified current, but the scale is normally calibrated to indicate *r.m.s. values*. When a moving coil meter has been modified in the manner described it can be used for a.c. measurements up to frequencies of the order of 100 kc/s.

Moving Iron Instruments

15. There are two types of moving iron instrument which differ according to whether deflection is produced by magnetic attraction or repulsion. The former makes use of the attraction of iron in the electromagnetic field of a current-carrying solenoid; the latter depends upon the mutual repulsion of two similar, magnetised pieces of iron within an energised solenoid, one being fixed and the other movable.

16. **Attraction type.** This instrument is illustrated in Fig. 7. The current to be measured passes through a coil wound on a non-magnetic former, thereby establishing a magnetic field whose strength is proportional to the current. An eccentrically pivoted soft iron vane, which carries a light pointer, is magnetised by induction from the field and assumes *opposite* polarity, irrespective of the direction of the current. It is consequently attracted into the open end of the coil, thus moving the pointer over the graduated scale. The controlling force is provided by a hairspring (as shown) or by the pull of gravity on the soft iron vane. Damping is provided by a piston moving in a dashpot.

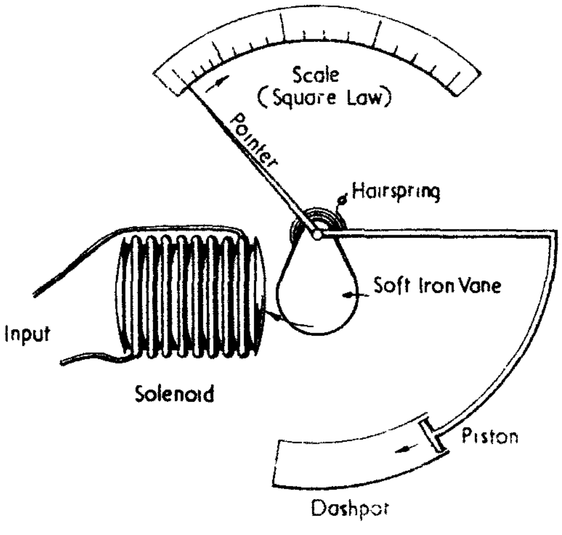


Fig. 7 ATTRACTION TYPE MOVING IRON INSTRUMENT

17. The force of attraction between the soft iron vane and the coil (i.e. the deflecting force) is proportional to the strength of the field due to the current in the coil, and to the strength of the induced magnetism in the soft iron vane. *Both* are proportional to the coil current. The deflecting force is, therefore, proportional to the *square* of the current. As a result, the graduations of the scale are not uniform, but are crowded at the lower end. A scale of this type is generally known as a "*square-law*" scale. The meter scale is fundamentally a square-law one, but by suitable shaping of the vane, the scale may be made to approach a linear law.

18. **Repulsion type.** This is illustrated in Fig. 8. Two pieces of soft iron are arranged inside and parallel to the axis of a circular magnetising coil, which is carrying the current to be measured. One of the pieces (A) is of uniform breadth and is attached to a pivoted spindle which also carries the

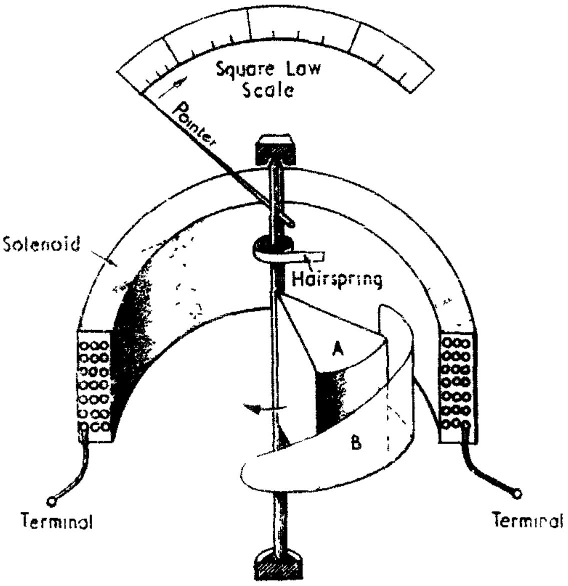


Fig. 8. REPULSION TYPE MOVING IRON INSTRUMENT

pointer. The other piece (B), which is *fixed* to the case, is curved to a circular arc and is tapered in breadth. Under the magnetising force of the current in the coil both pieces of iron are magnetised in the *same sense* and, since like poles repel each other, the moving iron (A) will be repelled from the wide to the narrow end of the fixed iron (B). The controlling force is provided by hairsprings, and damping is by the air dashpot method (not shown).

19. The force of repulsion is proportional to the *product* of the magnetic field strengths of the two iron pieces. Since each of the field strengths is proportional to the coil current, the movement of the pointer is proportional to the *square* of the current, and the scale is square-law.

Note. Since the meter reads $(\text{current})^2$, the scale must be calibrated to give the square root of this. A meter whose scale has been graduated in this way will read the true value for d.c. or the r.m.s. value for a.c.

20. **Advantages of the moving iron instrument.**

(*a*) Cheap, simple and robust.

(*b*) Can be used to measure d.c. or *low frequency* a.c. since the deflecting force is in the same direction irrespective of the direction of the current.

(*c*) Can be used as either ammeters or voltmeters; ammeters have a coil of few turns of thick wire to provide a low resistance; and voltmeters use a coil of many turns of fine wire to give a high resistance.

21. **Disadvantages of the moving iron instrument.**

(*a*) Affected by external magnetic fields.

(*b*) Subject to errors from hysteresis.

(*c*) Uneven scale; can be made more linear by suitably shaping the soft iron pieces.

(*d*) Sensitivity and accuracy are poorer than that of the moving coil meter.

Hot-wire Instruments

22. The hot-wire instrument shown in Fig. 9 measures current by using the heating effect. The construction consists of a wire W, which has a high melting point and a large coefficient of expansion (e.g., platinum-silver), stretched between a fixed point A and a zero-adjusting screw B. One end of a phosphor-bronze wire P is attached to W at C the other end being fixed to an insulated post. A silk fibre is attached to P at D, and the fibre passes round a pulley E which carries a pointer. The silk is kept in tension by a spring F. When a current is passed through W, the wire expands and sags. The sag in W is taken up by the spring F acting through the silk fibre and P, and the pulley rotates, moving the pointer over the scale. A small expansion of the wire W is greatly magnified and conveyed to the pointer. Eddy current damping is introduced by an aluminium disc H (carried on the spindle of the pulley) moving between the poles of a permanent magnet G. The heating effect of the current is proportional to the *square* of the current so that the expansion of W, and consequently the pointer deflection, also varies as I^2. The scale is, therefore, square-law.

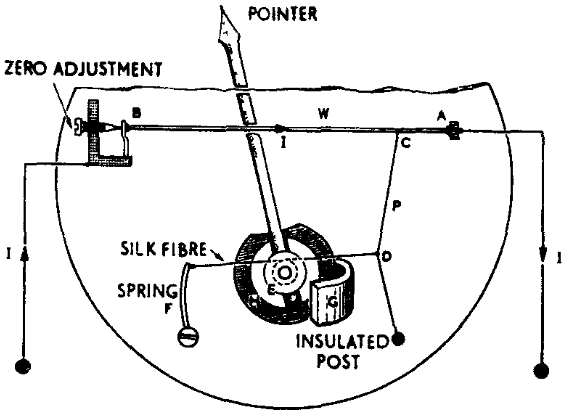


Fig. 9. HOT-WIRE INSTRUMENT

23. The advantages and disadvantages of the hot-wire ammeter are listed below:—

(*a*) *Advantages.*

(i) Can be used for d.c. or moderately high-frequency a.c. measurements since the heating effect is independent of the current direction.
(ii) Efficient damping.

(*b*) *Disadvantages.*

(i) Fragile and easily overloaded.
(ii) Suffers from zero drift due to thermal and elastic fatigue in the wire.
(iii) The power absorbed is high.
(iv) Sluggish in action.
(v) Square-law scale.

Thermo-junction Instruments

24. If two dissimilar metals are joined at one end and the junction heated, a *thermo-*

electric e.m.f. is set up and current will flow in any circuit connected to the other ends of the two metals. The usual "*thermo-junction*" used in meters consists of bismuth and antimony, but other combinations may be used.

25. This principle is used in the thermo-junction meter illustrated in Fig. 10. The current to be measured passes between A and B, raising the temperature of the heater wire. The thermo-junction can be spot-welded to the heater wire at C, or simply placed close to the heater without actually touching it. In either case the junction is normally surrounded by a glass capsule which is evacuated to reduce heat losses. As the temperature of the heater wire (and hence that of the thermo-junction) rises with increased current, so the e.m.f. across the two dissimilar metals rises. The current resulting from the thermo-junction e.m.f. is indicated by a moving coil meter, and the meter is calibrated to read directly the current in the external circuit of which the heater wire AB forms a part. Since the heating effect is independent of the direction of the current, the thermo-junction meter is suitable for both a.c. and d.c. The meter is so built that the junction is neatly encased in the instrument and it often resembles an ordinary moving coil meter though, as it depends for its action on the heating effect of a current, its scale is *square-law*. The thermo-junction meter has all the advantages of the moving coil meter, with the added advantage that it can be used for the measurement of *radio frequency current*.

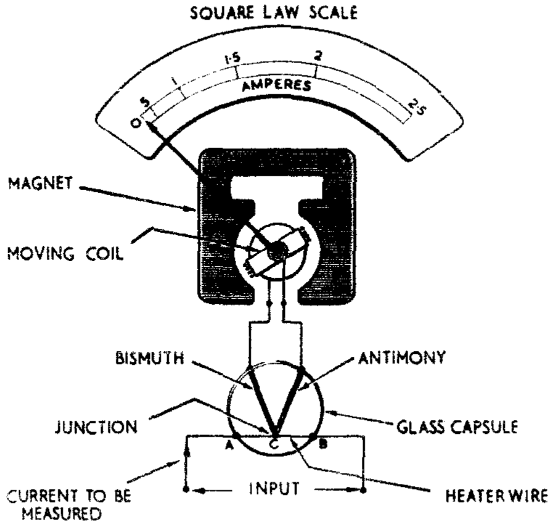


Fig. 10. THERMO-JUNCTION INSTRUMENT

Electrostatic Voltmeters

26. The action of this instrument depends on the force of attraction which exists between two bodies having opposite electric charges. This principle cannot be applied to the measurement of current, but it is frequently used in the measurement of high voltage. One type of electrostatic voltmeter is illustrated in Fig. 11. It consists of a spring-loaded pivoted spindle on which is mounted a pointer and a light aluminium vane C, the latter being so positioned that it can rotate between two fixed plates A and B. The fixed plates are electrically common and are insulated from the metal body of the instrument. Also mounted on the spindle is an additional small vane D which moves between the poles of a permanent magnet M to give eddy current damping. The hairspring tends to hold vane C away from the fixed plates and acts as the controlling force. One terminal of the meter is connected to the fixed plates, the other terminal being connected to the moving vane C via the metal base plate and the spindle.

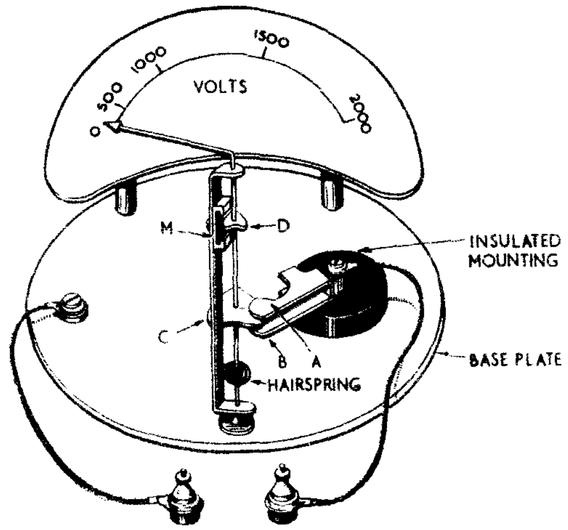


Fig. 11. ELECTROSTATIC VOLTMETER

27. When a voltage is being measured, a difference of potential is established between vane C and the fixed plates, and they acquire equal and opposite charges in a manner similar to that of a capacitor. The electrostatic attraction between the oppositely-charged plates causes vane C to move into

the space between the fixed plates, and the pointer moves over the scale. This movement continues until the deflecting force is balanced by the controlling force of the hairspring, when the pointer comes to rest. The equal charges on the fixed and moving plates are proportional to the voltage being measured, but the force of attraction between the plates is proportional to the *product* of the charges (Coulomb's law). The movement of vane C and the pointer is thus proportional to the *square* of the applied voltage, giving a square-law scale. Since the deflection depends on V^2 the instrument can be used both for direct and alternating voltage measurement. When calibrated for d.c. measurement it reads r.m.s. values on a.c.

28. The great advantage of the electrostatic voltmeter is that the power consumption is negligible. It is not affected by external magnetic fields. It must, however, be screened against the effect of external *electrostatic* fields. The number of vanes on the instrument varies with the range; on a typical 150V instrument there are 13 moving vanes arranged in a manner similar to that of an ordinary variable capacitor; on a 3,500V instrument there is a single moving vane (Fig. 11). Electrostatic voltmeters are used mainly for high voltage measurement.

Summary

29. The main points concerning the meters discussed in this Chapter are summarised in Table 1.

Type	Scale	Remarks	Main Use
Moving coil	Linear	Sensitive and accurate. D.C. only	Most d.c. circuits
Moving coil (Rectifier)	Linear. If calibrated on d.c., reading must be multiplied by 1·11 to give r.m.s. value on a.c.	D.C. and a.c. up to frequencies of the order of 100 kc/s.	In testmeters
Moving iron (both types)	Square-law. If calibrated on d.c., gives r.m.s. value on a.c.	D.C. and low frequency a.c.	A.C. power supply circuits.
Hot-wire	— ditto —	D.C. and a.c. up to moderately high frequencies. Sluggish in action	Not often used
Thermo-junction	— ditto —	D.C. and a.c. up to very high frequencies	Radio frequency currents
Electrostatic voltmeter	— ditto —	D.C. and a.c. Negligible power consumption	High voltage measurement

TABLE 1. AMMETERS AND VOLTMETERS

SECTION 6

CHAPTER 2

TEST INSTRUMENTS

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PART 1, SECTION 6, CHAPTER 2

TEST INSTRUMENTS

Introduction

1. The primary concern of the radio fitter is to maintain the equipment for which he is responsible in an efficient state. This involves the location and rectification of faults in the equipment and this can be achieved by careful observation and routine testing using appropriate test equipment. The quantities which normally require to be measured during "fault-finding" are current, voltage and resistance at various points in a circuit. Current and voltage measurements are carried out with ammeters and voltmeters, such as those described in Chap. 1. The instruments used for resistance measurement—together with the multi-purpose test instruments which combine the measurement of current, voltage and resistance—are described in this Chapter.

2. Instruments used for measuring resistance are of three main classes:—

(*a*) General purpose instruments which are used to measure *medium* values of resistance fairly accurately. The "*simple ohmmeter*" satisfactorily fulfils this function.

(*b*) Continuity testers, to indicate *very low* values of resistance such as those which occur when testing the continuity of a cable. The "*bonding tester*" does this accurately.

(*c*) Insulation testers, to measure *very high* values of resistance such as those which occur when testing the insulation between cables. The "*megger*" is the instrument generally used for this.

Simple Ohmmeter

3. Approximate measurements of resistance may be carried out by means of the simple and convenient arrangement illustrated in Fig. 1. A sensitive milliammeter is connected in series with a single 1·5V cell and a variable resistor R. Terminals X and Y are short-circuited and R adjusted to give full scale reading on the meter. This adjustment allows for variation in the p.d. of the battery with use. If now the short-circuit is removed, and a resistor of *known* value is connected between X and Y, the resultant *decrease* of current is a measure of the value of the resistance, and the current scale can be calibrated in ohms. The *greater* is the resistance, the *less* the deflection. Such an arrangement is called an ohmmeter and is invariably included as an integral part of the multi-range test sets described later.

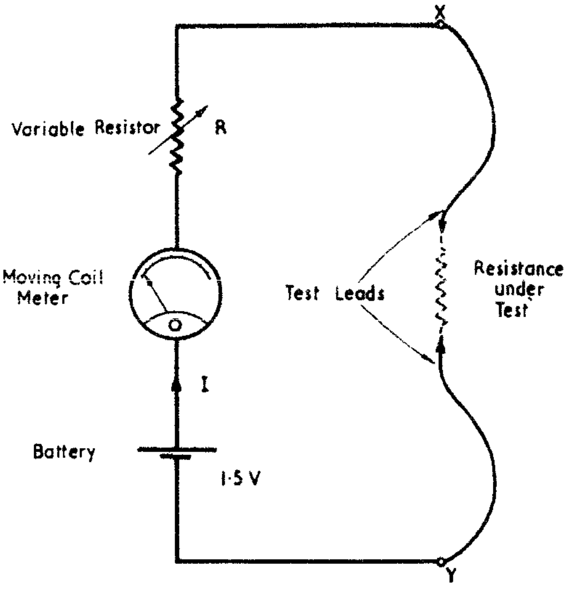


Fig. 1. SIMPLE OHMMETER CIRCUIT

Multimeters

4. These are multi-purpose, multi-range testmeters for the measurement of voltage, current and resistance values, the basic measuring unit being a sensitive moving coil meter. For the first two of these functions it is simply a question of arranging suitable switches to bring in the appropriate series multiplier or shunt resistors. A second set of voltage and current circuits *with a rectifier* may be provided for a.c. measurements. For resistance measurement an ohmmeter assembly, with battery and variable resistor, is switched in.

5. A typical multimeter uses a moving coil meter which itself has a resistance of 50 ohms and gives f.s.d. with 2 milliamperes. Fig. 2 shows the d.c. voltmeter arrangement. Terminal X goes to a suitable switch which makes contacts A, B, C, D or E, and terminal Y goes directly to the meter. The series multiplier resistors are arranged in cumulative steps, with the results of Table 1.

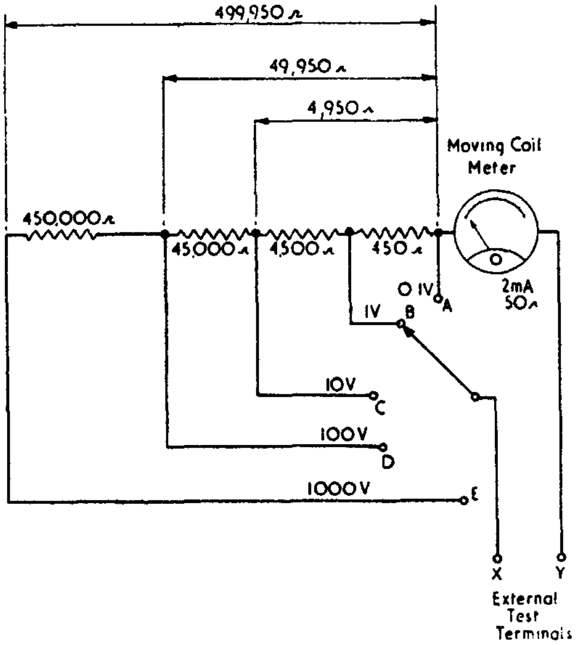

499,950Ω
49,950Ω
4,950Ω
Moving Coil Meter
450,000Ω
45,000Ω
4,500Ω
450Ω
0·1V
A
1V
B
2mA
50Ω
10V
C
100V
D
1000V
E
X
Y
External Test Terminals

Fig. 2. D.C. VOLTMETER ARRANGEMENT

Position of Switch	Series Multiplier (Ohms)	Total Resistance (Ohms)	P.d. between X and Y for f.s.d.
A	0	50	50 × 0·002 = 0·1V
B	450	500	500 × 0·002 = 1V
C	4,950	5,000	5,000 × 0·002 = 10V
D	49,950	50,000	50,000 × 0·002 = 100V
E	499,950	500,000	500,000 × 0·002 = 1,000V

TABLE I. VOLTAGE MEASUREMENT.

6. For the d.c. ammeter ranges, the universal shunt principle is used. The full resistance of the tapped shunt (Fig. 3) is 12·5 ohms, which is one quarter of the meter resistance. Thus, when the switch is in position A, *one fifth* of the total current passes through the meter, and f.s.d. corresponds to 5 × 0·002 = 0·01 A. The tappings are at $\frac{1}{10}$, $\frac{1}{100}$ and $\frac{1}{1,000}$ of the total shunt resistance, giving the ranges shown in Table 2.

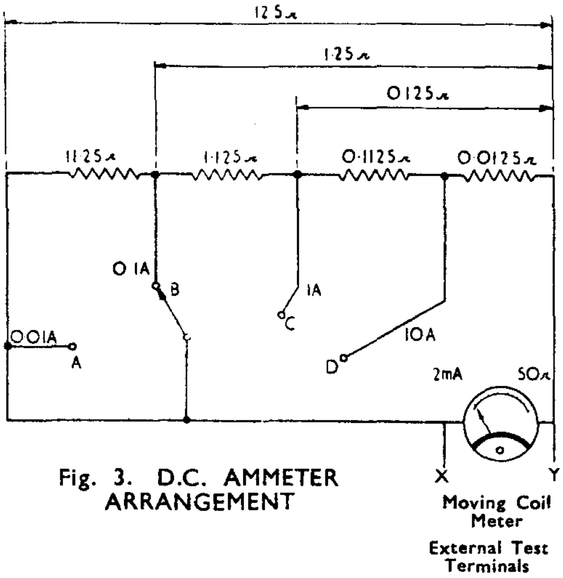

12·5Ω
1·25Ω
0·125Ω
11·25Ω
1·125Ω
0·1125Ω
0·0125Ω
0·1A
B
1A
C
0·01A
A
D
10A
2mA
50Ω
X
Y
Moving Coil Meter
External Test Terminals

Fig. 3. D.C. AMMETER ARRANGEMENT

Position of Switch	Fraction of Shunt in Parallel with Meter	Current between X and Y for f.s.d.
A	All	0·01A
B	$\frac{1}{10}$	10 × 0·01 = 0·1A
C	$\frac{1}{100}$	100 × 0·01 = 1A
D	$\frac{1}{1,000}$	1,000 × 0·01 = 10A

TABLE 2. CURRENT MEASUREMENT.

7. Where a.c. measurements are included, the moving coil meter is connected to the output of a bridge rectifier, the input to which is from a *transformer* (see Sect. 7). The current and voltage a.c. ranges are then obtained by adjusting the transformer on each switch position.

8. When the meter is switched to read resistance, a battery is connected in series with an adjustable resistance network and the meter, as in the simple ohmmeter.

Testmeter Type D

9. The Testmeter Type D, commonly known by its trade name of "Avometer" is a multimeter of the type described in the preceding paragraphs, and is illustrated in Fig. 4. It is a 34-range instrument giving the ranges shown in Table 3.

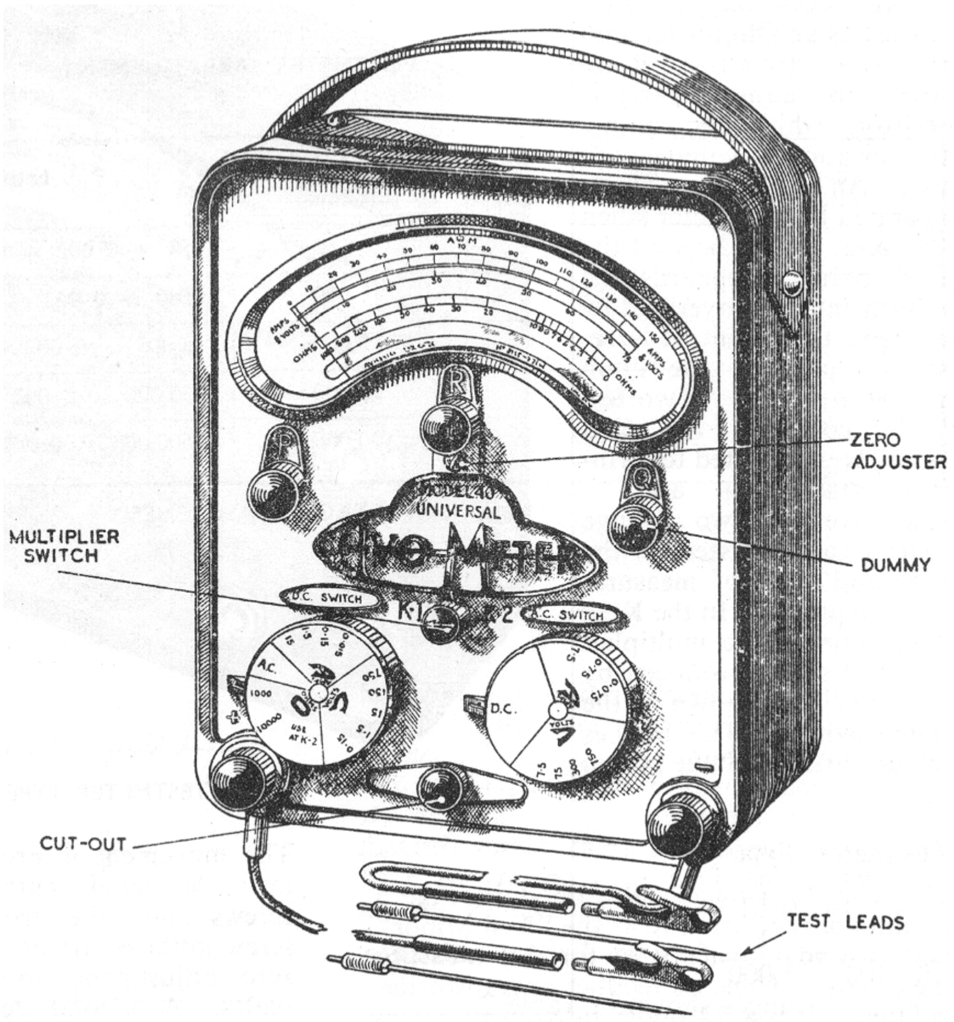

ZERO ADJUSTER
MULTIPLIER SWITCH
DUMMY
CUT-OUT
TEST LEADS
MODEL 40 UNIVERSAL
AVO-METER
D.C. SWITCH
A.C. SWITCH
K1
K2
A.C.
D.C.

Fig. 4. TESTMETER TYPE D

Voltage		Current	
D.C.	A.C.	D.C.	A.C.
Ten ranges, 0 — 150mV to 0 — 1,500V	Eight ranges, 0 — 7·5V to 0 — 1,500V	Eight ranges, 0 — 15mA to 0 — 30A	Six ranges, 0 — 75mA to 0 — 15A
	Resistance		
—	0 — 1,000 Ω 0 — 10,000 Ω		—

TABLE 3. RANGE MEASUREMENTS.

10. The scale has three sets of calibrations marked in Amps, Volts and Ohms (hence the name "Avo"). A 1·5V dry cell supplies the voltage when the testmeter is used as an Ohmmeter, and the two controls P and R are used to adjust variable resistors which compensate for variation of voltage with use. An automatic cut-out, operated by the pointer when the latter travels beyond the f.s.d. point, safeguards the movement from overload by opening the circuit; it is re-set by pressing the knob shown in Fig. 4. Two test leads, coloured red and black, are connected to terminals marked + and − respectively. Two range switches are adjusted for the type and range of measurement required. In the K = 2 position of the multiplier switch, f.s.d. is produced by *twice* the value shown on the range switch. This applies to current and voltage ranges only.

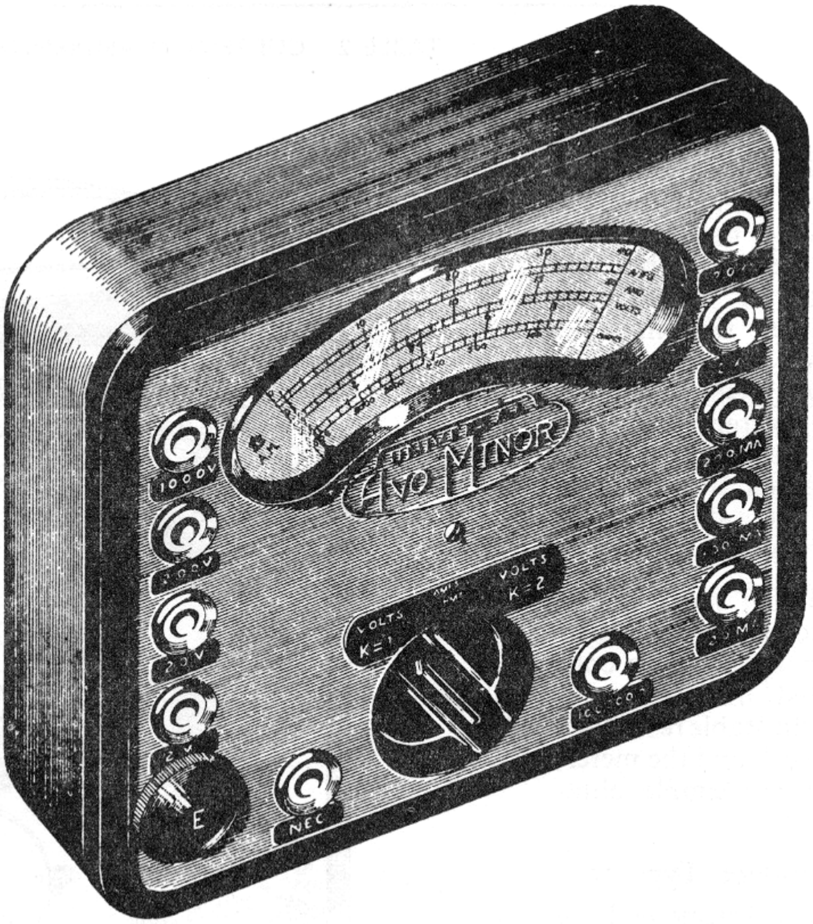


Fig. 5. TESTMETER TYPE E

Testmeter Type E

11. This instrument, illustrated in Fig. 5, is commonly referred to as the "Avo Minor", and is a multimeter used for d.c. measurement only. Thus, no rectifier or transformer is fitted. It is a 14-range instrument giving the ranges shown in Table 4.

12. The front of the testmeter carries eleven protected sockets and a three-position switch. The movement is protected by a fuse held in a detachable carrier marked E, which screws into the front panel. A slotted screw in the centre of the front panel provides zero adjustment on the volts and amps scales. A second zero adjuster, used to compensate for battery voltage variations when measuring resistance, is located on the bottom edge of the testmeter (not seen in Fig. 5). The socket marked NEG is

Voltage	Current
Eight ranges, 0 — 2V to 0 — 2,000V	Five ranges, 0 — 20mA to 0 — 20A
	Resistance
	0 — 10,000Ω
Multiplier switch on the panel doubles *voltage* indications in position K = 2	

TABLE 4. RANGE MEASUREMENTS.

common to all ranges and carries one of the test leads, the other test lead being plugged into the socket covering the desired range. The three-position switch must be correctly set.

13. The simple ohmmeter, as used in Testmeters Types D and E, requires continual adjustment if the error due to variation in supply voltage is to be eliminated. This error occurs since the deflecting force varies with the applied voltage, but the controlling force exerted by the springs of the instrument remains unaffected. This unsatisfactory feature is overcome in the *ratiometer* type of ohmmeter.

The Ratiometer Principle

14. In the ratiometer the controlling force is an electrical force which is derived from the same supply voltage as the deflecting force. Any variation in supply voltage affects both forces equally, and error due to variation in the supply voltage is eliminated. The ratiometer is illustrated in Fig. 6. It is essentially a moving coil instrument in which the usual solid cylindrical iron core is replaced by a *hollow* concentrator which is mounted eccentrically to give a varying air gap. The flux density between the pole faces and the concentrator therefore varies from a maximum at the lower pole faces to a minimum at the upper portion. In addition to the normal deflecting coil A, a second coil B is wound on the same former-unit, but at an angle to coil A. Only one side of coil B is exposed to the magnetic flux, the other side being screened from the flux by the concentrator. The coils are wound in such a way that when they are connected to a single source they will produce *opposing* torques. The instrument has no hairspring control so that with both coils disconnected from the supply the pointer will "wander", having no fixed position.

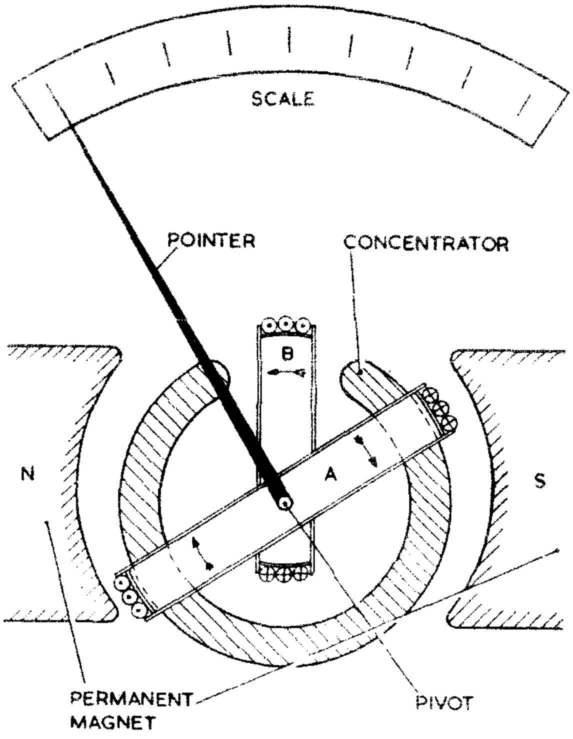


Fig. 6. RATIOMETER TYPE OF OHMMETER

15. The coils are wound with very fine wire and can carry only a small current. Each coil is therefore connected in series with a resistor in order to limit the current. Coil B, with its associated resistor X, is connected across the supply voltage. Coil A, with its

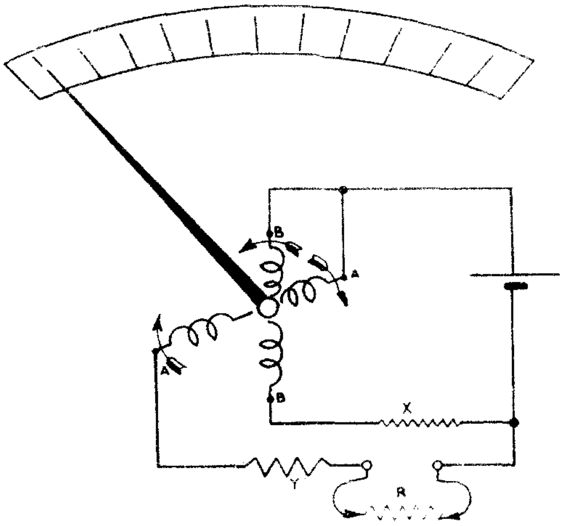


Fig. 7. CIRCUIT OF RATIOMETER

resistor Y, is also connected across the supply when an external resistance R is connected across the test leads (Fig. 7). With a given current in coil A, the torque exerted will be relatively constant, irrespective of its position in the air gap, since a weakening of flux at one side of the coil is compensated by a corresponding strengthening of flux at the opposite side. The torque due to coil B, with a given current, will vary according to the position of the unscreened side of the coil, being zero at the position shown in Fig. 6 and increasing steadily as the unscreened side of the coil moves into the decreasing air gap.

16. (*a*) With the test leads open-circuited there is no circuit through coil A. Coil B exerts an anti-clockwise torque, and when it reaches the magnetic neutral position (Fig. 6), the torque ceases and the pointer takes up a position at the left hand side of the scale, reading infinity.

(*b*) With the test leads connected to a circuit, current is established in coil A. This current is inversely proportional to the resistance of the circuit under test, and the resulting torque moves the former unit *clockwise* and the pointer traverses the scale. As the unscreened side of coil B moves into the air gap an *anti-clockwise* torque is developed, and this increases steadily as coil B moves into the decreasing air gap. The torque due to coil A remains constant. When the torques due to each coil are equal, the coil former stops and the pointer indicates the external circuit resistance on the scale.

Bonding Tester

17. This is a ratiometer type of instrument designed to measure very low values of resistance. It is intended primarily for testing the continuity of "bonded" connections in aircraft to ensure that the resistance between such connections does not exceed a certain authorised maximum value. It can also be used for continuity testing of other circuits. The simple ohmmeter is not sufficiently accurate to perform this function satisfactorily. The bonding tester consists of a wooden case in which is housed a ratiometer type of ohmmeter and a 1·2V battery, a 6 ft. length of twin flexible cable (A) carrying a double-spike probe, and a 60 ft. length of similar cable (B) attached to a single-spike probe (Fig. 8).

18. Reference to Fig. 9 will show that with both probes connected to the bond under test, the low resistance coil A is energised by current flowing from the cell to the single spike, through the bond under test to the right hand spike of the double probe, to the coil, and so to the negative pole of the cell. That is, coil A is *in series* with the bond under test. The high resistance coil B is *in parallel* with the bond under test; it will therefore carry a current proportional to the *p.d.* across the bond. The position taken up by the pointer is determined by the ratio between opposing torques, i.e., by the ratio

$$\frac{\text{Voltage across bond}}{\text{Current in bond}} = \text{Bond resistance.}$$

Megger

19. A distinct difference exists between resistance measurement as applied to conductor resistance, and resistance measurement of insulation. The ability of an

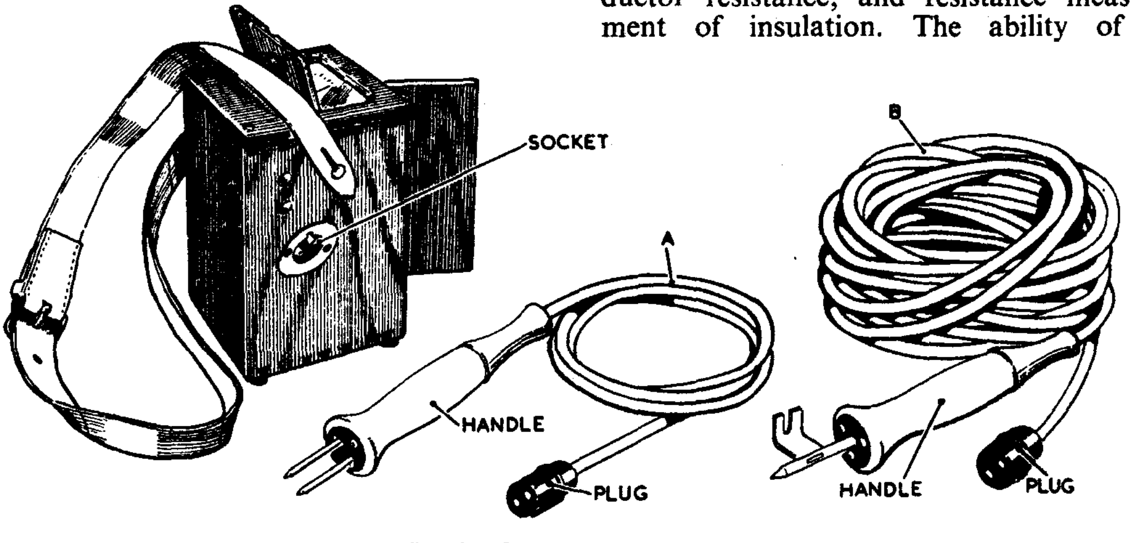


Fig. 8. BONDING TESTER

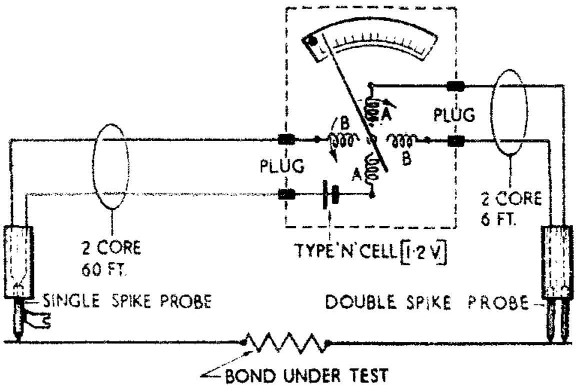


Fig. 9. CIRCUIT OF BONDING TESTER

insulator to resist the passage of current is affected by the voltage applied to it, and in extreme cases the insulation may break down completely under increasing voltage. Insulation resistance measurement is usually carried out with a testing voltage considerably in excess of the normal voltage of the circuit. If the resistance value remains high under high voltage conditions, it is safe to assume that there is little risk of failure in operation at the normal supply voltage.

20. Several types of insulation testers are available, but the majority consist of a ratiometer type of ohmmeter with a hand-driven generator as the source of supply. Typical of those approved for insulation testing of radio installations and equipments is the Insulation Tester Type C, more generally known as the "Wee Megger" (Fig. 10). The generator is of the two-pole permanent magnet type, driven through gears by a handle which folds back into a recess in the casing when not in use. A clutch is provided to restrict the output voltage of the generator to 250V d.c. at a handle speed of 160 r.p.m. The scale reads from zero to infinity with intermediate readings between 10,000 ohms and 20 megohms.

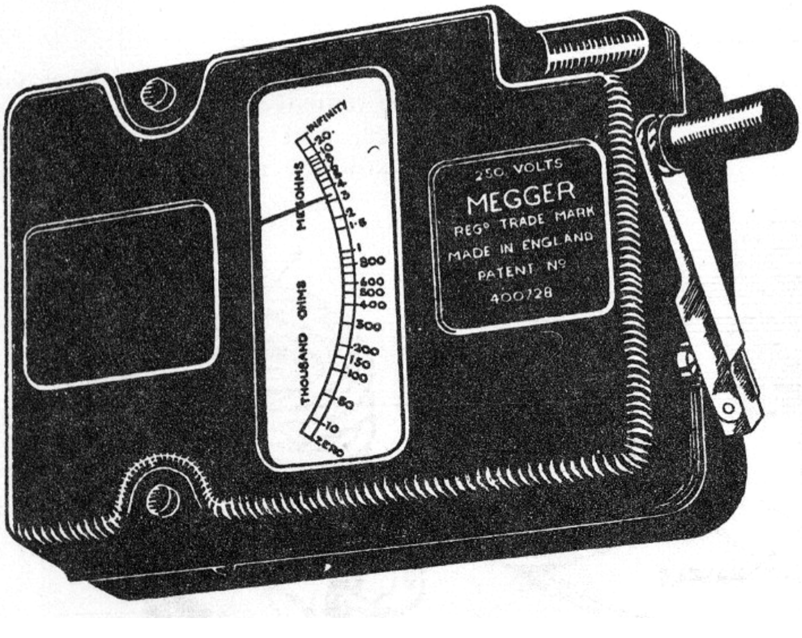
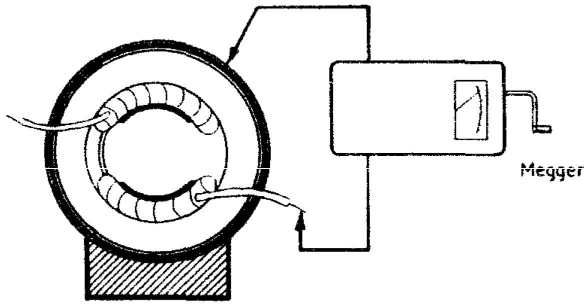


Fig. 10. INSULATION TESTER TYPE C

21. Insulation tests may be divided into two categories, examples of each being given below:—

(*a*) *Insulation to earth.* This test is to establish that a circuit is electrically isolated from metal which surrounds or supports it. One lead of the Megger is connected to the circuit and the other to a clean surface on the frame on which the circuit is installed (Fig. 11). The tester handle is turned at 160 r.p.m. and the insulation resistance between the circuit and the frame is read off the scale. It should not normally be less than several megohms.



Testing insulation between field coils and yoke of a small motor

Fig. 11. TESTING INSULATION TO EARTH

(*b*) *Pole to pole tests.* These tests verify that the insulation is satisfactory between one part of a circuit and another, e.g., the insulation between the cores of a cable (Fig. 12). The tester leads are connected to the conductors between which it is desired to test the insulation. On turning the Megger handle at approximately 160 r.p.m. the insulation resistance will be indicated. The minimum permissible insulation resistance for any given circuit or component is given in the appropriate servicing schedule.

Summary

22. The main points concerning the test instruments discussed in this Chapter are listed in Table 5.

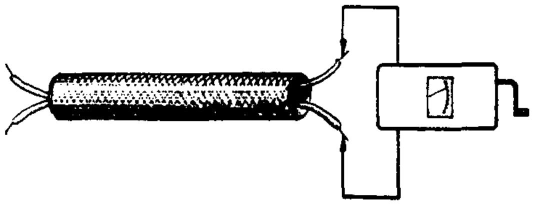


Fig. 12. TESTING INSULATION BETWEEN THE CORES OF A CABLE

Instrument	Details	Use
Testmeter Type D (Avometer)	Multimeter using shunts and multipliers to give current and voltage d.c. ranges: rectifier and transformer for a.c. ranges: simple ohmmeter for resistance measurement.	General purpose testing and fault-finding: moderate values of resistance where a high accuracy is not essential.
Testmeter Type E (Avominor)	Similar to the Testmeter Type D but with the a.c. ranges excluded.	D.c. measurements only during general purpose testing.
Bonding tester	Ratiometer type of ohmmeter.	Accurate measurement of very low resistance values.
Insulation Tester Type C (Wee Megger)	Ratiometer type of ohmmeter with a hand-driven generator supply.	Insultation testing and measurement of very high resistance values.

TABLE 5. SUMMARY OF TEST METERS.

SECTION 6

CHAPTER 3

MEASUREMENT OF POWER

	Paragraph
Introduction	1
Dynamometer Wattmeter	2
Power Ratios	6
The Decibel	9
Absolute Powers Expressed in Dbm	13
Current and Voltage Ratios	14
Decibel Meters	16
The Neper	17

MEASUREMENT OF POWER

Introduction

1. In a d.c. circuit, the product VI, where V is the steady p.d. across a load and I the steady current through it, measures the power consumption of the load. In an a.c. circuit, the product of the instantaneous values of voltage and current gives the *instantaneous* rate of working. The product of the r.m.s. values of V and I gives the *apparent power* in the circuit. Except in the case of a purely resistive load, there will be a phase difference θ between voltage and current. The *true power* in the circuit is then VI cos θ, where cos θ is the power factor of the load. An instrument designed to measure true power in an a.c. circuit is termed a *wattmeter.* It combines in itself the functions of voltmeter and ammeter, and responds to phase difference in such a way as to record the true power VI cos θ.

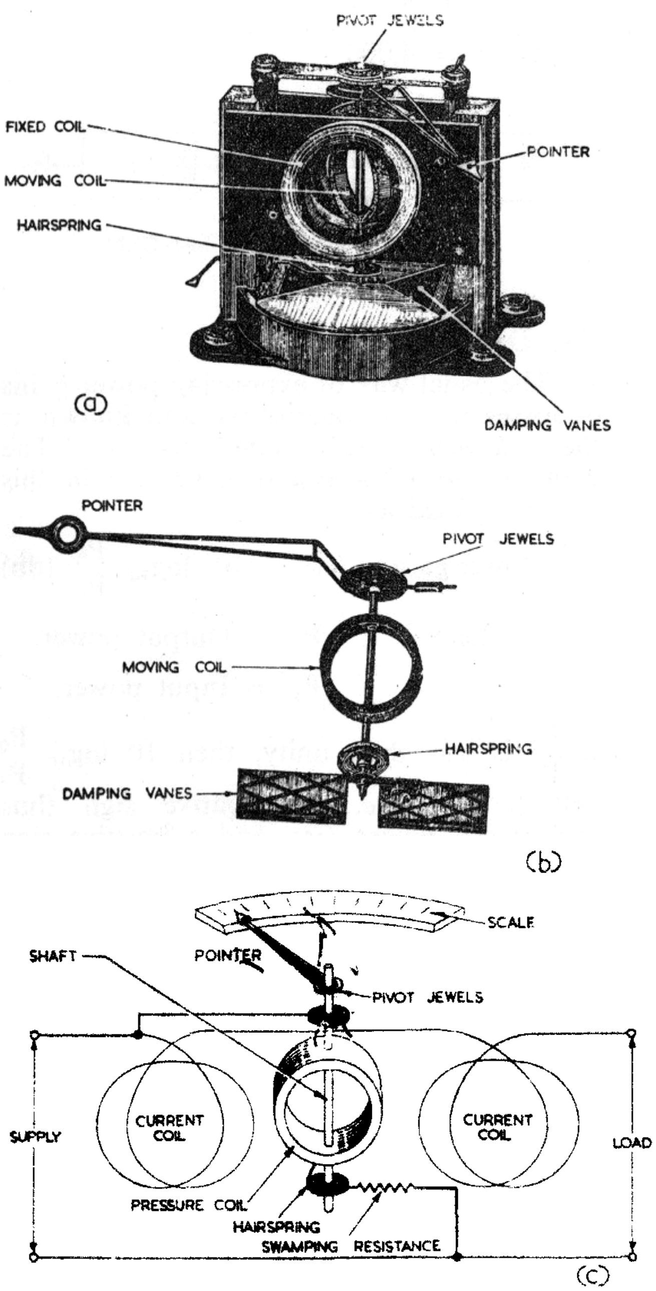

PIVOT JEWELS
FIXED COIL
MOVING COIL
HAIRSPRING
POINTER
DAMPING VANES
(a)
POINTER
PIVOT JEWELS
MOVING COIL
HAIRSPRING
DAMPING VANES
(b)
SCALE
SHAFT
POINTER
PIVOT JEWELS
SUPPLY
CURRENT COIL
CURRENT COIL
LOAD
PRESSURE COIL
HAIRSPRING
SWAMPING RESISTANCE
(c)

Fig. 1. THE DYNAMOMETER WATTMETER

Dynamometer Wattmeter

2. This type of instrument is in general use for the measurement of power at *low frequencies* and is the only one considered in this Chapter. The complete assembly of a typical dynamometer wattmeter is shown in Fig. 1(*a*), the assembled moving system (i.e., the moving coil, damping vane, control springs and pointer) being shown in Fig. 1(*b*). Fig. 1(*c*) shows the connections in diagrammatic form. The fixed coil assembly (the "*current*" coil) consists of two air-cored coils connected in series with the load. The moving coil (the "*pressure*" coil) is of fine wire, wound on a light frame which is mounted on a spindle and pivoted to turn in the magnetic field produced by the current coil. The movement is controlled by two hairsprings of phosphor-bronze, which also serve as the connections to the pressure coil. Air damping is provided.

3. A simplified circuit diagram of the dynamometer wattmeter is given in Fig. 2. The low resistance current coil L_1, is connected *in series* with the load and carries the whole of the load current. The magnetic field of this coil is thus proportional to the instantaneous current. The high resistance pressure coil L_2 is connected in series with a high resistance, *across* the load voltage.

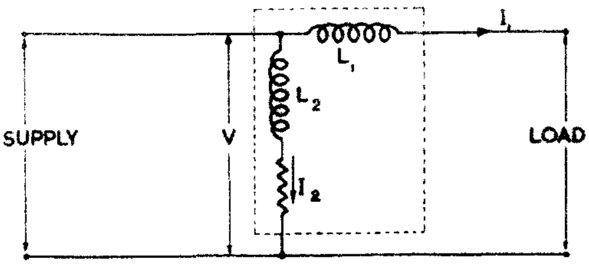

I_1
L_1
L_2
SUPPLY
V
LOAD
I_2

Fig. 2. SIMPLIFIED CIRCUIT OF THE DYNAMOMETER WATTMETER

The reactance of this circuit can therefore be neglected at low frequencies, and the current in the pressure coil is *in phase with*, and proportional to, the load voltage. The connections of the fixed coil L_1 resemble those of an ammeter and the connections of the moving coil L_2 those of a voltmeter.

4. The torque exerted upon the moving coil is proportional to the product of the currents in each of the coil assemblies; that is, to the product of the instantaneous current and the instantaneous voltage in the load, namely the instantaneous power. The *average* value of this torque over any number of complete cycles gives an indication of the true power (VI cos θ) and the scale is graduated accordingly.

5. At high frequencies, the increased reactance of the instrument introduces additional phase shift and the meter becomes increasingly inaccurate. The usual practice at high frequencies is to measure the current in a known resistance in the circuit. Alternatively, the voltage across a standard resistance inserted in the circuit can be measured. The power is then calculated from the relationship:—

$$P = I^2 R$$

$$P = \frac{V^2}{R} \text{ (watts).}$$

The ammeter or voltmeter used will normally be a rectifier type of instrument, and the scale may be calibrated directly in watts (when used with a standard resistance).

Power Ratios

6. No device can deliver more power than is supplied to it. *Amplifiers* are designed to increase the power of an a.c. by drawing additional power from an independent supply. If the power supplied to operate the amplifier (the input) is 2 watts, and the power delivered from the amplifier to a load (the output) is 40 watts, the power ratio or "*gain*" is $\frac{40}{2} = 20$. The power drawn from the supply is $40 - 2 = 38$ watts.

7. Power losses occur under certain conditions, and circuits called *attenuators* are designed to reduce the power of an a.c. If the input power to the attenuator is 5 watts and the output power is 3 watts, then the power ratio, or "*loss*" is $\frac{3}{5} = 0{\cdot}6$.

8. If power is delivered at the rate of P_1 watts to an amplifier or attenuator, and taken from it at the rate of P_2 watts, the ratio $\frac{P_2}{P_1}$ is the "power gain" (greater than unity) or the "power loss" (less than unity) of the system. Consider three amplifiers connected "in cascade" or "in tandem" as shown in Fig. 3; that is, the output of one is connected to the input of the next. The overall power gain $\frac{P_4}{P_1}$ is obtained by *multiplying together* the individual power gains :—

$$\frac{P_4}{P_1} = 4 \times 100 \times 10$$

$$\therefore \text{Gain} = 4{,}000$$

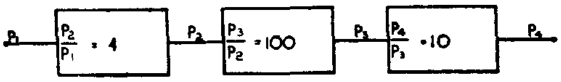


Fig. 3. AMPLIFIERS IN TANDEM

The Decibel

9. The usual way of expressing power gains or losses is by a logarithmic unit known as the "*decibel*" (abbreviated to db). The gain or loss of a system expressed in this unit is defined as:—

$$\text{Power gain or loss} = 10 \log_{10} \frac{P_2}{P_1} \text{ (db)}$$

where P_2 = Output power.

P_1 = Input power.

If $\frac{P_2}{P_1}$ is less than unity, then $10 \log_{10} \frac{P_2}{P_1}$ will be *negative*. A negative sign thus indicates a power *loss*, and a positive sign a *gain*. The overall power gain in decibels of the system shown in Fig. 3 is:—

$$\text{Gain} = 10 \log_{10} 4 + 10 \log_{10} 100 + 10 \log_{10} 10$$

$$\text{Gain} = 6 + 20 + 10$$

$$\therefore \text{Gain} = +\ 36 \text{ db.}$$

10. As an example, consider an amplifier with an input of 1mW giving an output of 10W. The power gain in decibels is:—

$$\text{Gain} = 10 \log_{10} \frac{P_2}{P_1}$$

$$\text{Gain} = 10 \log_{10} \frac{10}{10^{-3}}$$

$$\text{Gain} = 10 \log_{10} 10^4$$

$$\text{Now } \log_{10} 10^4 = 4$$

$$\therefore \text{Gain} = +40 \text{ db.}$$

Thus, + 40 db. indicates a power gain of $\frac{10}{10^{-3}} = 10{,}000$.

11. Given the gain of an amplifier in decibels the ratio of output to input powers can be obtained.

$$\text{Power gain in db} = 10 \log_{10} \frac{P_2}{P_1}$$

$$\therefore \log_{10} \frac{P_2}{P_1} = \frac{\text{Power gain in db.}}{10}$$

$$\therefore \frac{P_2}{P_1} = \text{Antilog}_{10} \frac{\text{Power gain in db}}{10}$$

Thus, in an amplifier which has a power gain of +20 db, the power output for an input of 100mW is:—

$$\frac{P_2}{P_1} = \text{Antilog}_{10} \frac{20}{10}$$

$$\frac{P_2}{P_1} = \text{Antilog}_{10} 2$$

$$\frac{P_2}{P_1} = 100$$

$$P_2 = 100 \times P_1$$

$$P_2 = 100 \times 10^{-1}$$

$$\therefore P_2 = 10 \text{ W.}$$

12. There are obvious advantages in using the decibel in power transfer. Multiplication and division are replaced by addition and subtraction, and + and - signs distinguish gains and losses. However, the the original reason for using logarithmic units was that the human ear responds to power *ratios* in such a way as to interpret them as *differences* in signal strength; *squaring* a power *doubles* the sound intensity, and so on. Thus, the decibel is a natural unit for expressing sound intensities since it is logarithmic, and it is now universally employed for measuring gain and loss in all cases of power transfer.

Absolute Powers Expressed in Dbm

13. Instead of recording the gain or loss of an amplifier or attenuator, it may be required to measure the *actual power level* at some stage in the circuit. This means comparing the observed power level P_2 with some *standard* power level P_1. The standard usually adopted is 1 mW. Power levels expressed in decibels with reference to a standard power of 1 mW are written as so many "*dbm*". Using this standard, any power P can be expressed as $10 \log_{10} P$(dbm). Thus:—

$$1 \text{ W} = 10 \log_{10} 1{,}000 = +30 \text{ dbm.}$$

$$5\text{mW} = 10 \log_{10} 5 = +7 \text{ dbm.}$$

$$5\mu\text{W} = 10 \log_{10} \frac{5}{1{,}000} = -23 \text{ dbm.}$$

Current and Voltage Ratios

14. Consider two equal resistances each of R ohms carrying currents of r.m.s. values I_1 and I_2, and having voltages across them of r.m.s. values V_2 and V_2 respectively. The powers developed in these two resistors are:—

$$P_1 = I_1^2 R = \frac{V_1^2}{R}$$

$$P_2 = I_2^2 R = \frac{V_2^2}{R}$$

$$\therefore \frac{P_2}{P_1} = \left(\frac{I_2}{I_1}\right)^2 = \left(\frac{V_2}{V_1}\right)^2$$

The power ratio $\frac{P_2}{P_1}$ expressed in decibels is:—

$$10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \left(\frac{I_2}{I_1}\right)^2$$

$$\therefore 10 \log_{10} \frac{P_2}{P_1} = 20 \log_{10} \left(\frac{I_2}{I_2}\right)$$

$$10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \left(\frac{V_2}{V_1}\right)^2$$

$$\therefore 10 \log_{10} \frac{P_2}{P_1} = 20 \log_{10} \left(\frac{V_2}{V_1}\right)$$

Thus, *provided that the two resistances are equal* through which the two currents I_1 and I_2 (or across which the two voltages V_1 and V_2) are measured the gain or loss of a circuit in decibels is:—

$$\text{Gain or loss} = 10 \log_{10} \frac{P_2}{P_1}$$

$$\text{Gain or loss} = 20 \log_{10} \frac{I_2}{I_1}$$

$$\text{Gain or loss} = 20 \log_{10} \frac{V_2}{V_1}$$

15. In Sect. 5, Chap. 2 it was stated that the bandwidth of a series tuned circuit was the separation between two frequencies either side of resonance at which the current has

fallen to 70% of its peak value. In terms of decibels, the current ratio is:—

$$20 \log_{10} \frac{0 \cdot 707}{1} = -3 \text{ db.}$$

The bandwidth is then expressed as so many kc/s at "3 *db down*". Another figure at which the bandwidth is commonly measured is 6 *db down.* At this latter figure the current has fallen to *half* its peak value.

Decibel Meters

16. By comparing the voltages they cause when connected in turn across a standard impedance, the powers of two sources can be compared. A high impedance voltmeter (Fig. 4) is connected across the impedance and the meter can be calibrated to read power levels above and below 1 mW directly in dbm. The standard impedance usually chosen is a pure resistance of 600 ohms. The r.m.s. voltage V which must be applied across a resistance of 600 ohms in order to dissipate the reference power of 1 mW is 0·775 volts. Thus, when the voltmeter registers 0·775 volts, the reference power of 1 mW is developed and the pointer indicates O dbm on the scale. When the voltmeter registers r.m.s. voltage V_1, the power level is $20 \log_{10}\frac{V_1}{0 \cdot 775}$ (dbm). For instance the power dissipated in the 600 ohms resistance when the voltmeter reads 2 volts r.m.s. is:—

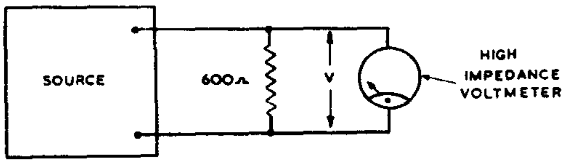


Fig. 4. POWER COMPARISON

$$20 \log_{10} \frac{2}{0.775} = 20 \times 0 \cdot 417$$
$$= + 8 \cdot 23 \text{ dbm.}$$

The scale is calibrated directly in dbm in accordance with Fig. 5. The instrument used up to frequencies of the order of 50 kc/s will be a rectifier type, moving-coil meter; at higher frequencies, special *valve-voltmeter* circuits are used (see Sect. 18).

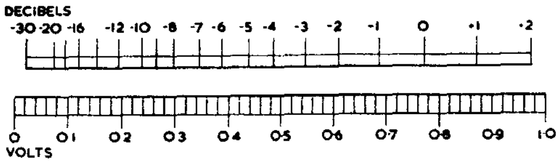


Fig. 5. CALIBRATION OF VOLTMETER TO READ DBM.

The Neper

17. This is a unit based on the *natural* logarithm of the ratio of two *current* values, regardless of the resistance value of the circuits. It can be used to express power and voltage ratios when the resistances of the components are equal. It is mainly used in calculations concerned with filters and attenuators in line transmission. The gain or loss of a circuit in nepers is:—

$$\text{Gain or loss} = \log_e \frac{I_2}{I_1}$$

$$\text{Gain or loss} = \log_e \frac{V_2}{V_1}$$

$$\text{Gain or loss} = \tfrac{1}{2} \log_e \frac{P_2}{P_1}$$

Note. There are 8·686 decibels to the neper.

SECTION 7

TRANSFORMERS

Chapter 1 . . . . Coupled Circuits
Chapter 2 . . . . Iron-cored Transformers
Chapter 3 . . . . Transductors

SECTION 7

CHAPTER 1

COUPLED CIRCUITS

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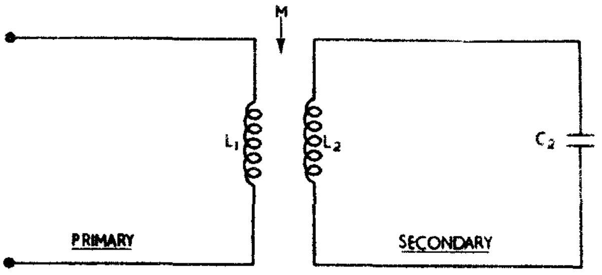
PART 1, SECTION 7, CHAPTER 1

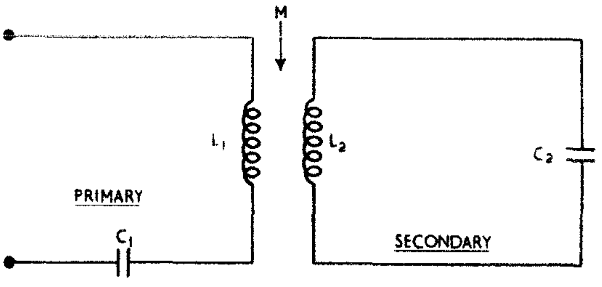
COUPLED CIRCUITS

Introduction

1. Two a.c. circuits are said to be *coupled* when they are so linked that energy is transferred from one circuit to the other. For example, when mutual inductance exists between coils that are in separate circuits, these circuits are 'inductively coupled'. The effect of the mutual inductance is to make possible the transfer of energy from one circuit to the other by *transformer* action. That is, an alternating current established in the first or *primary* circuit produces magnetic flux which is linked with, and induces a voltage in, the coupled or *secondary* circuit. This does not, of course, apply to d.c. circuits since the flux must be *changing* for electromagnetic induction to occur. Two examples of inductively coupled circuits commonly encountered in radio equipments are shown in Fig. 1.



(a) TUNED SECONDARY



(b) TUNED PRIMARY, TUNED SECONDARY

Fig. 1—INDUCTIVELY COUPLED CIRCUITS

Coupling Coefficient

2. It was shown in Sect. 2, Chap. 2 that the proportion of the flux from one circuit which is linked with another determines the extent of the coupling between them. The greatest possible mutual linkage between two circuits occurs when *all the flux* from each embraces every turn of the other. With two coils of self-inductance L_1 and L_2 having mutual inductance M:—

$$M = k\sqrt{L_1 L_2}$$

$$\therefore k = \frac{M}{\sqrt{L_1 L_2}} \quad .. \quad (1)$$

The ratio $\frac{M}{\sqrt{L_1 L_2}} = k$ is termed the *coupling coefficient* between two mutual inductively coupled circuits. It is always less than unity since when $k = 1$, $M = \sqrt{L_1 L_2}$ and there is no leakage of flux between the two circuits ; this is a condition which cannot exist in practice.

Mutual Inductive Coupling

3. Fig. 2 shows two series resonant circuits in which the inductances L_1 and L_2 are placed sufficiently close together to be coupled via the mutual inductance M between them. The values of L and C in each circuit are such that both circuits have the same resonant frequency.

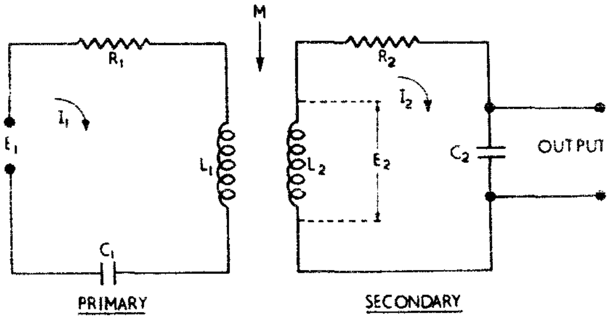


Fig. 2—TWO SERIES TUNED CIRCUITS COUPLED BY MUTUAL INDUCTANCE

4. With the secondary on open circuit, the primary current is dependent entirely on the primary impedance, i.e., $I_1 = \frac{E_1}{Z_1}$. With the secondary on load the effect of the secondary current is to modify the primary circuit by the addition of a 'reflected impedance'. Thus when an e.m.f. of r.m.s. value E_1 is applied to the primary circuit an alternating current of r.m.s. value I_1 will be

established and will set up an alternating magnetic flux around the coil L_1. Some of this flux links with the turns of L_2 and induces an e.m.f. of r.m.s. value E_2 in it, so that a current $I_2 = \frac{E_2}{Z_2}$ is established in the secondary when it is on load. The secondary current in turn induces a voltage back into the primary in such a direction as to tend to oppose the primary voltage (Lenz's law). The two circuits thus interact so as to affect each other to an extent depending on the coupling between them.

Reflected Impedance

5. The current in the primary circuit is not the same as it would be in the absence of the secondary current. The effect of the presence of the secondary is as though an impedance $\frac{(\omega M)^2}{Z_2}$ had been added in series with the primary, where $\omega = 2\pi f$, M = mutual inductance, and Z_2 = series impedance of the secondary circuit considered by itself $= \sqrt{R_2^2 + X_2^2}$. The equivalent impedance $\frac{(\omega M)^2}{Z_2}$ is termed the *reflected impedance,* and since Z_2 contains resistance R_2 and reactance X_2 so also does the reflected impedance. The effect of the reflected impedance is:—

(*a*) To *increase* the primary *resistance.*

(*b*) To *reduce* the primary *reactance.*

6. When M is small or Z_2 is large the reflected impedance $\frac{\omega^2 M^2}{Z_2}$ will be small and the primary circuit will be virtually unaffected by the secondary. When M is large or Z_2 is small, the reflected impedance will be large, and the secondary circuit will affect the primary to a considerable extent. This is especially the case when the secondary is at resonance to the applied frequency for then $Z_2 = R_2$ and $\frac{\omega^2 M^2}{Z_2}$ is a maximum.

7. Off resonance, the effect of the reflected impedance is to increase the primary resistance and to reduce the primary reactance. When both circuits are resonant with the applied frequency, the reflected impedance is purely resistive and its effect is to increase the primary resistance only.

Untuned Primary and Tuned Secondary

8. This is a type of circuit often used in the initial (r.f.) stages in receivers and is as shown in Fig. 3(a). Transformer action results in the primary current inducing a voltage in the secondary, the magnitude of which depends on the coupling coefficient. The resulting secondary current will depend on the secondary impedance and is given by $I_2 = \frac{E_2}{Z_2}$. It varies as in a normal series tuned circuit, so that the variation of output voltage ($V_c = I_2 X_{c_2}$) as the frequency varies about resonance will be as shown in Fig. 3(b).

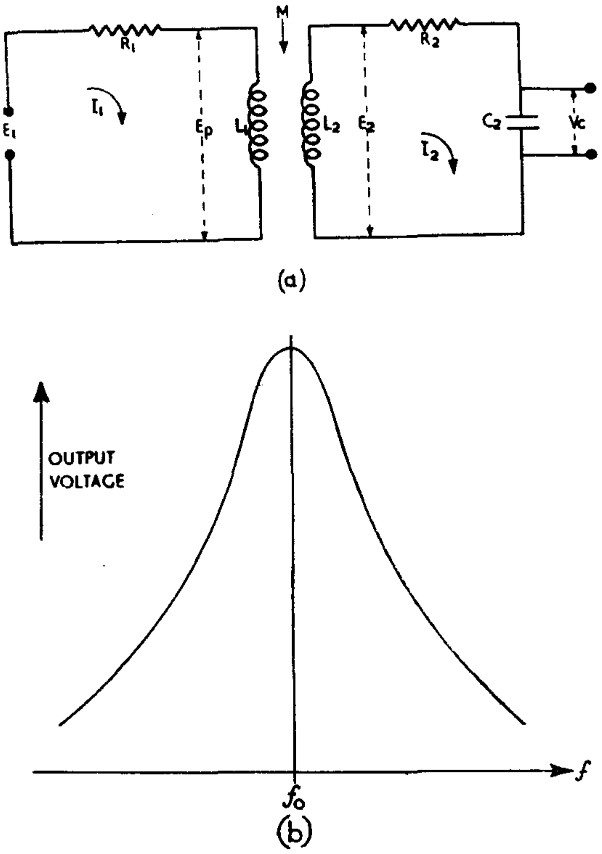


Fig. 3—R.F. TRANSFORMER—UNTUNED PRIMARY, TUNED SECONDARY

9. In practice, the input of E_1 volts to the r.f. transformer of Fig. 3(a) would produce an output voltage V_c of about the same value. This is because the high internal resistance of the supply makes the p.d. across L_1 much less than the e.m.f. E_1. The secondary voltage E_2 would be smaller still (since loose coupling is usual in r.f. transformers) and the Q of the secondary would

cause the output voltage to be of the same order as E_1. The output voltage can be made larger than the supply voltage by tuning the primary as well as the secondary.

Tuned Primary and Tuned Secondary

10. This type of circuit (Fig. 4) is used extensively in radio equipments. When two resonant circuits having equal Q values are tuned to the same frequency and coupled together, the resulting behaviour depends very largely upon the value of the co-efficient of coupling k.

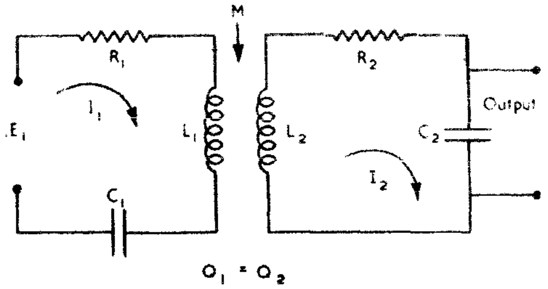


Fig. 4—R.F. TRANSFORMER—TUNED PRIMARY, TUNED SECONDARY

11. **Loose coupling.** This is the term used to denote a low value of k, when there is little interaction between the two circuits. As the frequency of the applied voltage is varied through the resonant point of the two circuits, the current in the primary increases to a maximum and falls off again according to the normal series resonance curve. The voltage induced in the secondary by this primary current will vary in the same manner and the secondary current will vary even more sharply, since the selectivity of the secondary is proportional to the *product* of the Q values of primary and secondary. Its peak value is however small because of the low value of k. These points are illustrated in Fig. 5. Loose coupling is usual in r.f. voltage transformers used in the initial stages of receivers in order to improve the selectivity of such circuits.

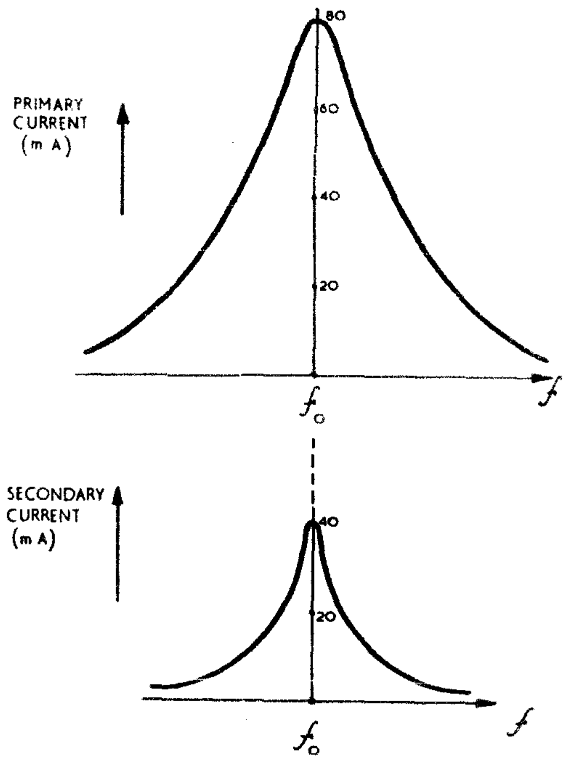


Fig. 5—LOOSE COUPLING

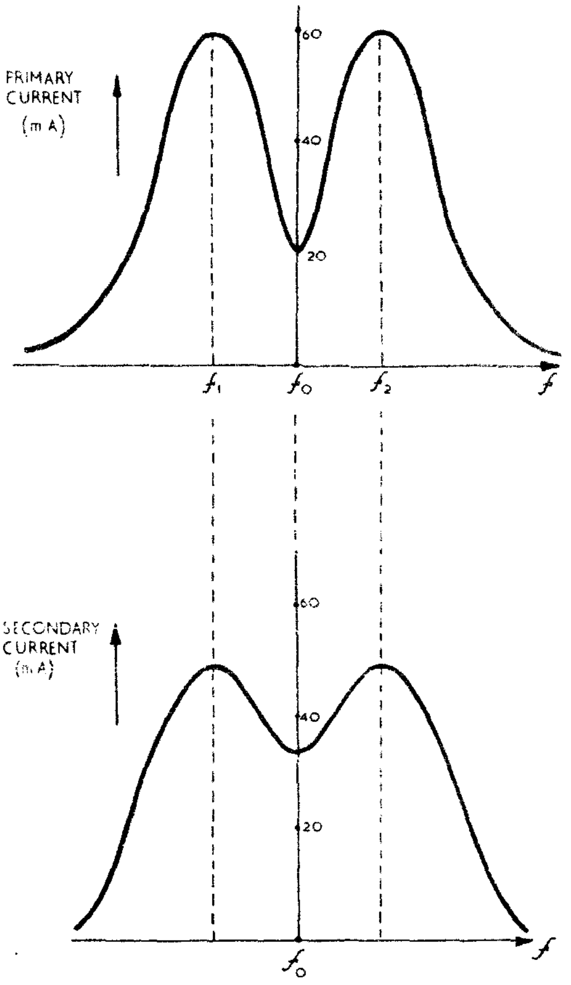


Fig. 6—TIGHT COUPLING

12. **Tight coupling.** This is the term used to denote a high value of k, with appreciable interaction between the circuits. The variations in primary and secondary currents

as the applied frequency is varied about resonance are shown in Fig. 6:—

(*a*) *At resonance* in a series tuned circuit the reactance is zero ($X_L = X_c$) and the impedance $Z = R$. The primary current however, is *not* a maximum because the effective primary impedance has been *increased* by $\frac{\omega^2 M^2}{Z_2}$ and this is large because M is large (a high k) and Z_2 is small (equals R_2 at resonance).

(*b*) *Above resonance* a series tuned circuit behaves as an inductance. The primary inductive reactance is, however, *reduced* by the reactive component of the reflected impedance. It is in fact possible for this component of the reflected impedance to equal the primary reactance considered by itself. Under these conditions the effective primary reactance *again becomes zero*, giving a 'resonant' condition. In addition, the resistive component of the reflected impedance is *reduced* from that at resonance, because of the increase in Z_2, with the result that the primary current may be greater than before.

(*c*) *Below resonance* a series tuned circuit behaves as a capacitance and the situation is similar to (*b*) above, with the primary capacitive reactance being neutralized by the reactive component of the reflected impedance. The net effect of the reflected impedance is to lower the primary current at the resonant frequency and to raise the current at frequencies somewhat off resonance.

(*d*) The 'double-humping' is less pronounced in the secondary than in the primary. This is due to the fact that at the resonant frequency, Z_2 equals R_2 only, whereas the effective *primary* resistance is much higher because of the reflected impedance.

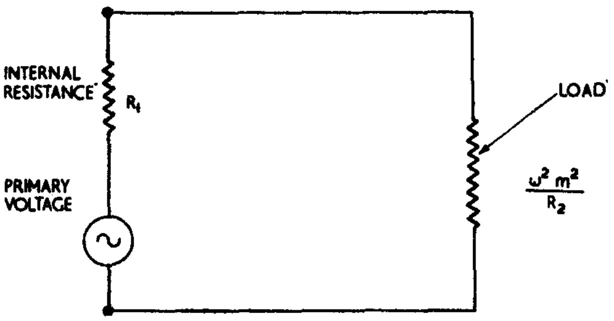


Fig. 7—CONDITIONS FOR MAXIMUM POWER TRANSFER

13. **Critical coupling.** At resonance the impedance of the secondary circuit equals R_2. The effective primary impedance is then $R_1 + \frac{\omega^2 M^2}{R_2}$. The whole primary circuit can be regarded as a generator of internal resistance R_1 transferring energy to the load resistance $\frac{\omega^2 M^2}{R_2}$ as in Fig. 7. Maximum power is transferred to the load when the internal resistance and the external resistance are equal. With coupled circuits, maximum power is transferred from the primary to the secondary when R_1 equals $\frac{\omega^2 M^2}{R_2}$ and this is the condition for optimum or *critical coupling*, at which the secondary current has a single maximum value as shown in Fig. 8. By making R_1 equal to $\frac{\omega^2 M^2}{R_2}$, and sub-

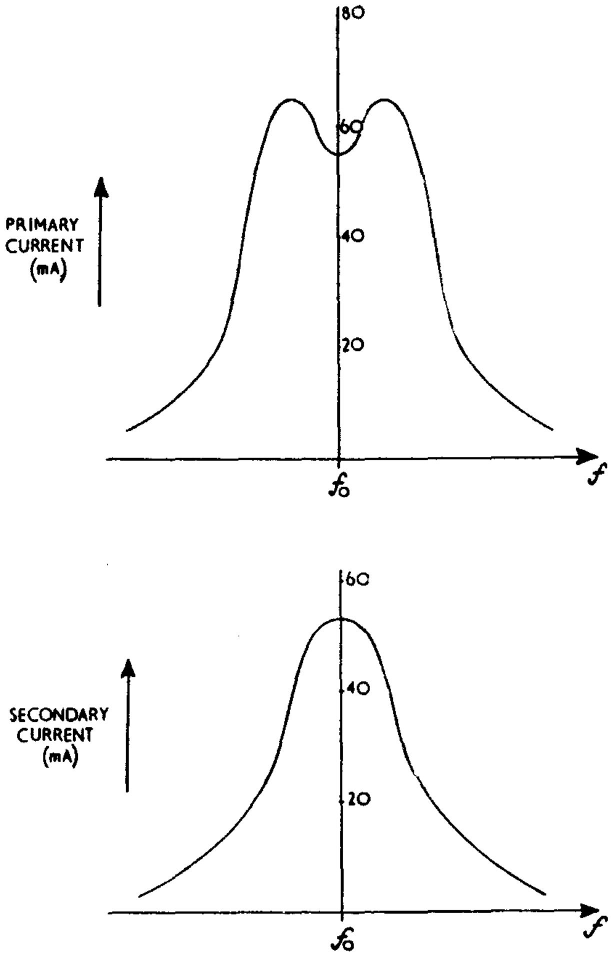


Fig. 8—CRITICAL COUPLING

stituting for M in terms of k it can be shown that for critical coupling:—

$$k = \frac{1}{\sqrt{Q_1\ Q_2}}$$

$$\text{where } Q_1 = \frac{\omega L_1}{R_1}$$

$$Q_2 = \frac{\omega L_2}{R_2}$$

For identical circuits, $Q_1 = Q_2$

$$\therefore\ k_{crit} = \frac{1}{Q} \quad \cdot\cdot \quad \cdot\cdot \quad (2)$$

As normal values of Q in radio circuits range between 100 and 200, the critical value of k for maximum energy transfer is of the order of 0·01 to 0·005.

14. **Summary.** The value of k can be increased by moving the coupled coils closer together. The effect of varying k from a small value, through its critical value, to a high value for a given coupled circuit is as shown in Fig. 9:—

(*a*) *Small k.* If k is very small, the primary response curve (a) is very nearly as if there were no secondary present. Little energy is transferred to the secondary so the secondary current curve (a), while showing a peak at the resonant frequency, is small. As k is increased, more energy is drawn from the primary, and the peak of the primary current curve (b) is lowered while its general shape is broadened and flattened. The secondary current curve (b) is higher and broader, still reaching its maximum value at the resonant frequency.

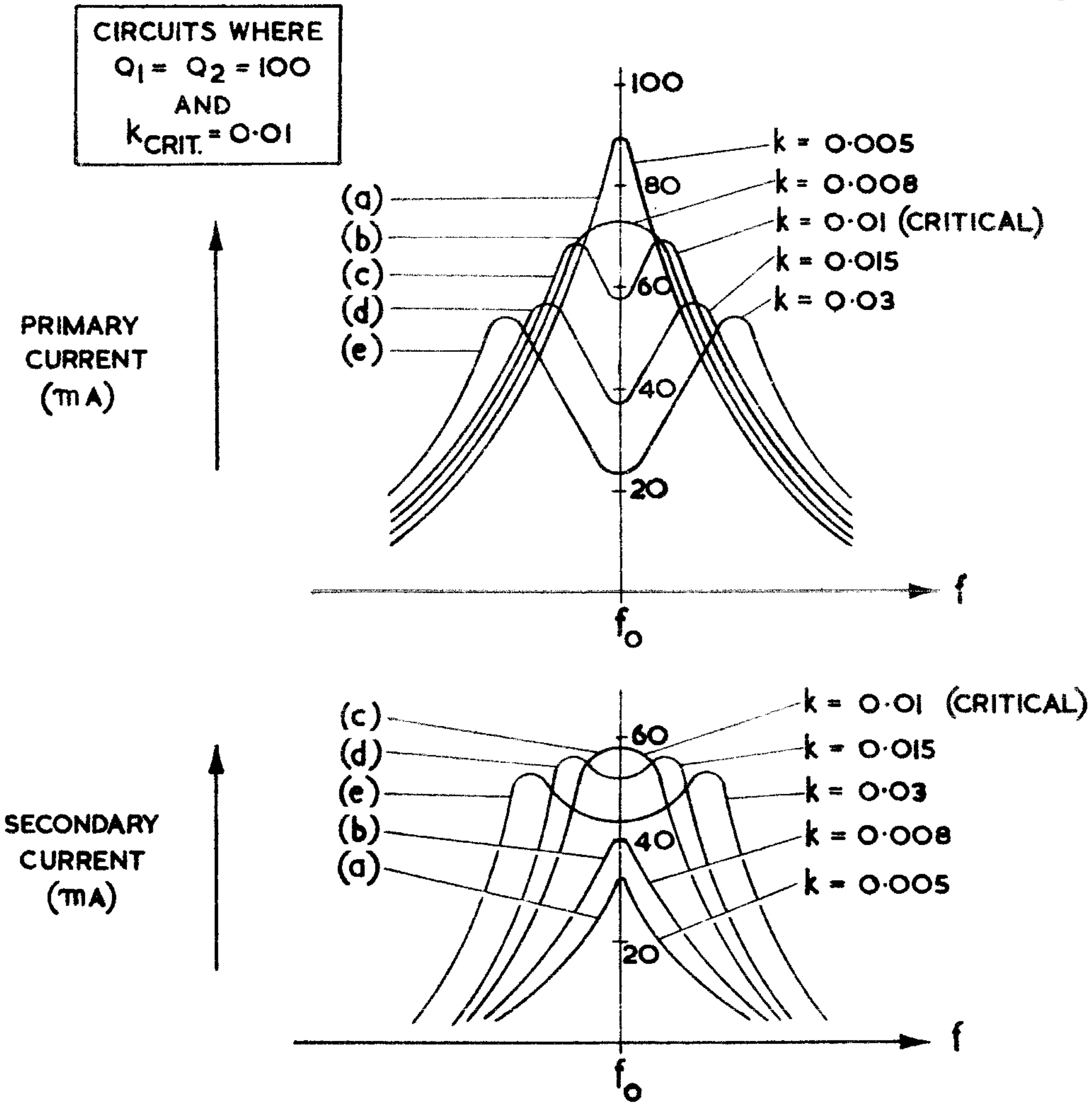


Fig. 9—EFFECT OF VARIATION OF k

(*b*) *Critical k.* This condition is shown by the curve marked (c) in Fig. 9. Two peaks are now evident in the primary curve, while the secondary curve still shows a single maximum, the greatest possible.

(*c*) *Large k.* The primary and secondary response curves for conditions where k is greater than the critical value are shown by curves (d) and (e) of Fig. 9. Double peaks appear in both primary and secondary curves, though the 'dip' between them is much more marked in the primary for reasons given in Para. 12(*d*).

Frequencies of Peaks

15. Where the value of k is greater than the critical value, double-humping occurs in both primary and secondary response curves. The peaks occur at a *virtual* resonant frequency when the primary reactance is zero. The positions of the two virtual resonant peaks can be found from the equation:—

$$f = \frac{f_o}{\sqrt{1 \pm k}} \quad .. \quad .. \quad (3)$$

where f_o is the normal resonant frequency, f is the frequency of maximum primary response, and k is the coupling coefficient. The frequencies at which peaks occur in the primary circuit are, therefore:—

$$f_1 = \frac{f_o}{\sqrt{1 + k}} \text{ (below resonance)}$$

$$f_2 = \frac{f_o}{\sqrt{1 - k}} \text{ (above resonance)}$$

Bandwidth

16. If the bandwidth of a coupled circuit is considered as the spacing between the frequencies at which the peaks occur in the primary circuit:—

$$\text{Bandwidth} = f_2 - f_1$$

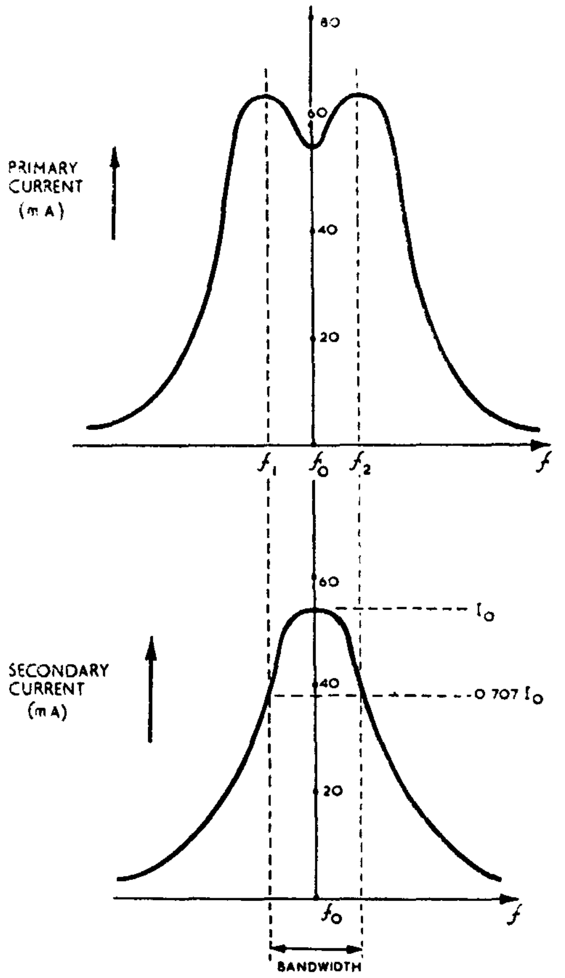
$$= \frac{f_o}{\sqrt{1 - k}} - \frac{f_o}{\sqrt{1 + k}}$$

$$\therefore \frac{\text{Bandwidth}}{f_o} = \frac{1}{\sqrt{1 - k}} - \frac{1}{\sqrt{1 + k}}$$

And if k is small:—

$$\frac{\text{Bandwidth}}{f_o} = k$$

$$\therefore \text{Bandwidth} = kf_o \quad .. \quad .. \quad (4)$$



HALF POWER BANDWIDTH = $f_2 - f_1$ AT CRITICAL COUPLING

Fig. 10—k, Q, AND BANDWITH

At critical coupling for identical circuits:—

$$k = \frac{1}{Q}$$

$$\therefore \text{Bandwidth} = \frac{f_o}{Q} \quad .. \quad .. \quad (5)$$

This is the half-power bandwidth as defined in Sect. 5, Chap. 2 and is as shown in Fig. 10.

17. **Wide-band r.f. transformers.** For coupled circuits, having identical primary and secondary circuits with Q values of 100, the critical value of k is 0·01. From equation (4), it is seen that the bandwidth is then 1 per cent of the resonant frequency. In some cases, bandwidths of up to 10 per cent of the resonant frequency are necessary. For example, a circuit resonant at 50 Mc/s but giving full response at frequencies from 47·5 Mc/s to 52·5 Mc/s

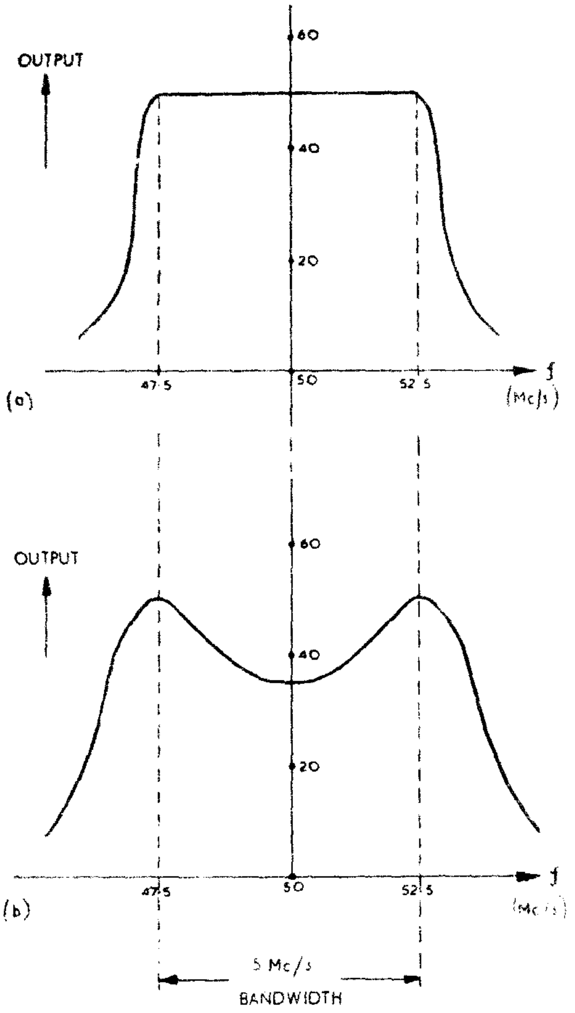


Fig. 11—WIDEBAND R.F. TRANSFORMER

(a bandwidth of 5 Mc/s) may be required as shown in Fig. 11(a). By increasing the value of k beyond the critical value it is possible to produce a circuit having a bandwidth of 5 Mc/s, but the response falls off rapidly between these points (Fig. 11(b)). The use of damping resistors across the primary and secondary will produce a flat top, but the overall response is considerably reduced as shown in Fig. 12(a). The method of obtaining a response of the kind illustrated in Fig. 11(a) is to combine the response curves of several pairs of coupled circuits. Each pair is coupled tightly enough to give two resonant peaks about 1 Mc/s apart. Slight damping may also be included to give a reasonably flat-topped response. If five such circuits are tuned to 48, 49, 50, 51 and 52 Mc/s respectively, the overall response curve will be reasonably flat over the bandwidth required, as shown in Fig. 12(b).

R.F. Power Transformers

18. The r.f. transformers considered up to the present have been concerned with the transfer of power at a *low level* over a desired band of frequencies. Such transformers are termed r.f. *voltage* transformers since the main consideration is the voltage developed across the output terminals. R.F. *power* transformers are normally used to transfer r.f. power at a *high level* from one part of a circuit to another. A typical use is in the transfer of power from a transmitter to an aerial system, as shown in Fig. 13. Maximum power is delivered to the aerial system when the impedance of this load is correctly 'matched' to the output impedance of the transmitter. Correct matching is achieved by:—

(*a*) Varying the coupling (Fig. 13(a)).

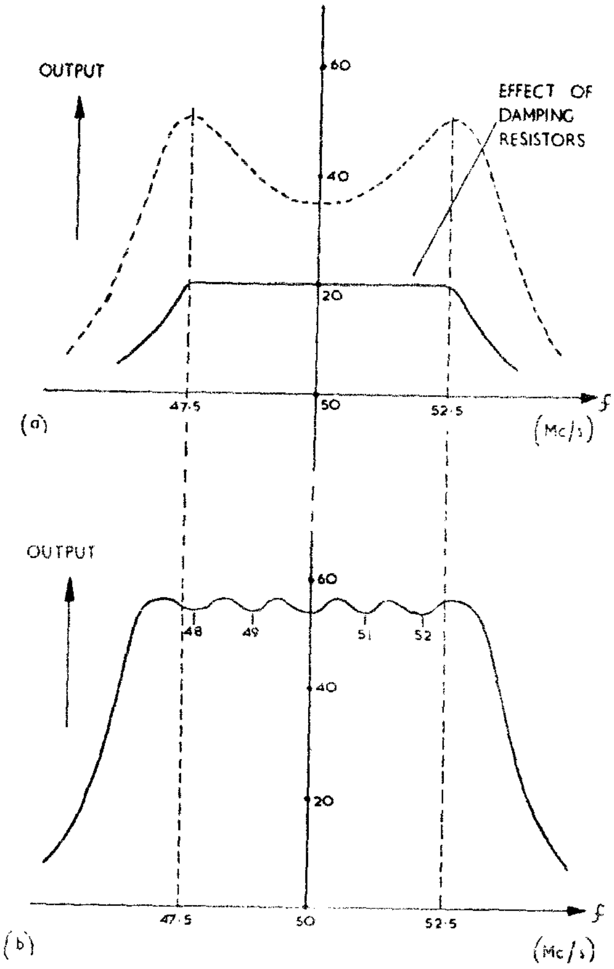


Fig. 12—METHODS FOR INCREASING THE BANDWIDTH

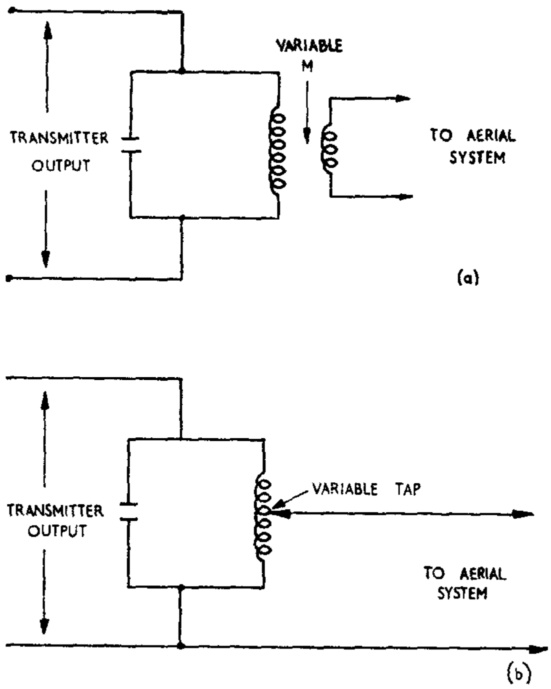
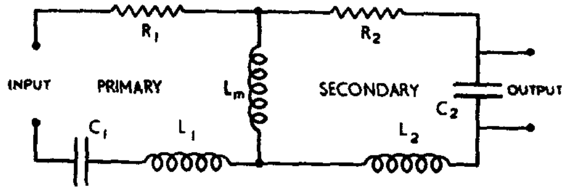

VARIABLE
M
TRANSMITTER
OUTPUT
TO AERIAL
SYSTEM
(a)
VARIABLE TAP
TRANSMITTER
OUTPUT
TO AERIAL
SYSTEM
(b)

Fig. 13—R.F. POWER TRANSFORMERS

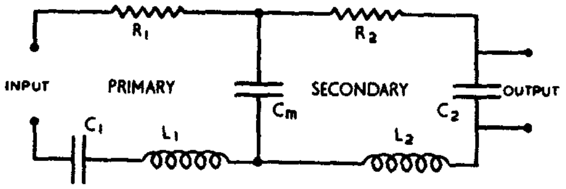
(*b*) Using an auto-transformer as in Fig. 13(b). (See Chap. 2 on auto-transformers).

Other Forms of Coupling

19. Energy can be transferred from one circuit to another by a variety of coupling methods in addition to the mutual inductive coupling method just considered. One method is to use an impedance which provides a current path *common* to both circuits (Fig. 14). The common impedance is *in parallel* with the secondary and the voltage developed across this impedance by the primary current acts as the applied e.m.f. to the secondary. Another form of coupling which is often encountered is 'top-end capacitive coupling' (Fig. 15). The two circuits are joined by a coupling capacitance that is *in series* with the secondary across the primary. A portion of the primary voltage depending on the value of the coupling capacitor, is then developed across the secondary.


R₁
R₂
INPUT
PRIMARY
Lm
SECONDARY
OUTPUT
C₁
L₁
C₂
L₂

(a) COMMON INDUCTIVE COUPLING


R₁
R₂
INPUT
PRIMARY
SECONDARY
OUTPUT
Cm
C₁
L₁
C₂
L₂

(b) COMMON CAPACITIVE COUPLING

Fig. 14—COMMON IMPEDANCE COUPLING

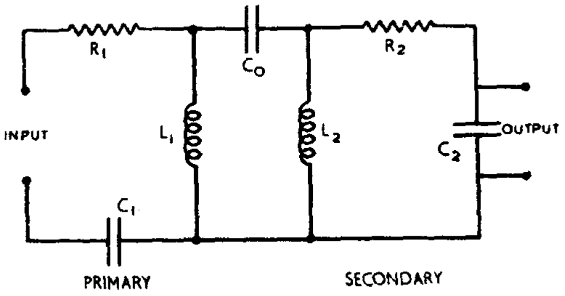

R₁
C₀
R₂
INPUT
L₁
L₂
OUTPUT
C₂
C₁
PRIMARY
SECONDARY

Fig. 15—TOP-END CAPACITIVE COUPLING

20. In top-end capacitive coupling, the primary and secondary inductance values can be varied by iron dust cores to alter the resonant frequency of the coupled circuits, and the two coils are normally placed at right angles to each other to avoid mutual inductive coupling (Fig. 16). The main practical difference between inductive and capacitive coupling is that in the latter the coupling is almost wholly capacitive and

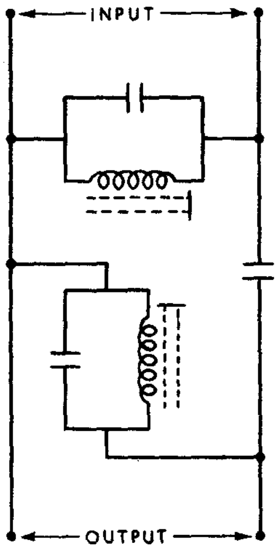

INPUT
OUTPUT

Fig. 16—PRACTICAL ARRANGEMENT OF TOP-END CAPACITIVELY COUPLED CIRCUITS

can be adjusted within fine limits, whereas in the former, unless an electric screen is provided between primary and secondary, both inductive and capacitive coupling are present and adjustment is difficult.

21. Other forms of coupling employ a combination of the methods described, but the behaviour of all coupled circuits follows the same general character as that discussed for mutual inductive coupling.

Screening

22. It is normal to confine the electric and magnetic fields produced by a r.f. transformer by means of a screening can, in order to prevent mutual interference between such fields at other parts of the circuit. The screening can, which is normally fixed to the chassis, gives combined electric and magnetic shielding. The electric lines of force terminate on the earthed can so that no electric field from the r.f. transformer exists outside the can. At the same time, eddy currents are induced in the can by the alternating magnetic field. These eddy currents produce a subsidiary magnetic field which is in such a direction as to neutralize the main field outside the can, giving magnetic screening. The dimensions of the screening can must be such as to alter the constants of the r.f. transformer as little as possible.

SECTION 7

CHAPTER 2

IRON-CORED TRANSFORMERS

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PART 1, SECTION 7, CHAPTER 2

IRON-CORED TRANSFORMERS

Introduction

1. A transformer consists essentially of an arrangement in which two coils are magnetically coupled to one another. If a *varying* current is passed through one coil, known as the primary, the changing magnetic flux linking with the second coil, called the secondary, induces a voltage therein. Thus, the mutually coupled circuits considered in Chap. 1 are transformers. At power and audio frequencies *iron cores* are generally used and the iron-cored transformer is then a special case of a mutually coupled circuit. The principles of iron-cored and air-cored transformers are the same, but it is convenient to treat them in a different manner for reasons which will become obvious in later paragraphs.

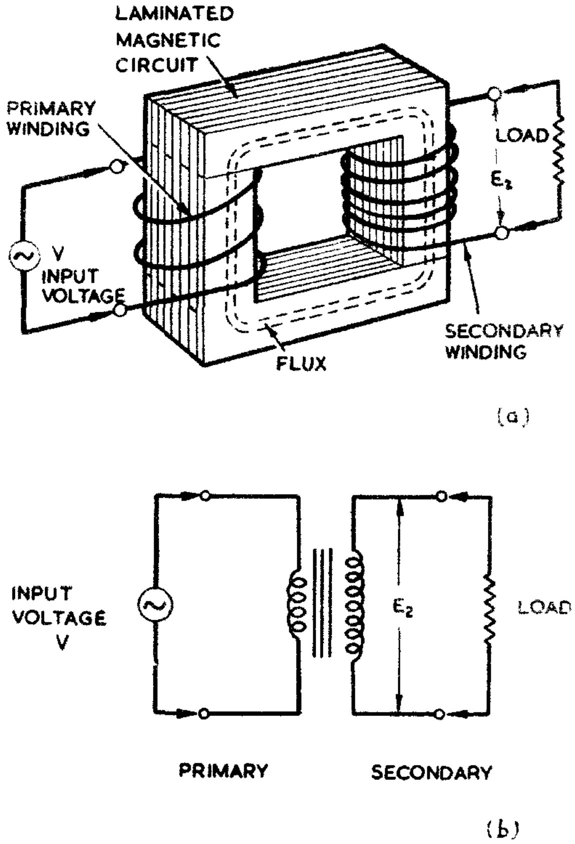


Fig. 1—IRON-CORED TRANSFORMER

2. An iron-cored transformer consists of two insulated coils wound separately over a closed magnetic circuit of low reluctance (Fig. 1). An alternating current generator is connected across the primary and the load is connected to the secondary. The alternating voltage applied to the primary establishes a current in this winding and an alternating magnetic flux is set up in the core. Most of the flux links with the secondary winding to induce a voltage in the secondary and this is available at the secondary terminals. If these terminals are closed by a load circuit, a secondary current is established and energy is expended in the load. The power transmitted to the load in the secondary must in the first place, *be drawn from the generator* by the primary circuit.

3. The effects of using iron are considerable:—

(*a*) The reluctance of the magnetic circuit is low and its permeability high, so that the magnetic flux density and the inductance of each coil are greatly increased.

(*b*) Nearly all the flux from the primary links with the secondary because of the low reluctance of the iron path in relation to that of air. Thus, the coupling coefficient k is very nearly equal to unity and for practical purposes can be considered so. Therefore, the mutual inductance M is large ($M = k\sqrt{L_1 L_2}$).

(*c*) Hysteresis and eddy current losses are introduced.

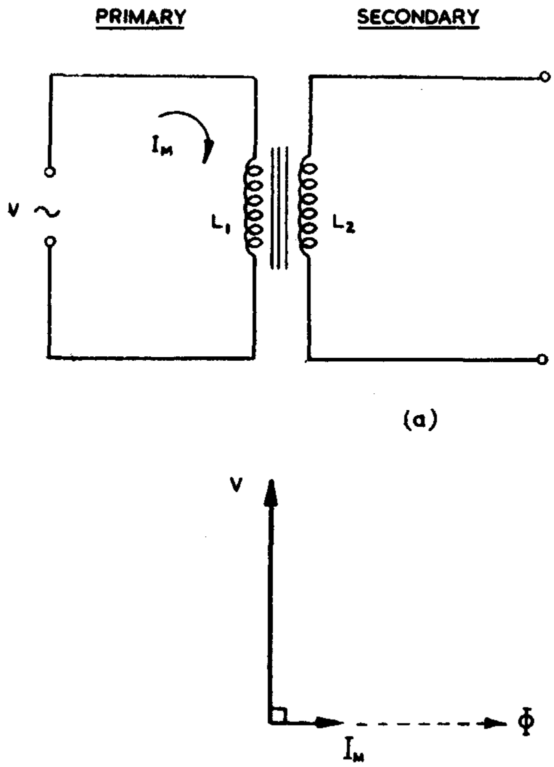
Magnetising Current

4. Fig. 2(a) shows the circuit of an iron-cored transformer in which the primary circuit resistance is so small as to be neglected, the primary inductance is L_1 and the secondary is open-circuited (off load). The primary circuit is not affected by the presence of the secondary since no current can be established in the secondary on open circuit. Thus, the current in the primary is dependent only on the applied voltage V and on the primary impedance. This current is termed the *magnetising current* I_M and is given by:—

$$I_M = \frac{V}{X_{L1}}$$

$$\therefore I_M = \frac{V}{\omega L_1} \quad \ldots \quad \ldots \quad (1)$$

The primary impedance is here considered to be a pure inductance so that I_M *lags* V by 90°. The flux Φ is proportional to the magnetising current and is *in phase* with it, so that Φ lags V by 90°. The phase relationship between V, I_M and Φ is as shown in Fig. 2(b).



(a)

(b)

Fig. 2—MAGNETISING CURRENT

5. The primary inductance of a transformer is normally high because of the presence of the iron core. The magnetising current is, therefore, normally small. If however, the frequency of the applied voltage is *lower* than that for which the transformer is designed, the magnetising current $I_M = \frac{V}{\omega L_1}$ may become excessive and may cause damage to the primary winding. A transformer can be used at a frequency *higher* than that for which it is designed without any such danger.

Transformation Ratio

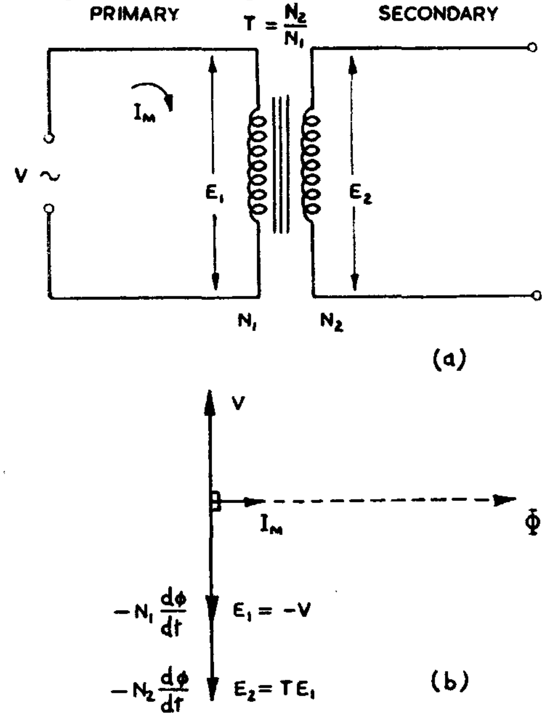
6. Fig. 3(a) shows a transformer with N_1 turns on the primary winding and N_2 on the secondary. The transformer is off load. When an alternating voltage V is applied to the primary a magnetising current I_M is established. The flux resulting from this current is changing at the rate $\frac{d\phi}{dt}$ webers per second, and since this flux is the same for primary and secondary windings (assuming k = 1), the primary induced voltage is,

$$E_1 = -N_1 \frac{d\phi}{dt} \text{ (volts),}$$

and the secondary induced voltage is,

$$E_2 = -N_2 \frac{d\phi}{dt} \text{ (volts).}$$

The two voltages E_1 and E_2 must be *in phase* or 180° out of phase depending on the winding directions since the same flux is cutting both windings.



(a)

(b)

Fig. 3—TRANSFORMATION RATIO

The *magnitude* of the primary and secondary induced voltages depends, however, on the primary turns N_1 and the secondary turns N_2 respectively. Further, E_1 and E_2 are 180° out of phase with the applied voltage V, since E_1 is the *back e.m.f.* in the primary and is equal and opposite to V. The vector diagram showing these relationships is given in Fig. 3(b). Now:—

$$\frac{E_2}{E_1} = \frac{-N_2 \, d\phi/dt}{-N_1 \, d\phi/dt}$$

$$\therefore \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

This relationship gives the transformation ratio T:—

$$T = \frac{N_2}{N_1} = \frac{E_2}{E_1} \quad .. \quad .. \quad (2)$$

7. The derivation of the name 'transformer' will now be clear; by choosing a suitable ratio for $\frac{N_2}{N_1}$ an alternating voltage can be transformed to any other required voltage of the same frequency. The transformation can be either *step-up* or *step-down*. A step-up transformer is one where T is greater than 1; a step-down transformer has T less than 1. A step-up ratio of 2 : 1 means that the secondary winding has twice as many turns as the primary and T = 2. A step-down ratio of 2 : 1 means that the secondary winding has half the turns of the primary and T = ½. An r.m.s. voltage of 230 V applied to each transformer in turn would give outputs of 460 V and 115 V r.m.s. respectively.

Transformer With More Than Two Windings

8. A transformer with three secondary windings is illustrated in Fig. 4. The same theory applies as for transformers with one primary and one secondary winding. Thus, the secondary voltages are proportional to the transformation ratio between the respective secondary winding and the primary. Hence:—

$$E_2 = T_2\,V = \frac{N_2}{N_1}\,.\,V$$

$$E_3 = T_3\,V = \frac{N_3}{N_1}\,.\,V$$

$$E_4 = T_4\,V = \frac{N_4}{N_1}\,.\,V$$

A typical transformer of this type has $V = 230$ V, $E_2 = 400$ V, $E_3 = 200$ V, and $E_4 = 6$ V (all r.m.s. values).

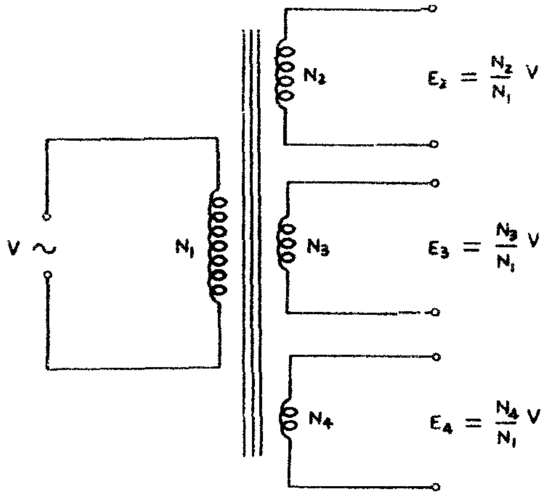

V ~
N_1
N_2
$E_2 = \frac{N_2}{N_1} V$
N_3
$E_3 = \frac{N_3}{N_1} V$
N_4
$E_4 = \frac{N_4}{N_1} V$

Fig. 4—TRANSFORMER WITH THREE SECONDARY WINDINGS

Primary No-load Current

9. So far, the only current considered has been the magnetising current I_M. This lags 90° on the applied voltage when the primary circuit is assumed to be a pure inductance.

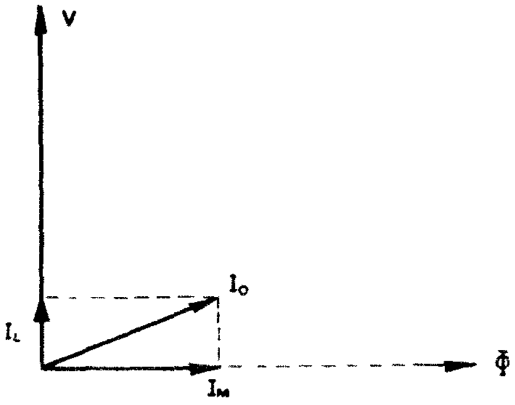


Fig. 5—PRIMARY NO-LOAD CURRENT

However, even when there is no secondary load current, power is being drawn from the supply because of hysteresis and eddy current losses in the iron core. The total primary no-load current is therefore made up of two components:-

(*a*) A magnetising current I_M which establishes magnetic flux in the core and which *lags* 90° on V.

(*b*) A loss component I_L representing hysteresis, eddy current and other power losses, which is *in phase* with V.

The primary no-load current I_0 is then the vector resultant of I_M and I_L and lags in phase by *less* than 90° on V. Fig. 5 shows the vector diagram.

Transformer on Load

10. If the applied voltage V is constant, then the flux Φ is constant whatever load is connected to the secondary. Assuming no resistance in the transformer, the applied

voltage must be exactly equal and opposite to the back e.m.f. developed across the primary. Hence, if V is constant the back e.m.f. must be constant. But the back e.m.f. is proportional to the rate of change of flux, so that the flux must be such that its rate of change is a sine wave of constant amplitude. Thus, *the flux itself must be a sine wave of constant amplitude irrespective of the load on the secondary.*

11. When a load is connected to the transformer, a current $I_2 = \frac{E_2}{Z_2}$ is established in the secondary. This current will set up a magnetic flux of its own and since, from Para. 10, the magnetising flux must remain *constant* if the applied voltage is constant, some action must take place to nullify the flux due to I_2. In fact, a primary current I_1 flows. If I_1 is to have the opposite effect to I_2 it must be 180° out of phase. Its magnitude is determined by the fact that the two effects are to be equal. Now, the secondary flux Φ_2 is proportional to $I_2 N_2$. The flux produced by I_1 must equal this.

$$\therefore I_1 N_1 = I_2 N_2$$

$$\therefore \frac{I_1}{I_2} = \frac{N_2}{N_1} = T \quad .. \ (3)$$

$$\text{Thus, } T = \frac{E_2}{E_1} = \frac{I_1}{I_2} \quad .. \ (4)$$

From this it is seen that the *secondary* voltage E_2 is T times the primary voltage E_1, and the *primary* current I_1 is T times the secondary current I_2.

12. From equation (4) :—

$$\frac{E_2}{E_1} = \frac{I_1}{I_2}$$

$$\therefore E_2 I_2 = E_1 I_1$$

$$\therefore \text{Output power} = \text{Input power} \quad .. \ (5)$$

The perfect transformer introduces no loss and the efficiency is 100 per cent. In practice efficiencies of 99 per cent can be obtained.

Types of Load

13. **Pure resistance.** Fig. 6(a) shows a transformer with a purely resistive load connected to the secondary. The vector diagram is given in Fig. 6(b), and is constructed as follows:—

(*a*) The flux Φ is the reference vector since the flux is common to primary and secondary. The magnetising current I_M which produces this flux is in phase with Φ and lags 90° on the applied voltage V.

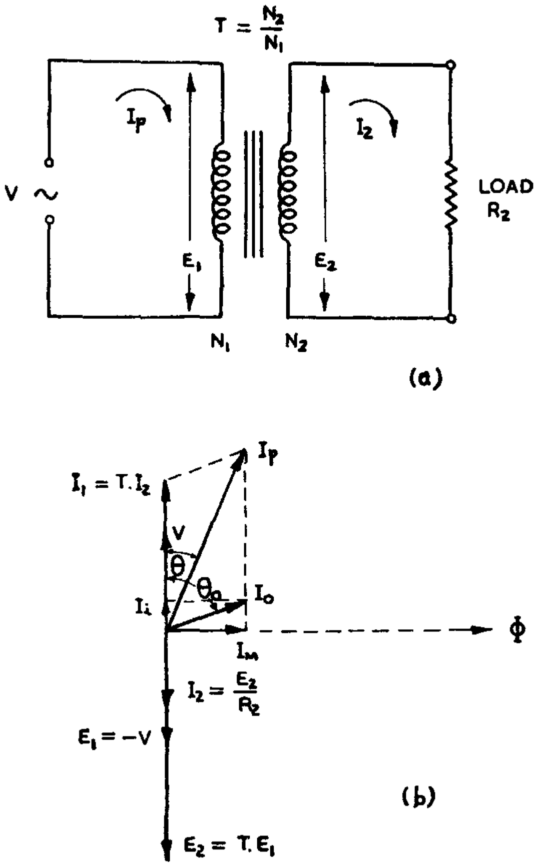


Fig. 6—TRANSFORMER WITH A RESISTIVE LOAD

(*b*) A loss current I_i is established in phase with V, and the total no-load current I_0 is the vector resultant of I_M and I_i as shown.

(*c*) The changing flux induces a back e.m.f. $E_1 = -V$ in the primary and an e.m.f. E_2 in the secondary. E_1 and E_2 are 180° out of phase with V, and $E_2 = T E_1 = -T V$.

(*d*) A secondary load current $I_2 = \frac{E_2}{R_2}$ is established *in phase* with E_2, since the load is a pure resistance.

(*e*) A primary current I_1 of magnitude $T I_2$ is established to produce a flux equal and opposite to that produced by I_2. I_1 is 180° out of phase with I_2.

(*f*) The total primary load current I_P is then the vector resultant of the no-load current I_0 and the current I_1. I_P is seen to lag V by an angle θ which is less than the original angle θ. Thus, the phase difference between primary current and applied voltage is altered by the secondary load.

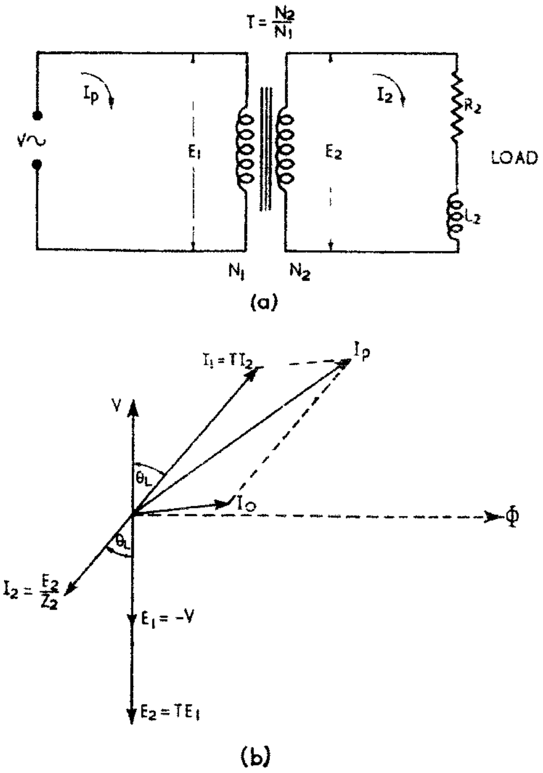


Fig. 7—TRANSFORMER WITH A RESISTIVE-INDUCTIVE LOAD

14. **Reactive load.** Fig. 7(a) shows a transformer with a resistive-inductive load connected to the secondary. The vector diagram is shown in Fig. 7(b) and is constructed in a manner similar to that given in Para. 13. However, the secondary current I_2 now *lags* E_2 by an angle θ_L and since I_1 is 180° out of phase with I_2, then I_1 lags V by the same angle θ_L. The resultant primary load current I_p is the vector resultant of I_o and I_1.

Impedance Transformation

15. Any impedance connected across the secondary will reflect an equivalent shunt impedance across the primary.

From equation (4):—

$$T = \frac{E_2}{E_1} = \frac{I_1}{I_2}$$

$$\therefore T^2 = \frac{E_2}{E_1} \times \frac{I_1}{I_2}$$

But $E_2 = I_2 Z_L$,

where Z_L = load impedance

And $E_1 = I_1 Z_p$,

where Z_p = effective primary impedance.

$$\therefore T^2 = \frac{I_2 Z_L}{I_1 Z_p} \times \frac{I_1}{I_2}$$

$$= \frac{Z_L}{Z_p}$$

$$\therefore Z_p = \frac{Z_L}{T^2} \quad \ldots \quad \ldots \quad (6)$$

Thus, a secondary load resistance R_L is equivalent to a resistance $\frac{R_L}{T^2}$ shunting the primary winding and a secondary load reactance X_L is equivalent to a reactance $\frac{X_L}{T^2}$ shunting the primary. A transformer does not alter the *angle* of an equivalent primary impedance reflected from the secondary, so that an inductive reactance in the secondary appears as an equivalent inductive reactance in the primary. The equivalent circuit for Fig. 8(a) is illustrated in Fig. 8(b). The internal impedance of the supply is Z_g and the impedance of the primary winding is assumed to be so *large* in relation to the reflected impedance shunting it that it is neglected.

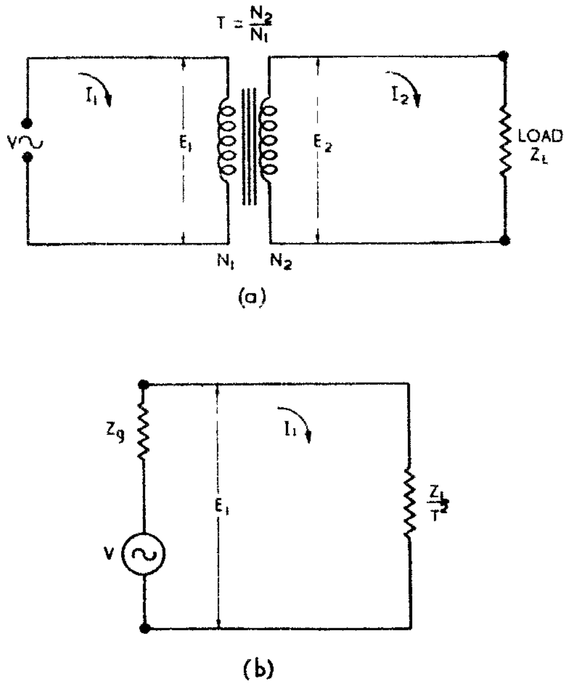


Fig. 8—REFLECTED IMPEDANCE

16. The maximum power transfer theorem states that the power taken by a load from a generator is greatest when the impedances of

the generator and the load are equal. In a.c. circuits where the two impedances are not equal, a transformer of suitable transformation ratio can be interposed between the generator and the load. The value of T must be such that the equivalent primary impedance reflected from the secondary load equals the internal impedance of the generator. Maximum transfer of power to the load is then obtained. This process is known as '*impedance matching*'. Fig. 9 illustrates a typical example. The load and generator are shown in Fig. 9(a). If direct connection were made to the load, the power developed in the 4 ohms resistor would be negligible. A transformer is, therefore, interposed. If the impedances are to match:—

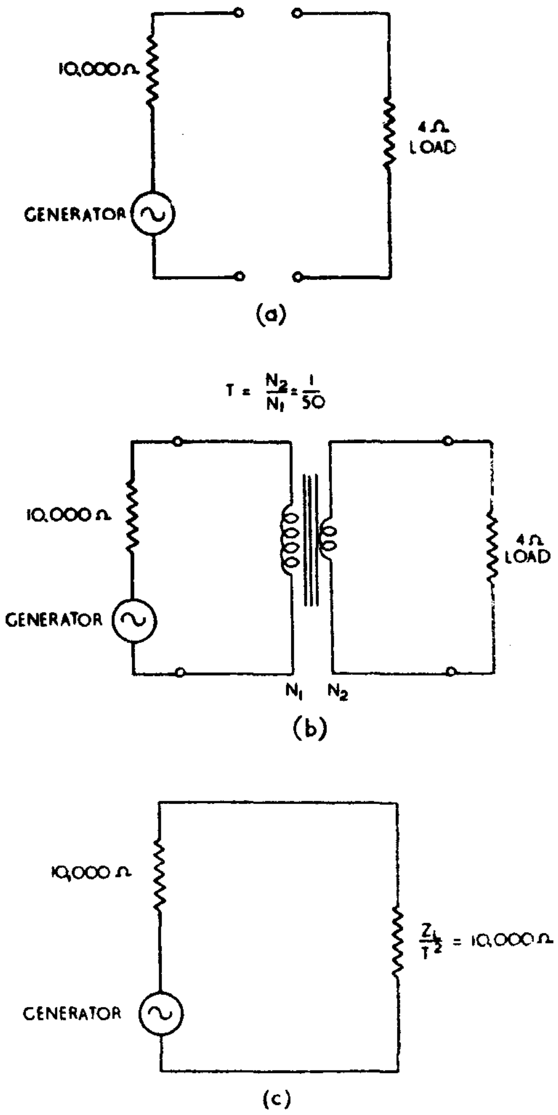


Fig. 9—IMPEDANCE MATCHING

$$T^2 = \frac{Z_L}{Z_P} = \frac{4}{10{,}000} = \frac{1}{2{,}500}$$

$$\therefore T = \frac{1}{50}$$

Thus a transformer having a *step-down* ratio of 50 : 1 will transform the 4 ohms in the secondary to 10,000 ohms in the primary, and maximum power will then be transferred to the load. The circuit arrangement is given in Fig. 9(b), and the equivalent circuit as seen from the generator in Fig. 9(c)

Transformer Losses

17. The various losses associated with a transformer are summarized below:—

(*a*) **Iron losses.**

(i) *Magnetising current.* In an 'ideal' transformer the primary inductance will offer an infinite impedance and no magnetising current will flow. In practice, however, a magnetising current *does* flow since the primary impedance is not infinite.

(ii) *Eddy current loss.* Resistive (heating) losses are caused by eddy currents circulating in the core of the transformer.

(iii) *Hysteresis loss.* Resistive (heating) losses occur in taking the core through its magnetisation cycle.

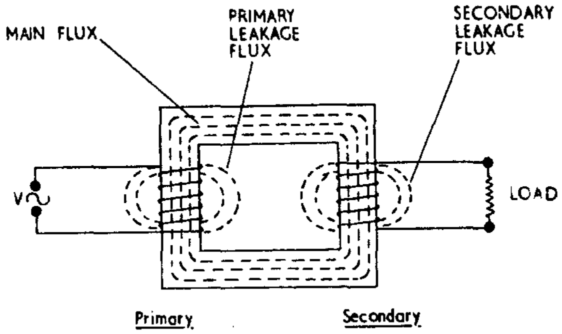


Fig. 10—LEAKAGE FLUX

(*b*) **Copper losses.** These are resistive (heating) losses due to the resistance of the windings, including the losses due to skin effect.

(*c*) **Flux leakage losses.** All the primary flux will not link with the secondary, and *vice versa.* This 'leakage flux' is shown in Fig. 10. The induced voltages are therefore smaller than those indicated by a coupling factor of unity.

(*d*) **Self-capacitance of windings.** Capacitive losses occur because of the capacitance between the turns in the windings. Such losses are important at audio frequencies.

18. Because of the factors above, the efficiency of a transformer, given by $\frac{\text{Output power}}{\text{Input power}} \times 100$ (per cent), is always less than 100%. For small transformers it is around 80 to 90 per cent, rising to 95 per cent for medium transformers and 99 per cent for large transformers.

Construction of Transformers

19. A transformer is constructed in such a way that the losses are kept to a minimum.

(*a*) *Magnetising current.* This is kept to a low value by using, where possible, a primary winding which has a high inductance and a low resistance.

(*b*) *Eddy current loss.* The core is built up of suitably shaped thin laminations, averaging about 0·012 inches thick, of silicon iron or other similar material. The surfaces of the laminations are oxidised to reduce eddy current losses to a minimum.

(*c*) *Hysteresis loss.* This is reduced by using a core material which has a low hysteresis loss, e.g., silicon iron, mumetal, or permalloy.

(*d*) *Copper losses.* The use of heavy gauge copper wire for the windings will reduce these losses.

(*e*) *Flux leakage losses.* The magnetic circuit is so constructed that it has a low value of reluctance, the laminations being arranged in such a way that air gaps are kept to a minimum.

(*f*) *Self-capacitance of windings.* Due attention is paid to the method of winding the turns.

20. Two forms of transformer magnetic circuit in common use are illustrated in Figs. 11 and 12. These are:—

(*a*) *Core type,* shown in Fig. 11 (a), in which parts of both primary and secondary windings are wound on opposite limbs to give a good flux linkage between the windings. The laminations are either L shaped or I and U shaped as shown in Fig. 11(b), and they are arranged in such a manner that all joints are staggered to give a low value of flux leakage (Fig. 11(c)). This type of magnetic circuit is used mainly for high voltage transformation at low power levels.

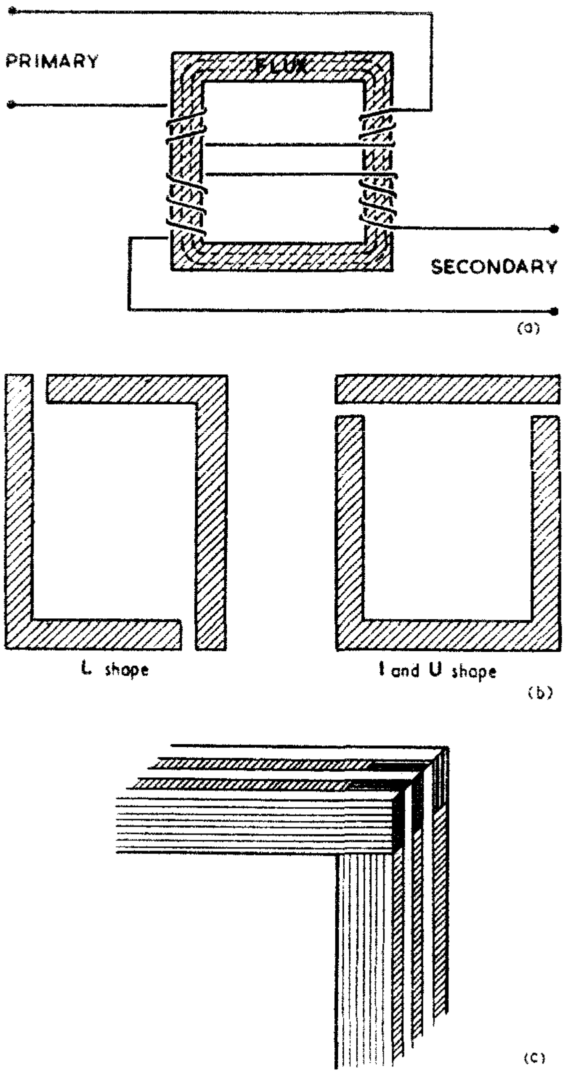


Fig. 11—CORE TYPE OF TRANSFORMER MAGNETIC CIRCUIT

(*b*) *Shell type,* shown in Fig. 12(a), in which all the windings are placed on the centre limb, the two outer limbs completing the magnetic circuit. To provide a constant value of reluctance at all points in the magnetic circuit, the cross-sectional area of the centre limb is made *twice* that of the remainder of the circuit. The

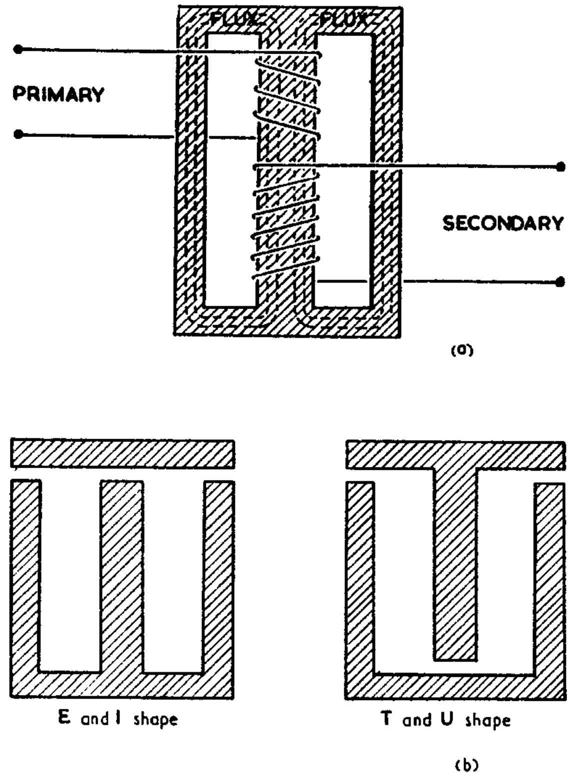


Fig. 12—SHELL TYPE OF TRANSFORMER MAGNETIC CIRCUIT

laminations are either E and I shaped, or T and U shaped as shown in Fig. 12(b), and the joints are staggered to give a low value of flux leakage. This type of magnetic circuit is in more common use and can be used at higher power levels than the core type.

21. The windings are usually of insulated copper wire wound on the limbs or limb of the core. The normal form of insulation for the wire is enamel, and the layers are interleaved with paper. Some thicker insulation is provided between the separate windings, and between the core and the first layer. In some transformers there are several secondary windings providing different output voltages, and the connecting leads to the separate windings are brought out to metal tags mounted on an insulated base. The circuit connections are made to these tags. Two methods of winding are in common use:—

(*a*) *Cylindrical.* The windings consist of concentric coils wound one inside the other and suitably insulated from each other as shown in Fig. 13(a).

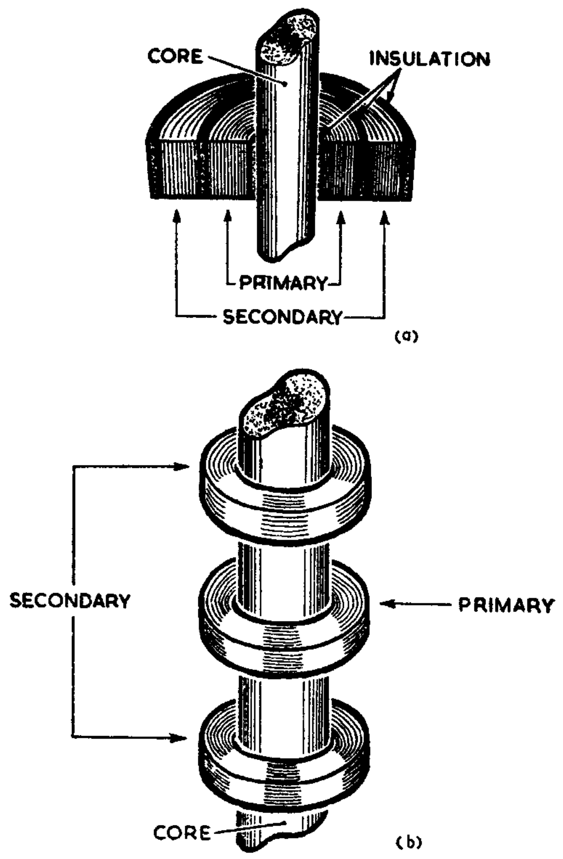


Fig. 13—METHODS OF WINDING

(*b*) *Sandwich.* These windings are separate ring coils assembled one on top of the other on the core as shown in Fig. 13(b).

22. Where self-capacitance between turns in a winding must be reduced to a minimum (e.g., in a.f. transformers) it is usual to use 'sectionalised windings' and also to insert an electric screen of copper foil between the windings. A sectionalised winding is one where the primary and the secondary windings are wound in small series-connected sections, similar to that of a pie-wound inductor, thereby reducing the self-capacitance. The sectionalised winding can be of cylindrical form (Fig. 14(a)) or sandwich form (Fig. 14(b)).

Power Transformers

23. One of the more important purposes of a transformer is the transmission of power. The transformer receives its supply from the a.c. mains or other source and transfers the power to the load at either a higher or a lower voltage level, depending on whether

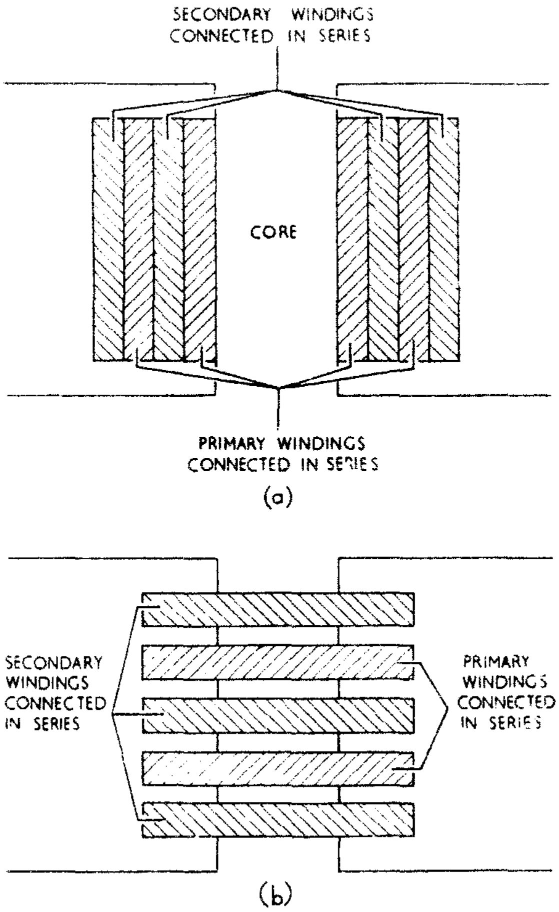

SECONDARY WINDINGS CONNECTED IN SERIES
CORE
PRIMARY WINDINGS CONNECTED IN SERIES
(a)
SECONDARY WINDINGS CONNECTED IN SERIES
PRIMARY WINDINGS CONNECTED IN SERIES
(b)

Fig. 14—SECTIONALISED WINDINGS

the transformer has a step-up or a step-down ratio. The input is normally 230V at 50 c/s and the primary winding has several tapping points in order to adjust the transformation ratio for any variation in input voltage (Fig. 15). Several secondary windings are usual. In certain installations, the frequency of the supply is higher than 50 c/s, being of the order of 400 c/s to 2,400 c/s. This simplifies design of the transformer and its associated circuits since the increased rate of change of flux will give higher induced voltages for the same number of turns. The number of turns in all the windings, and hence the size of the transformer, can therefore be considerably reduced.

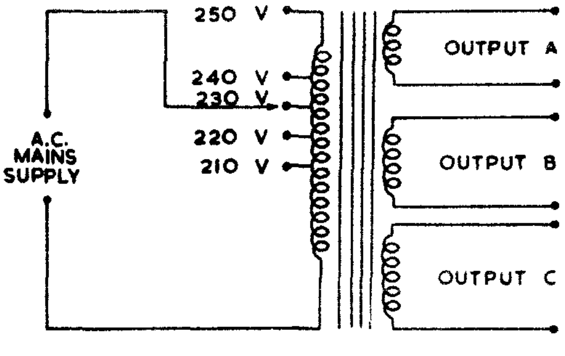

250 V
240 V
230 V
220 V
210 V
A.C. MAINS SUPPLY
OUTPUT A
OUTPUT B
OUTPUT C

Fig. 15—MAINS TRANSFORMER WINDINGS

24. Although the percentage loss of power in transformers is very small, the amount wasted in the form of heat may be quite large when transforming large powers. It is then necessary to limit the temperature rise by cooling. In small transformers, natural air circulation is sufficient to cool by conduction and convection the heat generated in the core and the windings. A power transformer of this type is shown in Fig. 16. In larger types, oil cooling is general and the transformer is immersed in a tank filled with insulating oil. The oil is carried by convection currents to the surface where it radiates its heat to the

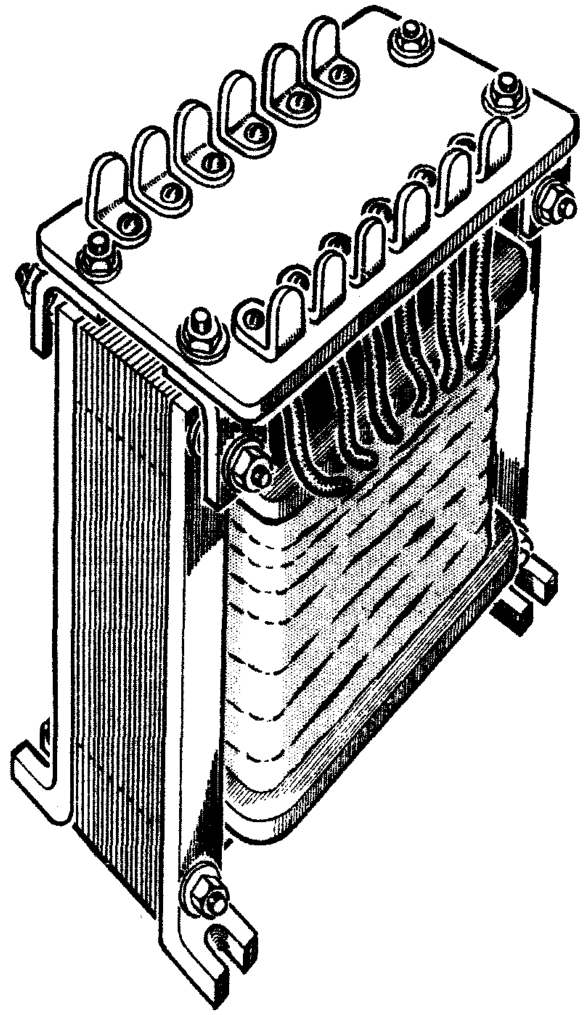


Fig. 16—AIR-COOLED POWER TRANSFORMER

atmosphere. The oil, apart from being a good cooling medium, assists in maintaining the insulation of the windings.

Audio Frequency (a.f.) Transformers

25. These transformers are used in radio equipments to provide voltage transformation and impedance matching. They must operate satisfactorily without undue distortion, over a fairly wide frequency range in the band 20 c/s to 20 kc/s.

26. **Distortion of waveform.** In a transformer, if the magnetising current I_M is sinusoidal so also is the magnetising force H. The flux Φ resulting from the magnetising force can be determined from a hysteresis loop for the core material. Fig. 17 shows the result for a typical material. The flux is seen to be distorted and lagging in phase on the magnetising force H because of the shape of the hysteresis loop. The curve for the rate of change of flux $\frac{d\varnothing}{dt}$ leads Φ by 90° and is considerably distorted because of the flattening at the peaks of the Φ curve. The secondary output voltage is proportional to $\frac{d\varnothing}{dt}$ and is similarly distorted, a pronounced third harmonic component being in fact produced. If the material is allowed to reach saturation the distortion is even more pronounced, and for this reason *air-gaps* are often included in transformer magnetic circuits. To reduce distortion, a material with a narrow hysteresis loop and a high saturation level is selected for the core material.

27. **Air-gaps.** Air-gaps in magnetic circuits are a virtual necessity for a.f. transformers in which the primary winding is carrying

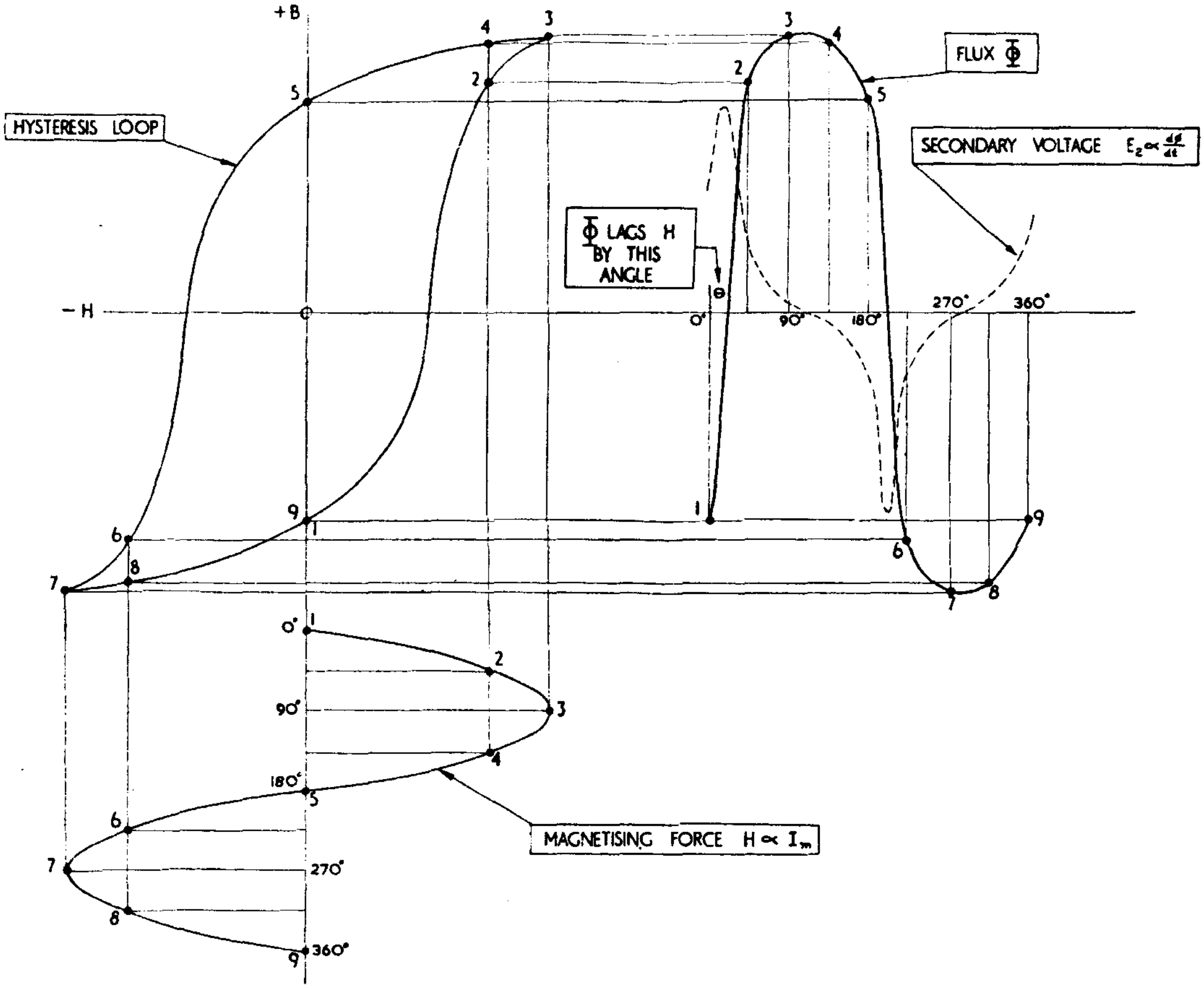
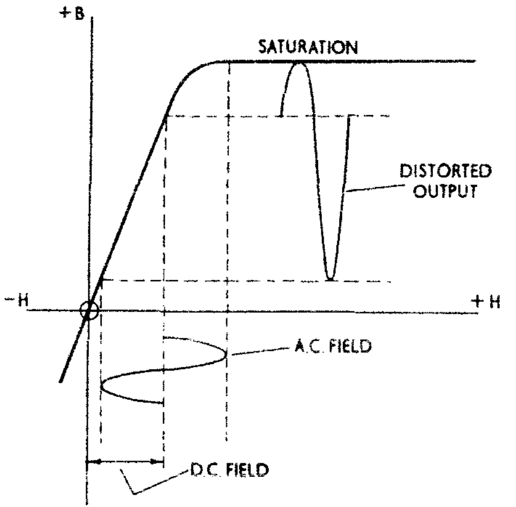
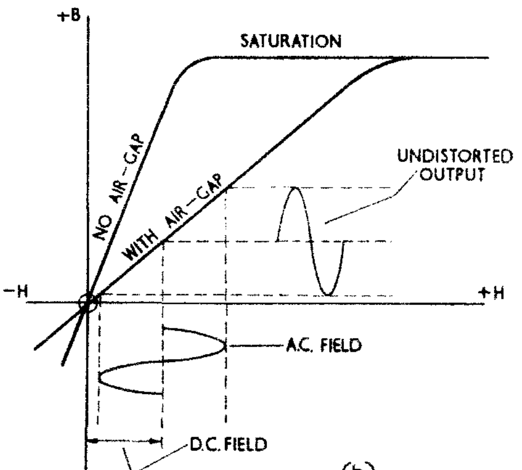

+B
HYSTERESIS LOOP
FLUX Φ
SECONDARY VOLTAGE $E_2 \propto \frac{d\Phi}{dt}$
Φ LAGS H BY THIS ANGLE
θ
−H
0°
90°
180°
270°
360°
MAGNETISING FORCE $H \propto I_m$
1
2
3
4
5
6
7
8
9

Fig. 17—DISTORTION OF WAVEFORM BY HYSTERESIS



(a) NO AIR-GAP



(b) WITH AIR-GAP

Fig. 18—EFFECT OF AIR-GAPS

d.c. as well as a.c. components. Unless this is done, severe distortion of the output waveform will result. Fig. 18(a) shows the effect of super-imposing an a.c. field on a d.c. field for a transformer which has no air-gap. With air-gaps in the magnetic circuit the slope of the hysteresis curve is reduced and the distortion reduced as shown in Fig. 18(b).

28. **Self-capacitance.** This is reduced in a.f. transformers by using sectionalised windings and by the insertion of an electric screen between the windings. This screen is earthed so that electric lines of force terminate on the screen, but it is so constructed that it has no effect on magnetic lines of force.

29. **Types of a.f. transformer.** Three types of a.f. transformer are common:—

(*a*) *Amplifier input transformers.* These are inserted in the input circuit of an amplifier for matching purposes and to step up the input voltage to a satisfactory level. The primary impedance must be high, and this is obtained by using a high permeability mumetal core and several thousand turns. Mumetal can be used in this case since the primary winding carries no direct current which would otherwise saturate the core and cause distortion. A step-up ratio is usual up to a limit of about 8 : 1.

(*b*) *Intervalve transformers.* These are inserted between two stages in an amplifier. Again the primary inductance must be large, but since the primary winding carries direct current in addition to the audio frequency current, a mumetal core cannot be used. To prevent magnetic saturation and resultant distortion, a silicon iron core *with an air-gap* is used. Because of the large number of primary turns, the step-up ratio is limited to about 4 : 1. A typical intervalve a.f. transformer is shown in Fig. 19.

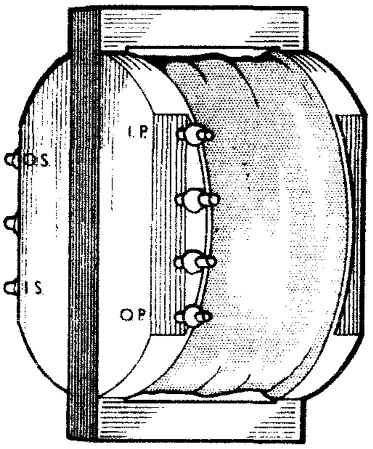


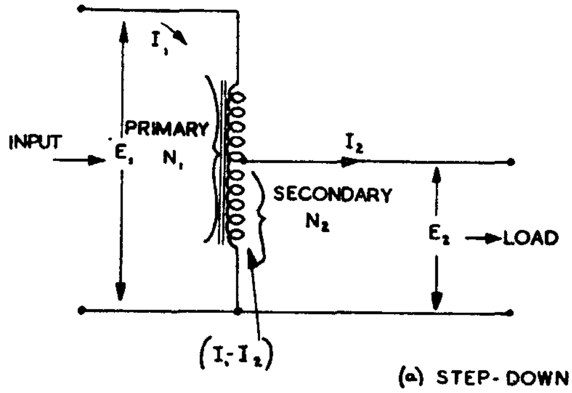
Fig. 19—TYPICAL A.F. TRANSFORMER

(*c*) *Amplifier output transformers.* These transformers transfer power from the amplifier to the load. They usually carry direct current in the primary so that a silicon iron core with an air gap is usual. The transformation ratio is usually *step-down* and depends on the impedance matching required.

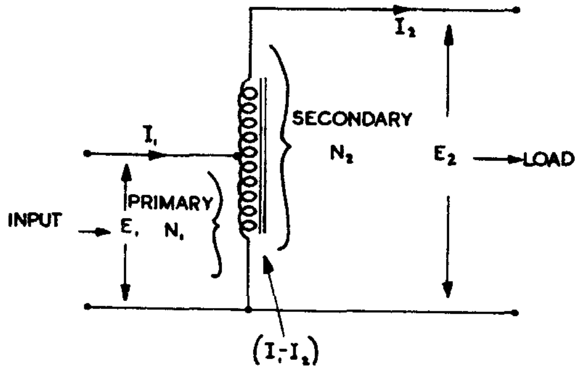
Auto-transformers

30. In the auto-transformer the primary is part of the secondary, or *vice versa* (Fig. 20).

On applying a voltage E_1 to the primary winding of N_1 turns, a current I_1 is established. The flux resulting from this current induces a voltage E_2 in the secondary winding of N_2 turns so that a current I_2 is established in the load. The auto-transformer can be either step-down as shown in Fig. 20(a), or step-up as shown in Fig. 20(b), depending on whether N_2 is less or greater than N_1. Thus, as with a normal transformer:—



(a) STEP-DOWN



(a) STEP-UP

Fig. 20—AUTO-TRANSFORMERS

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = T.$$

31. The main advantage of the auto-transformer is that copper and leakage losses are reduced, for the current in the secondary winding is the *difference* between I_1 and I_2 and this is obviously less than I_2. This advantage is greatest when N_1 and N_2 are nearly equal, for then I_1 and I_2 are also nearly equal, and the accompanying losses are small. Hence, auto-transformers are most useful when a *small* transformation ratio is required.

Variac

32. Fig. 21(a) illustrates the arrangement of a continuously variable auto-transformer known as a 'variac'. It consists of a layer toroidal winding round a ring-shaped core, the moving brush contact A sliding round a bared track on the toroid to alter the transformation ratio as required. In this way, a continuously adjustable output voltage is obtained. The construction is shown in Fig. 21(b).

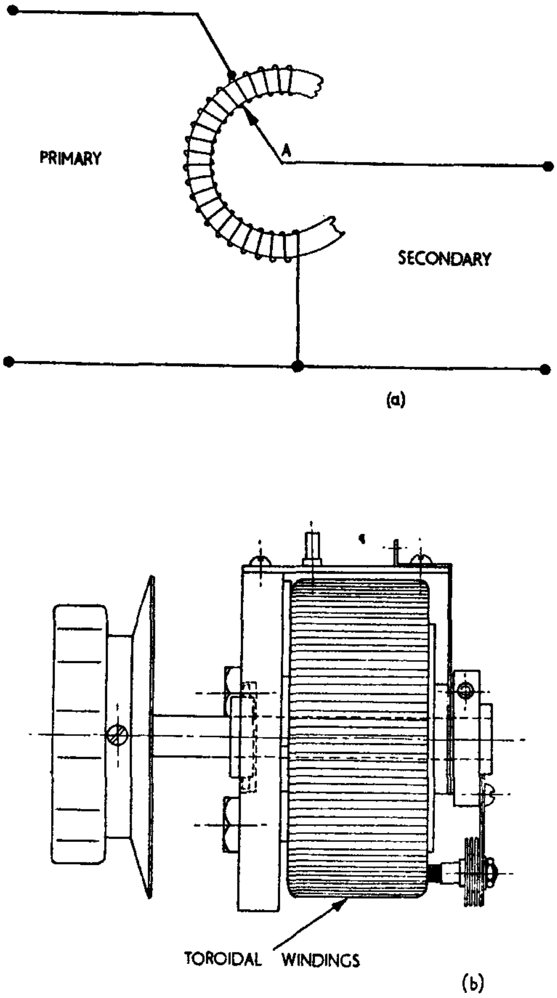


Fig. 21—VARIAC

Three-phase Transformers

33. Transformation of three-phase a.c. at one voltage to three-phase a.c. at another voltage can be effected either by means of three *separate* single-phase transformers, or by a *single* three-phase transformer. In the latter, the shell type iron core of *equal* limb area carries the primary and secondary

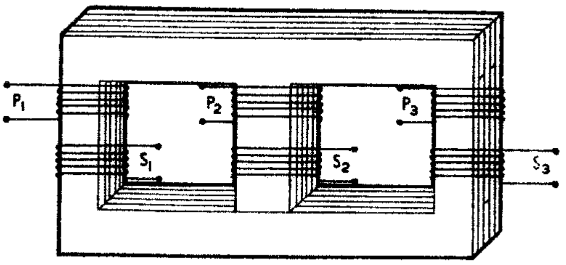


Fig. 22—THREE-PHASE TRANSFORMER

windings of each phase on each of the three limbs as shown in Fig. 22. In effect, each limb is used as a separate transformer for one phase.

34. All the variations possible with three-phase generator and load connections (see Sect. 5, Chap. 4) are available with the primary and secondary windings of a three-phase transformer. For example, the primary and secondary may both be star-connected (Fig 23(a)), in which case the line voltages are connected by the relation:—

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = T.$$

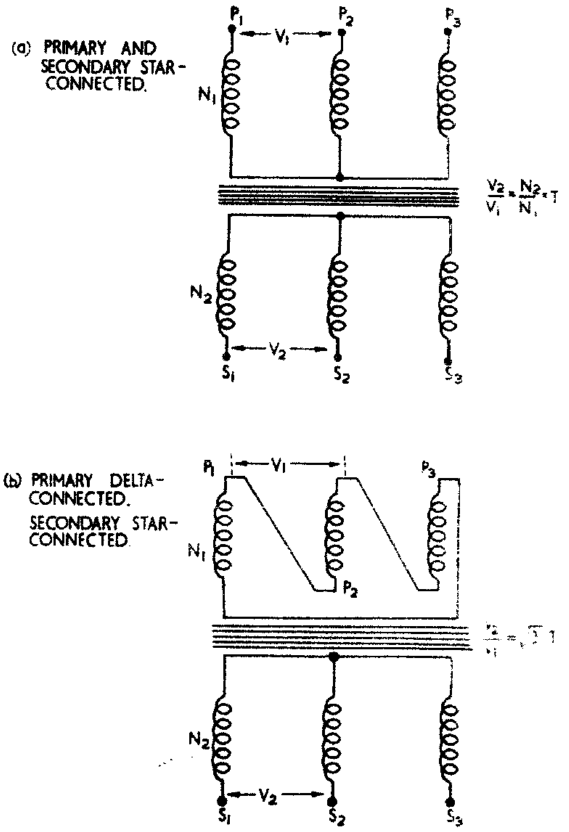

(a) PRIMARY AND SECONDARY STAR-CONNECTED.
P1
P2
P3
V1
N1
V2/V1 = N2/N1 = T
N2
V2
S1
S2
S3
(b) PRIMARY DELTA-CONNECTED. SECONDARY STAR-CONNECTED.
P1
V1
P3
N1
P2
N2
V2
S1
S2
S3

Fig. 23—CONNECTIONS TO A THREE-PHASE TRANSFORMER

Alternatively, the primary may be delta-connected and the secondary star-connected (Fig. 23(b)), when:—

$$\frac{V_2}{V_1} = \sqrt{3}\ T.$$

35. For the operation of certain radio equipments a *two-phase* power supply is required, and it is then necessary to convert from the three-phase mains supply to a two-phase supply. It is possible to do this by a suitable arrangement of transformers. A typical circuit, known as '*the Scott method*' is given in Fig. 24. Two transformers are used, with one winding on one transformer centre-tapped to one winding on the other, to give a star-connected arrangement. By applying the three-phase input to the terminals A, B and C a two-phase output is obtained across the terminals PQ and RS. Conversion from two-phase to three-phase can be obtained by reversing the procedure.

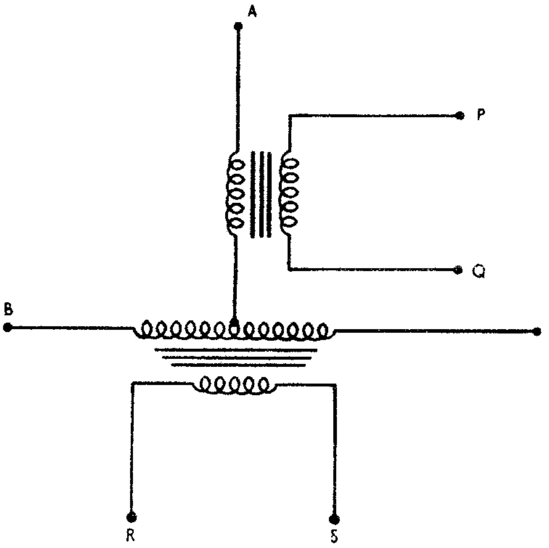


Fig. 24—SCOTT CONNECTIONS

Instrument Transformers

36. When measuring large currents in d.c. circuits it is usual to use shunts to limit the current through the meter to the f.s.d. value (see Sect. 6, Chap. 1). This is not convenient with a.c. instruments and it is, therefore, usual to pass the current to be measured through the primary of a 'current' transformer, the secondary being connected to the ammeter as shown in Fig. 25. The

step-up ratio *reduces* the secondary current in the ratio $\frac{N_2}{N_1}$ and by suitably adjusting this ratio the required f.s.d. current can be obtained. Measurement of high values of alternating voltage is normally obtained by using a *step-down* transformer.

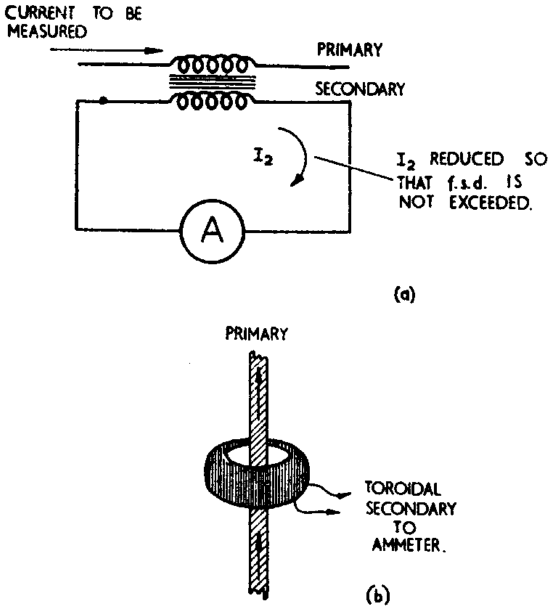

CURRENT TO BE MEASURED
PRIMARY
SECONDARY
I₂
I₂ REDUCED SO THAT f.s.d. IS NOT EXCEEDED.
A
(a)
PRIMARY
TOROIDAL SECONDARY TO AMMETER.
(b)

Fig. 25—'CURRENT' TRANSFORMER

Phase-shifting Transformers

37. Phase-shifting transformers serve the same purpose with respect to phase as do current and voltage transformers with respect to amplitude; they are used to alter the phase but not the amplitude of an alternating current. The phase-shifting transformer is similar in construction to an induction motor with a wound rotor (see Sect. 5, Chap. 4), except that the 'rotor' is set in any desired position but does not thereafter rotate. The stator windings are the primary of the transformer and the rotor is the secondary, the primary windings producing a rotating magnetic field. The primary flux threads the secondary, and whatever the position of the secondary the flux linked with it goes through one complete cycle for each rotation of the field. Since this is the same however the secondary is placed, the *magnitude* of the induced voltage does not depend on the position of the rotor. However, the *phase* of the secondary voltage depends on the position of the rotor with respect to the primary windings. In Fig. 26, if the secondary coil is in position 1 the induced voltage in it is zero when the rotating field is acting in the direction H_1, while if it is in position 2 this happens when the field is in the direction H_2. Rotating the secondary through angle θ thus causes a change of θ in the phase of the secondary voltage.

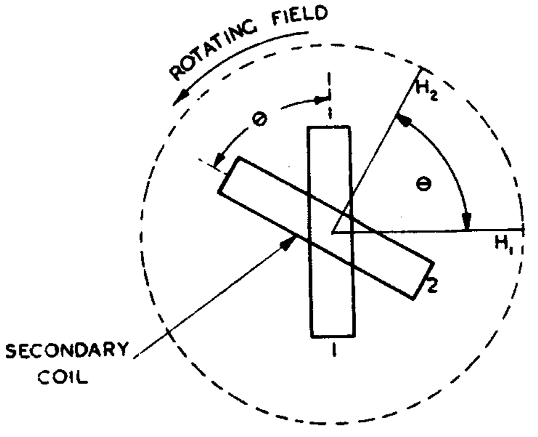


Fig. 26—PHASE SHIFTING TRANSFORMER

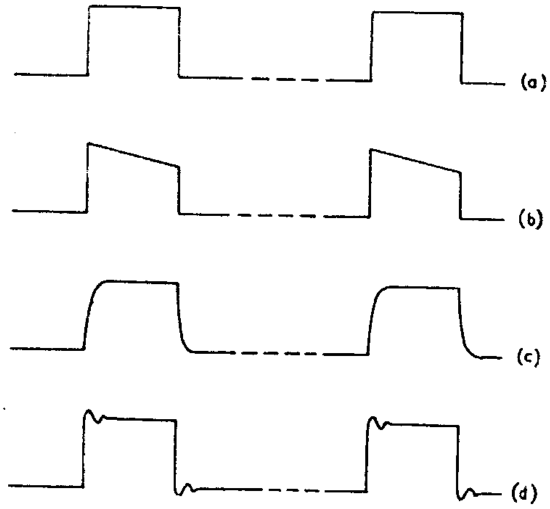


Fig. 27—DISTORTION IN A PULSE TRANSFORMER

Pulse Transformers

38. These are used to handle very short duration pulses of rectangular waveform as shown in Fig. 27(a). They must be so designed that distortion of the waveform by the transformer is kept to a minimum. Distortion may be due to several factors:—

(*a*) A low value of primary inductance will introduce distortion of the type shown in Fig. 27(b). The primary inductance is kept high and the distortion reduced by using a high permeability material for the core and a winding of many turns.

(*b*) Self-capacitance and resistive losses will introduce distortion of the type shown in Fig. 27(c). Self-capacitance is reduced by proper arrangement of the windings. Resistive losses are due mainly to eddy current and hysteresis losses and these are reduced by the use of very thin laminations and a material with a low hysteresis loss.

(*c*) Leakage inductance, together with the self-capacitance of the transformer, will introduce distortion of the type shown in Fig. 27(d). Such distortion is termed 'ringing' and is caused by resonance between the leakage inductance and the self-capacitance. Leakage inductance is reduced by coupling the primary and secondary windings as closely as voltage insulation requirements will permit, and by employing a magnetic core of special high permeability material.

Constant Voltage Transformers

39. This type of transformer is used to deliver a *stabilized* voltage to its load despite variations in the input voltage applied to it. Two forms of constant voltage transformer are commonly encountered:—

(*a*) Motor controlled tapped transformer.

(*b*) Saturated core transformer.

40. **Motor controlled tapped transformer.** This arrangement is normally used to compensate for large input voltage changes in high power circuits. An electric motor, fitted with a governor, is run from the same supply as that used for the input to the transformer. Due to the action of the governor the motor runs at approximately constant speed despite variations in the supply voltage. The movement of the governor in maintaining constant speed is used to operate a switch connected to 'taps' on the transformer windings. The transformation ratio is thereby automatically adjusted in such a way that the secondary output voltage is virtually unaffected by variations in the supply voltage.

41. **Saturated core transformer.** This type of transformer is constructed as shown in Fig. 28. It consists of a tuned secondary winding S *loosely* coupled to the primary winding P (by virtue of the magnetic shunt leakage path L) and a compensating winding C *tightly* coupled to the primary. When a low voltage is applied to the primary most of the flux threads the secondary, producing a secondary voltage proportional to the transformation ratio; little flux takes the shunt leakage path L since the air gap produces a relatively high value of reluctance. As the applied voltage increases, the circulating current in the tuned circuit increases and section B of the core begins to saturate so that the reluctance of this part of the core increases. The reluctance of the leakage path L is now low in relation to that of section B of the core and most of any further flux increase due to an increase in primary voltage takes the leakage path. The secondary voltage still rises very slowly, but this is compensated for by the small antiphase voltage acting in series with it from winding C. The constant output voltage may be taken from the whole or part of the secondary.

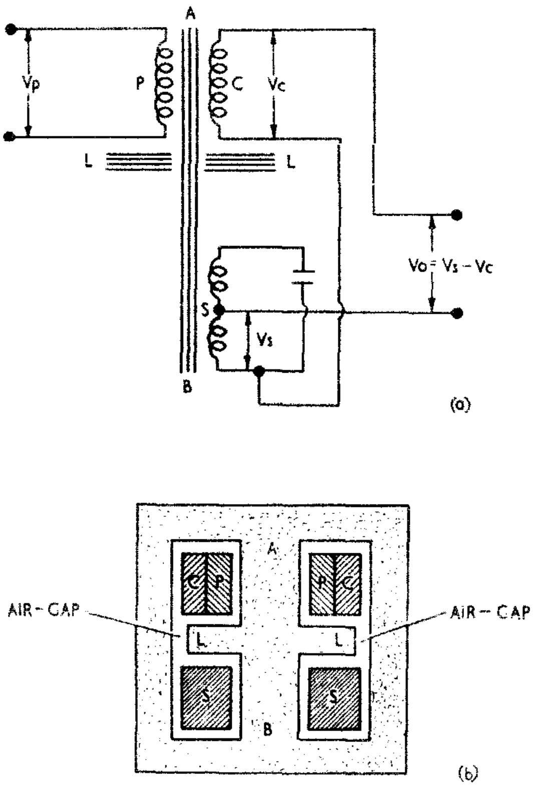


Fig. 28—SATURATED CORE TRANSFORMER

42. Because of the deliberate saturation of the core, distortion of the input waveform is inevitable. The resonant secondary circuit minimizes this effect, but where a pure a.c. waveform is required the secondary output voltage is passed through a suitable 'filter'. Fig. 29 shows how the secondary output voltage varies with input voltage for several load conditions.

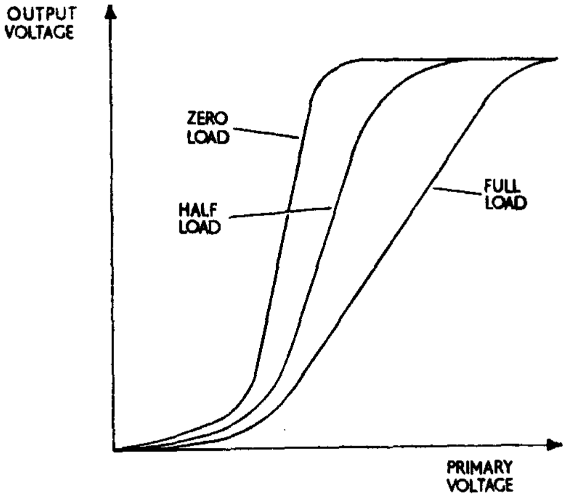


Fig. 29—VARIATION OF OUTPUT VOLTAGE WITH INPUT VOLTAGE

SECTION 7

CHAPTER 3

TRANSDUCTORS

Paragraph

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Saturable Reactor 2
A.C. Applied to a Saturable Reactor 5
D.C. Control of Saturation 9
Practical Form of Transductor 11
Operation of Transductor 14
Construction of Transductors 20
Advantages of Transductors 23
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PART 1, SECTION 7, CHAPTER 3

TRANSDUCTORS

Introduction

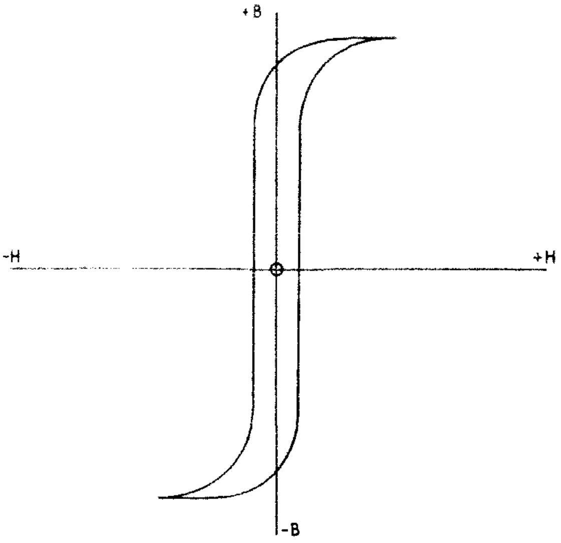
1. The transductor, or saturable reactor, is the main circuit element in 'magnetic amplifiers'. The magnetic amplifier will be considered in detail in Sect. 10, but since the transductor consists of two or more coils wound on a core of magnetic material, it has many of the features of an iron-cored transformer and it is convenient to discuss the basic principles of transductors in this Section.

Saturable Reactor

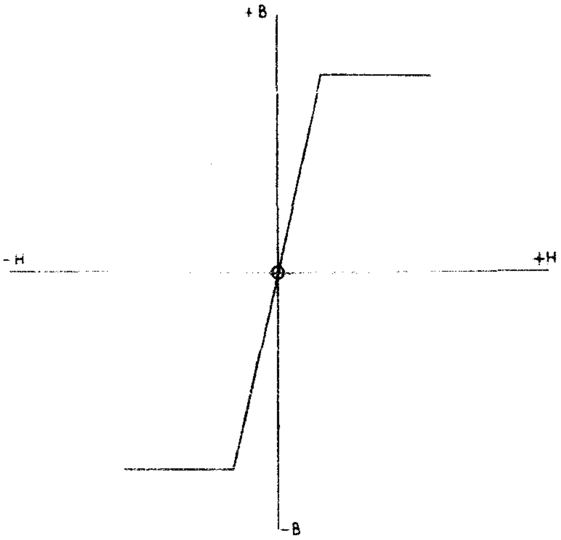
2. It was shown in Sect. 2, Chap. 2 that the inductance of a coil wound on a core of magnetic material is proportional to the absolute permeability of the core; that is, $L = N^2 \frac{\mu a}{l}$. However, the permeability of a magnetic circuit is *not* a constant (see Sect. 2, Chap. 1). Provided the core is not saturated, its permeability is high and so also is the inductance. If the core is saturated, the permeability falls off rapidly and the inductance falls to a low value.

3. If a coil, of negligible resistance, is wound on a core of magnetic material and connected to a source of alternating voltage V, the current established in the coil is $I = \frac{V}{\omega L}$. Thus, when the core is unsaturated and L is a large value, the current will be small; when the core is in the saturated condition so that L falls to a low value, the current will be large.

4. In most applications this change in inductance is undesirable, and core materials which gradually approach saturation are generally used in transformers and in electric machines. Transductors, however, operate *by virtue* of this saturation effect and the core materials used in transductors are those which saturate sharply for a small change in magnetising force, e.g., mumetal. The hysteresis loop for mumetal is given in Fig. 1(a), the ideal curve for a transductor material being as shown in Fig. 1(b).



(a) MUMETAL



(b) IDEAL

Fig. 1—HYSTERESIS LOOPS FOR TRANSDUCTOR MATERIALS

A.C. Applied to a Saturable Reactor

5. Fig. 2 shows an iron-cored inductance L connected in series with a resistance R across an a.c. supply of variable voltage V. The hysteresis loop for the core material may be

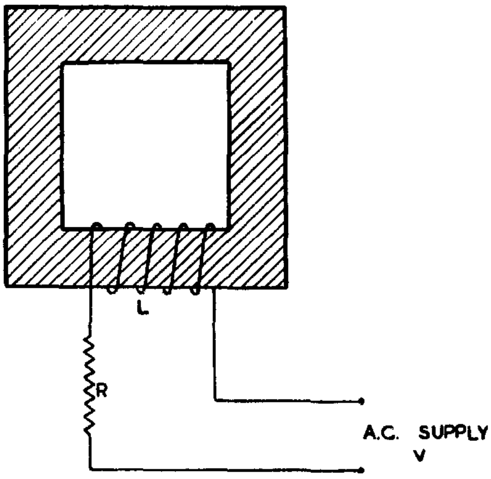


Fig. 2—A.C. APPLIED TO A SATURABLE REACTOR

assumed to have the ideal form of Fig. 1(b) so that the core will saturate at a low value of m.m.f.

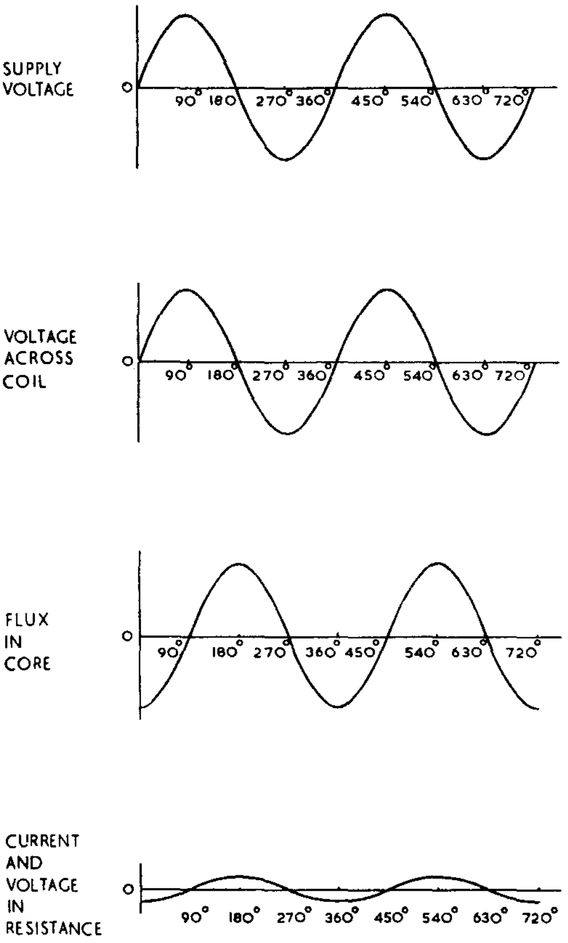


Fig. 3—AMPLITUDE INSUFFICIENT TO CAUSE SATURATION

6. Fig. 3 illustrates the conditions in the circuit when a low value of alternating voltage, insufficient to saturate the core, is applied. In the unsaturated state, the inductance L is high and the resistance R can be neglected in comparison. The impedance of the circuit is then high and inductive, so that the current established is small and lags on the applied voltage by 90°. The current is, in fact, merely the magnetising current necessary to produce the flux in the core. The flux itself is undistorted and lags the applied voltage by 90°.

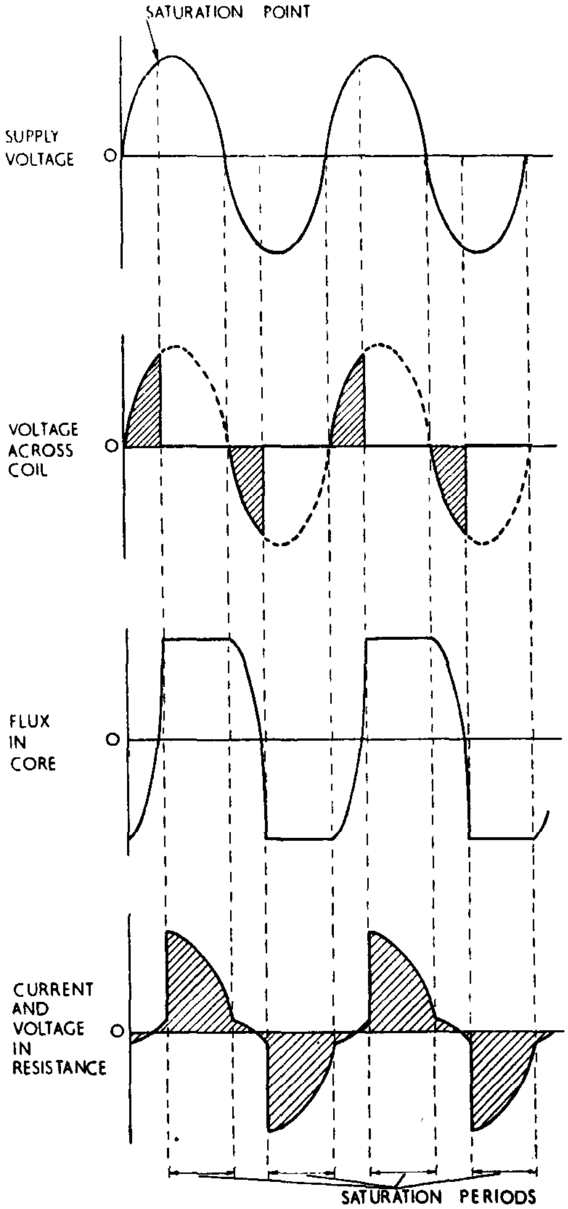


Fig. 4—EFFECT OF SATURATION OF THE CORE

7. Fig. 4 shows the effect of increasing the amplitude of the applied voltage to an extent sufficient to saturate the core. There is little change in the current in the circuit until the voltage reaches the point at which the core saturates. Saturation occurs sharply (because of the shape of the hysteresis loop for the core) and the core remains saturated for a short period on each half cycle. During that period the flux cannot change and, since the back e.m.f. is zero, the inductance may be considered to be short-circuited. From Kirchhoff's second law, the sum of the voltages across the coil and across the resistance must at every instant equal the applied voltage. Since the voltage across the coil has fallen to zero, the instantaneous voltage of the supply is suddenly applied to the resistance R and current flows in the resistance as a series of pulses. The duration of each pulse is equal to the saturation period and this, in turn, is dependent on the amplitude of the applied voltage. As the supply voltage is further increased, the core is held saturated for a longer period in each half cycle and the pulses of current are of longer duration.

8. Summarizing, it is seen that when the core is not saturated, the inductance of the coil is high and the major portion of the applied voltage is developed across the coil. During the saturation period, the inductance of the coil is low and major portion of the applied voltage is developed across the resistance.

D.C. Control of Saturation

9. While the saturation period can be varied by adjusting the amplitude of the supply voltage, it is much more convenient to provide control by an external circuit. Fig. 5(a) shows a saturable reactor with an

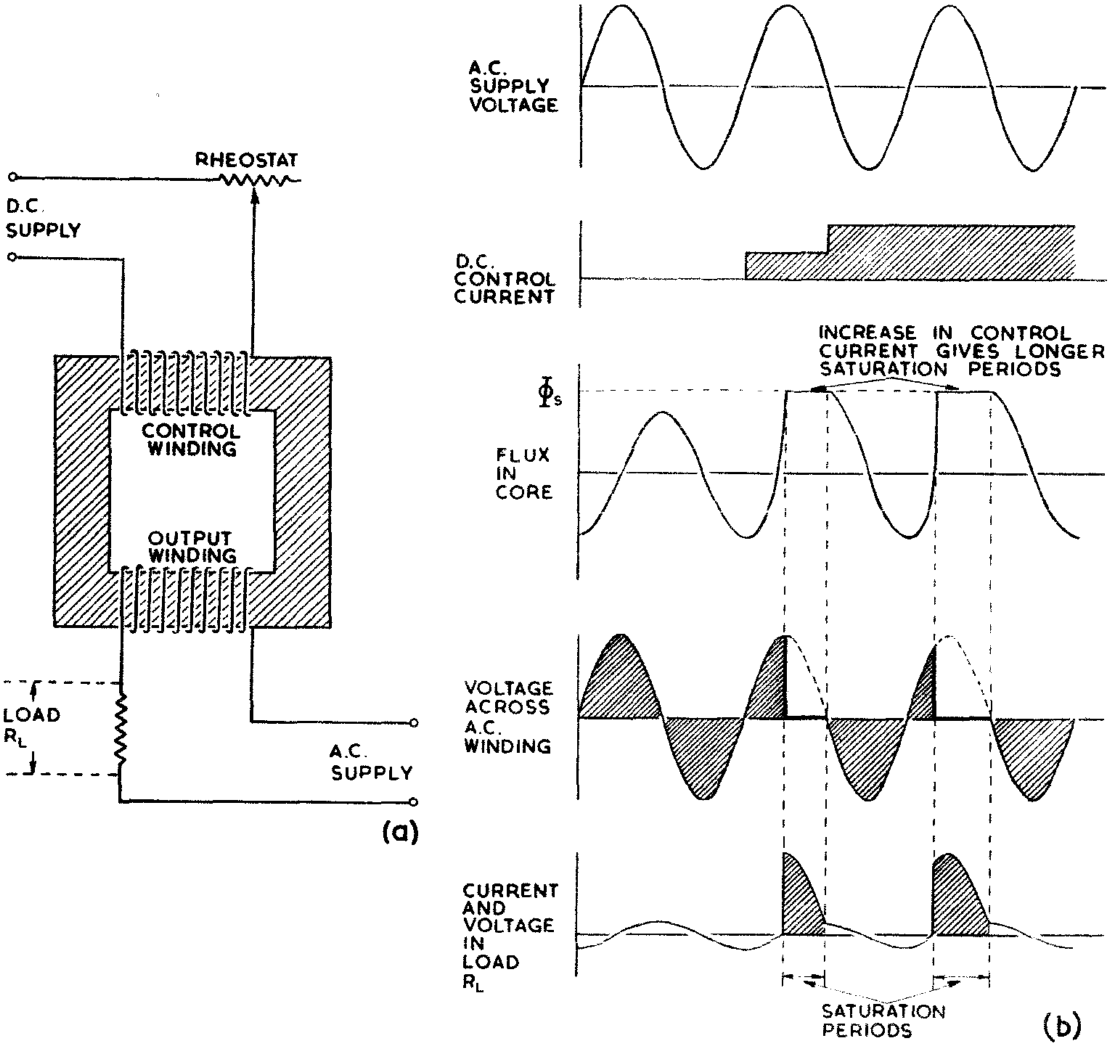

RHEOSTAT
D.C. SUPPLY
CONTROL WINDING
OUTPUT WINDING
LOAD R_L
A.C. SUPPLY
(a)
A.C. SUPPLY VOLTAGE
D.C. CONTROL CURRENT
INCREASE IN CONTROL CURRENT GIVES LONGER SATURATION PERIODS
Φ_s
FLUX IN CORE
VOLTAGE ACROSS A.C. WINDING
CURRENT AND VOLTAGE IN LOAD R_L
SATURATION PERIODS
(b)

Fig. 5—EFFECT OF CONTROL WINDING

additional winding, known as the *control winding*, which is connected to a source of d.c. supply via a rheostat. The amplitude of the a.c. supply voltage is fixed. With zero current in the control winding, the flux in the core almost, but not quite, reaches saturation point. A direct current in the control winding produces additional flux and, since this additional flux is unidirectional it causes the core to saturate on *every other* half cycle as shown in Fig. 5(b).

10. Variation of the control current causes the periods of saturation to be correspondingly increased or decreased. Thus, the pulses of current through the load resistance R_L, and the voltage developed across this resistance, are similarly varied. Quite small changes in the control current will cause large changes in the alternating current in the load resistance R_L. When a varying control signal is applied to the control winding an amplified signal appears in the a.c. circuit. This is the basis of the magnetic amplifier.

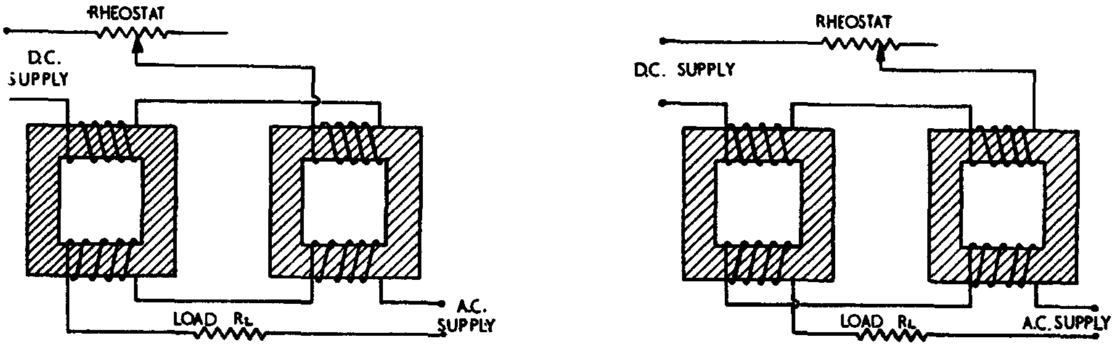
Practical Form of Transductor

11. The simple transductor described in Paras. 9 and 10 suffers from two disadvantages:—

(*a*) The control current in the d.c. winding affects only one half cycle of the a.c. supply.

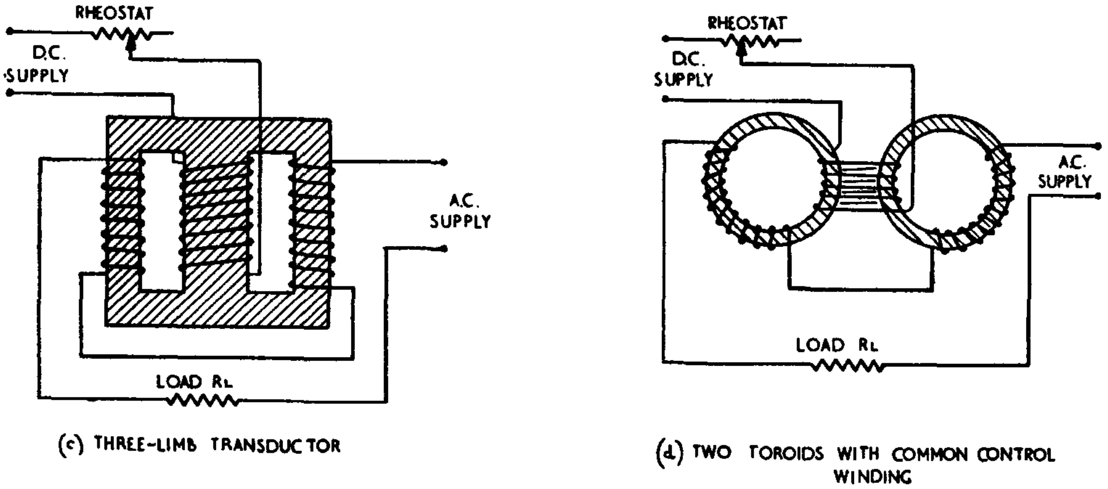
(*b*) The output and control windings are coupled by the same magnetic circuit and the alternating voltages induced in the control circuit by transformer action from the output winding may be sufficiently large to prevent effective control.

12. The first defect is remedied by providing *two* magnetic circuits which will saturate on alternate half cycles of the a.c. supply. The second defect is remedied by arranging the windings and cores so that the transformer effect is neutralized. Some typical arrangements are shown in Fig. 6.



(a) CONTROL WINDINGS IN OPPOSITION

(b) OUTPUT WINDINGS IN OPPOSITION



(c) THREE-LIMB TRANSDUCTOR

(d) TWO TOROIDS WITH COMMON CONTROL WINDING

Fig. 6—PRACTICAL FORMS OF TRANSDUCTOR

13. The transductor may be made in two identical portions as shown in (a) and (b) of Fig. 6. Voltages at the supply frequency induced in the control windings will be cancelled if the two halves of either the control winding or the output winding are wound in opposition. Alternatively, the magnetic circuits can be grouped so that the fluxes due to the alternating current in the output winding are in opposition through the control winding and no alternating voltage will be induced in that winding. This is the arrangement shown in (c) and (d) of Fig. 6. Although the output windings are shown connected in series in Fig. 6, they may be connected in parallel and still achieve the desired result.

Operation of Transductor

14. Fig. 7 illustrates an elementary transductor which consists of two identical inductors A and B enclosed by a common control winding. The output windings are series-connected and have the same number of turns so that the load current I_L produces equal fluxes Φ_A and Φ_B in cores A and B respectively. The common control winding is connected to a d.c. supply via a rheostat by means of which the control current may be varied. The output windings are connected in series with a load resistance R_L across an a.c. supply of voltage E. The amplitude of E is such that with no control current I_C in the control winding, the cores will at peak, be nearly but not quite saturated. In this unsaturated state the output windings have a very high impedance and the load current I_L will be very small.

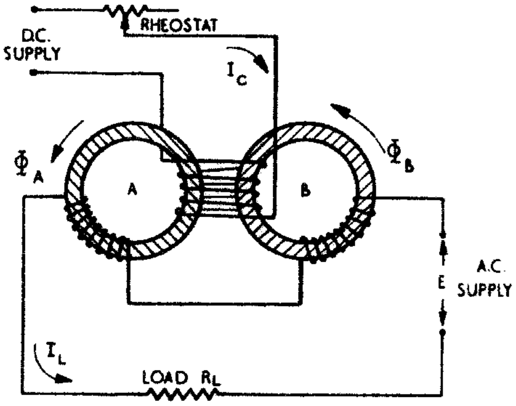


Fig. 7—CONNECTIONS TO AN ELEMENTARY TRANSDUCTOR

15. As the cores are so near to saturation, even a very small current I_C in the control winding will produce sufficient additional flux to cause both cores to saturate. However both cores do *not* saturate simultaneously since the flux produced by the direct current

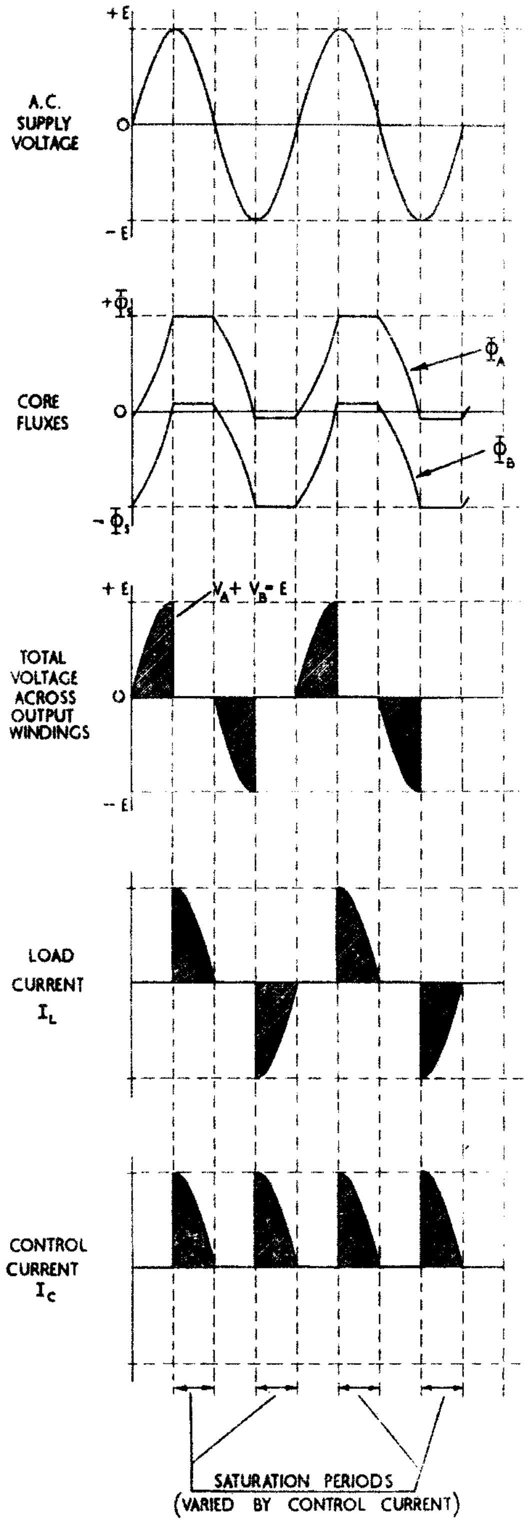


Fig. 8—OPERATION OF A SIMPLE TRANSDUCTOR

I_c is unidirectional, and will alternately add to and oppose the alternating fluxes in each core. That is, on one half cycle of the alternating voltage, the unidirectional flux will be in the same direction as the flux Φ_A in core A and, at the same time, it will be opposing the flux Φ_B in core B. On the next half cycle of the a.c. supply, the relative directions of the alternating fluxes will be changed, and the unidirectional control flux will now add to the flux Φ_B and oppose the flux Φ_A. With current I_c in the control winding, the fluxes will, therefore, be displaced as shown in Fig. 8, so that core A will be saturated on one half cycle of the a.c. supply and core B on the next.

16. In the unsaturated state the impedance of a transductor is high and, for the purpose of illustration, it may be assumed that there is zero load current unless the cores are saturated. Since the resistance of the load resistor R_L is very small compared with the impedance of the unsaturated transductor it may be ignored, and the voltage across each output winding will be equal to half the alternating voltage supply; that is, $V_A + V_B = E$. Assuming that the transductor is already in operation and that the hysteresis loop for each core is of the ideal form of Fig. 1(b), then the voltage, current and flux waveforms will be as shown in Fig. 8.

17. When flux Φ_A reaches the saturation point Φ_S there can be no further change of flux and the back e.m.f. falls to zero. Voltage V_A across inductor A falls sharply to zero and the winding behaves as though short-circuited. Inductor B however, is *not* saturated and behaves as a transformer together with the control winding for the period in which inductor A is saturated. Because of transformer action, the low impedance of the control circuit is reflected into the output winding B, making the voltage V_B very small and holding Φ_B constant at the particular value it has reached at that instant. With the voltages across the output windings A and B suddenly falling to zero, the full instantaneous supply voltage is applied to the load R_L and load current I_L flows. Since inductor B is behaving as a transformer, then to hold the flux Φ_B constant, a pulse of current must be established in the *control* circuit in order to produce a flux in opposition to that produced by the load current pulse.

18. A similar sequence of events occurs on the next half cycle of the a.c. supply. Inductor B saturates and inductor A behaves as a transformer giving the same results as before. Since the flux Φ_A is relatively in opposition to the flux Φ_B, the pulse of control current will be of the *same* polarity as before.

19. When the control current I_c is zero, the cores are unsaturated and the output current I_L is zero. As the control current is increased progressively in either direction, the cores saturate earlier and the output current increases. The control current may be increased until the a.c. supply voltage is applied to the load throughout the *complete* cycle. Thereafter, further increase in the control current will not result in greater output. A graph showing the relationship between the output current and the control current is given in Fig. 9. Amplification is obtained since a small variation in the control current will give a large variation in the output current.

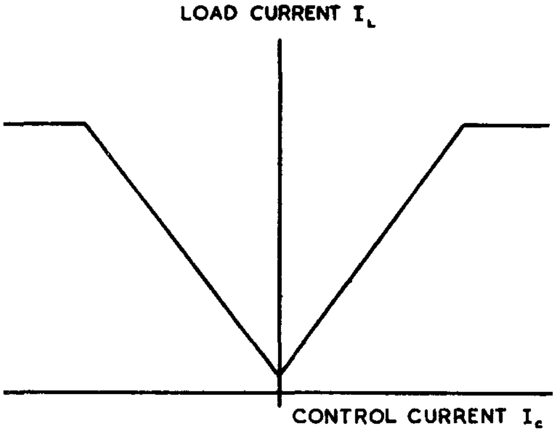


Fig. 9—CHARACTERISTICS OF A SIMPLE TRANSDUCTOR

Construction of Transductors

20. The simple form of transductor consisting of two separate identical cores, as illustrated in (a) and (b) of Fig. 6, is seldom used in practice because, although no alternating voltage is present in the complete d.c. winding, equal and opposite voltages are induced in each half of the control winding. These induced voltages necessitate adequate insulation which occupies considerable winding space. This disadvantage is avoided by using d.c. windings which enclose both magnetic circuits, as illustrated

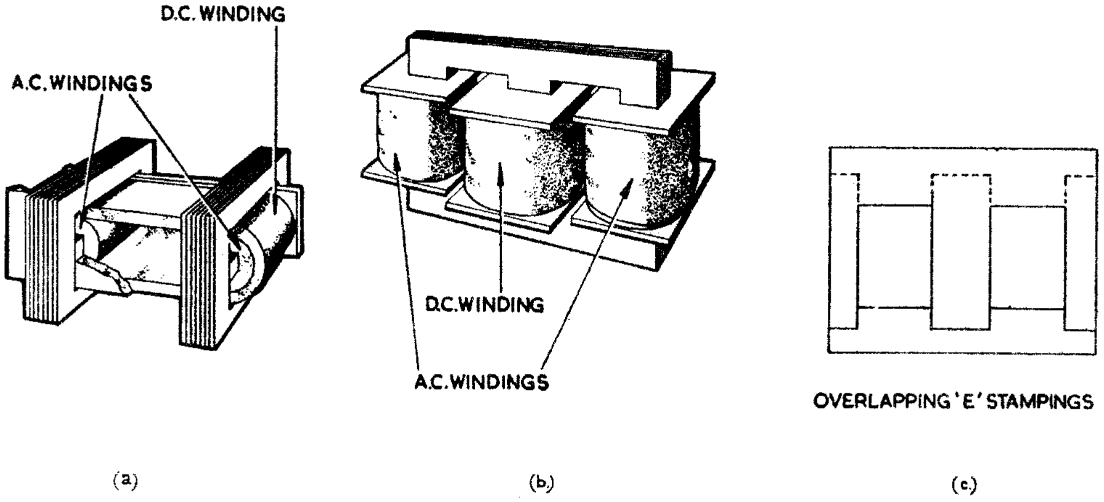


Fig. 10—TYPICAL TRANSDUCTOR CONSTRUCTION

in (c) and (d) of Fig. 6. Thus, during the period when neither magnetic circuit is saturated, no alternating voltage is induced in the control winding. Two typical constructional arrangements which are in common use are shown in (a) and (b) of Fig. 10.

21. The cores of the transductors are constructed of laminations in much the same way as normal transformer cores. Special precautions are taken to reduce air gaps to a minimum and it is normal to use special overlapping E type stampings to achieve this, as shown in Fig. 10(c).

22. Even when the greatest care is taken there will always be some residual air gap when interleaved laminations are used. Since a much greater m.m.f. is required to provide a given flux across even a small air gap than to provide that same flux in a continuous core, this residual air gap will have the undesirable effect of reducing the slope of the hysteresis loop. The only type of core which has no air gap is that constructed of ring stampings. Although a special winding technique is involved, the toroidal construction shown in Fig. 11 is the ideal arrangement.

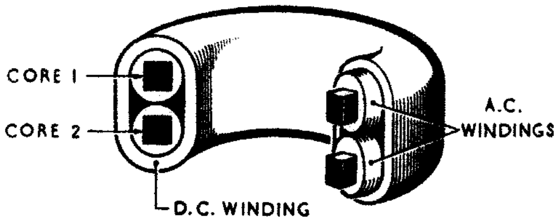


Fig. 11—TOROIDAL TYPE OF TRANSDUCTOR

Advantages of Transductors

23. (*a*) Its robust construction makes the transductor very reliable and ensures long life with no maintenance.

(*b*) The transductor is immediately available from the moment of switching on, i.e., no heating time is required, as it is for valves.

(*c*) The transductor has a high efficiency, of the order of 75 per cent.

(*d*) Power gains of the order of 10^7 can be obtained. The transductor shown in Fig. 12 gives an output of 70 watts for an input of 35 microwatts. This is a power gain of 2×10^6.

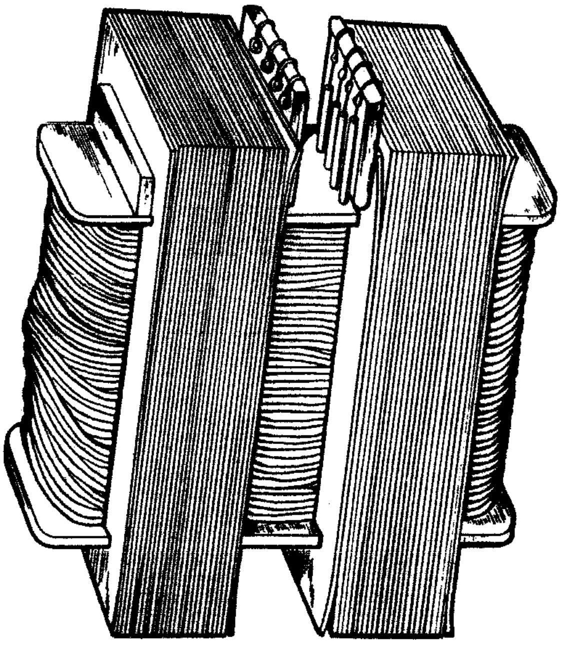


Fig. 12—A MODERN TRANSDUCTOR

Disadvantages of Transductors

24. (*a*) The value of the output current depends on the value of the control current and since the inductance of the control circuit causes the control current to lag behind the applied signal voltage, a change in signal voltage does not produce an instantaneous change of output current. In the transductor, the delayed response to an applied signal corresponds to the time constant of the control circuit and this may vary from a few milliseconds to about 1 minute.

(*b*) For a short time constant, the supply frequency should be high. However, the latter is limited to a few kc/s since magnetic materials able to give satisfactory results at higher frequencies are not normally available. Because of this the transductor is more suited to the amplification of signals which change slowly, i.e., it forms a useful alternative to the d.c. valve amplifier.

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Electrolysis	2	9	1	2
Electrolyte	2	9	1	2
Electrolytic capacitors	**1**	**4**	**2**	**13**
Electromagnet	**1**	**2**	**1**	**1**
Electromagnetic—induction	**1**	**2**	**2**	**1**
—radiation	3	16	1	7
—reflection (of waves)	3	16	2	24
Electromotive force (e.m.f.)	**1**	**1**	**1**	**27**
Electron,	**1**	**1**	**1**	**9**
free	**1**	**1**	**1**	**13**
valency	2	8	7	3
Electron—gun	2	8	5	2
—multiplier	2	8	6	7
—volt	2	8	1	4
Electron-coupled oscillator	2	12	1	30
Electronic—computer	3	20	1	2
—counter	3	18	1	15
—devices	2	8	1	1
—emission	2	8	1	3
Electrostatic—c.r.t.	2	8	5	4
—deflection	2	8	5	16
—focusing	2	8	5	14
—**screening**	**1**	**4**	**1**	**33**
—**voltmeter**	**1**	**6**	**1**	**26**
Electrostatics,	**1**	**4**	**1**	**1**
first law of	**1**	**4**	**1**	**2**
Element	**1**	**1**	**1**	**8**
Elliptical polarization	3	16	1	15
Emission, electron	2	8	1	3
secondary	2	8	1	6
Emitters	2	8	1	11
End-fire array	3	16	3	15
Energy,	**1**	**1**	**1**	**4**
electrical	**1**	**1**	**1**	**33**
reflection of (in line)	3	15	2	12
Energy—in electric field	**1**	**4**	**1**	**32**
—in magnetic field	**1**	**2**	**2**	**24**
Equivalent circuits	2	8	2	25
Error-measuring device	3	19	2	53

	BOOK	SECT.	CHAP.	PARA.
Error-rate damping	3	19	2	35
Extinction voltage	2	8	4	16
Extra high tension (e.h.t.)	2	9	1	1
F				
Facsimile telegraphy	3	13	1	13
Fading	3	17	1	18
Farad	**1**	**4**	**1**	**14**
Faraday's law	**1**	**2**	**2**	**1**
Feedback, Miller	2	10	3	38
negative	2	10	3	3
positive	2	10	3	1
undesired	2	10	3	37
Feedback—damping, velocity	3	19	2	26
—oscillators	2	12	1	7
—in computers	3	20	1	20
—in magnetic amplifiers	2	10	4	6
Feeders	3	15	3	32
Ferrite—core storage	3	20	2	29
—modulator	3	18	2	16
—rod aerial	3	16	5	13
Ferro-electric storage cell	3	20	2	31
Ferro-magnetism	**1**	**2**	**1**	**30**
Fidelity	3	14	1	3
Field—emission	2	8	1	6
—strength measurement	3	18	1	21
Field strength, electric	**1**	**4**	**1**	**19**
magnetic	**1**	**2**	**1**	**7**
Filter, band-pass	3	15	1	23
band-stop	3	15	1	25
capacitor input	2	9	3	7
choke input	2	9	3	13
crystal	3	15	1	28
detector	3	14	1	7
high-pass	3	15	1	15
low-pass	3	15	1	18
multi-section	3	15	1	32
pi-network	3	15	1	13
RC	3	15	1	4
T-network	3	15	1	13
Finite transmission line	3	15	3	1
Fleming's—left-hand rule	**1**	**3**	**2**	**5**
—right-hand rule	**1**	**3**	**1**	**5**
Flux, electric	**1**	**4**	**1**	**19**
magnetic	**1**	**2**	**1**	**7**
Flux density, electric	**1**	**4**	**1**	**20**
magnetic	**1**	**2**	**1**	**8**
Flux leakage	**1**	**7**	**2**	**17**
Focusing, electrostatic	2	8	5	14
magnetic	2	8	5	30
Focusing coil	2	8	5	30
Folded—dipole	3	16	2	15
—unipole	3	16	5	8
Force	**1**	**1**	**1**	**2**
Forward gain, aerial array	3	16	3	7
Fourier's theorem	**1**	**5**	**1**	**4**
Franklin—aerial	3	16	5	11
—oscillator	2	12	1	31
Free—electron	**1**	**1**	**1**	**13**
—oscillations	2	12	1	4
Frequency, fundamental	**1**	**5**	**1**	**5**
generator	**1**	**3**	**1**	**7**
intermediate	3	14	2	4
maximum usable	3	17	1	14
optimum working	3	17	1	14
resonant	**1**	**5**	**2**	**45**

	BOOK	SECT.	CHAP.	PARA.
Frequency—and wavelength	3	13	1	2
—changer	3	14	2	32
—changing	3	14	2	5
—instability	3	13	1	8
—measurement	3	18	1	7
—meters	3	18	1	10
—modulation	3	13	1	19
—monitor	3	18	1	18
—multiplication	3	13	1	11
—multipliers	2	11	2	26
—response measurement	3	18	2	8
—stability	2	12	1	24
—sweep generator	3	18	2	17
Frequency-modulated signal generator	3	18	2	14
Frequency-modulation measurements	3	18	4	14
Full-wave bridge rectifier	2	9	3	19
Full-wave rectifier	2	9	3	4
G				
Gain control,	3	14	1	11
automatic	3	14	2	52
Ganging (and tracking)	3	14	2	19
Gas-filled valves	2	8	4	2
Gates	3	20	2	42
Generator, constant current	2	8	2	26
constant voltage	2	8	2	25
d.c.	**1**	**3**	**1**	**1**
frequency sweep	3	18	2	17
losses in	**1**	**3**	**1**	**31**
self-excited	**1**	**3**	**1**	**31**
separately-excited	**1**	**3**	**1**	**29**
signal	3	18	2	1
single-phase a.c.	**1**	**5**	**4**	**3**
tachometer a.c.	3	19	2	54
tachometer d.c.	3	19	2	10
three-phase a.c.	**1**	**5**	**4**	**5**
Grid bias	2	8	2	6
Grid, control	2	8	2	1
screen	2	8	3	3
suppressor	2	8	3	17
Grid-dip meter	3	18	1	20
Grid stopper	2	10	3	38
Gripping rule	**1**	**2**	**1**	**13**
Ground wave	3	17	1	2
Grounded-grid triode	2	11	1	21
Growth of current (inductor)	**1**	**2**	**3**	**3**
H				
Half-adder	3	20	2	49
Half-wave—dipole	3	16	1	4
—phasing loop	3	15	4	19
—rectifier	2	9	3	3
Hard valve—	2	8	4	1
—stabilizer	2	9	3	23
Harmonics	**1**	**5**	**1**	**5**
Hartley oscillator	2	12	1	16
Heater (valve)	2	8	1	17
Heating effect (of current)	**1**	**1**	**1**	**19**
Heat-shielded cathode	2	8	4	9
Helical aerial	3	16	5	20
Henry	**1**	**2**	**2**	**9**

	BOOK	SECT.	CHAP.	PARA.
Heptode valve	2	8	3	31
Heterodyne—frequency meter	3	18	1	10
—principle	3	14	1	43
Hexode valve	2	8	3	30
High frequency—aerials	3	16	5	10
—effects (in valves)	2	8	3	35
High-pass filter	3	15	1	15
High tension (h.t.)	2	9	1	1
Highway (computer)	3	20	2	56
Hole (in semiconductor)	2	8	7	5
Horizontal polar diagrams	3	16	2	18
Horizontally polarized wave	3	16	1	14
Hot-cathode gas-filled valves	2	8	4	4
Hot-wire instrument	**1**	**6**	**1**	**22**
Hydrometer	2	9	1	24
Hysteresis, dielectric	**1**	**4**	**2**	**5**
magnetic	**1**	**2**	**1**	**34**
I				
Ideal filter	3	15	1	9
I.F. amplifier	3	14	2	41
Image interference	3	14	2	13
Impedance,	**1**	**5**	**2**	**19**
aerial	3	16	2	10
characteristic	3	15	2	9
dynamic	**1**	**5**	**3**	**12**
reflected	**1**	**7**	**1**	**5**
Impedance—matching	**1**	**7**	**2**	**16**
—triangle	**1**	**5**	**2**	**20**
Indicator, tuning	3	14	2	68
Indirectly-heated cathode	2	8	1	17
Induced grid noise	2	8	3	49
Inductance—	**1**	**2**	**2**	**7**
—in a.c. circuits	**1**	**5**	**2**	**4**
Induction, electromagnetic	**1**	**2**	**2**	**1**
Induction motor	**1**	**5**	**4**	**26**
Inductive reactance	**1**	**5**	**2**	**7**
Inductors,	**1**	**2**	**2**	**8**
power losses in	**1**	**5**	**2**	**40**
time constant of	**1**	**2**	**3**	**16**
types of	**1**	**2**	**4**	**4**
Inert cell	2	9	1	16
Infinite transmission line	3	15	2	3
In-phase current	**1**	**5**	**2**	**35**
Input devices, computer	3	20	2	73
Input impedance, line	3	15	2	8
Yagi array	3	16	3	4
Instability, frequency	3	13	1	8
Instantaneous value	**1**	**5**	**1**	**3**
Instrument, ratiometer	**1**	**6**	**2**	**14**
transformer for	**1**	**7**	**2**	**36**
types of	**1**	**6**	**1**	**4**
Insulator	**1**	**1**	**1**	**15**
Integral of error compensation	3	19	2	44
Integration (computer)	3	20	1	45
Integrator, Miller	3	20	1	47
Velodyne	3	20	1	50
Interconnection, three-phase	**1**	**5**	**4**	**12**
Interelectrode capacitance	2	8	2	37
Interference, adjacent channel	3	14	2	11
second channel	3	14	2	13
Interference fading	3	17	1	18
Intermediate frequency (i.f.),	3	14	2	4
choice of	3	14	2	17
Internal resistance	**1**	**1**	**2**	**18**

	BOOK	SECT.	CHAP.	PARA.
Interpoles, generator	**1**	**3**	**1**	**21**
motor	**1**	**3**	**2**	**14**
Intervalve coupling, a.f. amplifier	2	10	1	8
r.f. amplifier	2	11	1	10
Intervalve transformer	**1**	**7**	**2**	**29**
Intrinsic semiconductor	2	8	7	5
Inverted-V aerial	3	16	4	6
Inverter, rotary	**1**	**3**	**2**	**37**
Ion—	**1**	**1**	**1**	**13**
—trap (c.r.t.)	2	8	5	13
Ionosphere	3	17	1	8
Ionospheric characteristics	3	17	1	16
Ionization—	**1**	**1**	**1**	**13**
—potential	2	8	4	3
I-pot	3	20	1	41
Iron losses	**1**	**7**	**2**	**17**
Iron-cored—inductor	**1**	**2**	**4**	**6**
—transformer	**1**	**7**	**2**	**1**
J				
"j" operator	**1**	**5**	**5**	**1**
Joule	**1**	**1**	**1**	**6**
Junction—diode	2	8	7	18
—transistor	2	8	7	23
Junctions, metal-to-semi-conductor	2	8	7	9
p-n	2	8	7	14
K				
Kalium cell	2	9	1	14
Keying	3	13	1	14
Kinetic energy	**1**	**1**	**1**	**4**
Kirchhoff's laws	**1**	**1**	**2**	**25**
L				
Laminations	**1**	**2**	**4**	**5**
Lead inductance, valve	2	8	3	36
Lead-acid secondary cell	2	9	1	17
Leaky grid detector	3	14	1	28
Lecher bar frequency meter	3	18	1	23
Lecher bars	3	15	4	22
Leclanche cell	2	9	1	13
Lenz's law	**1**	**2**	**2**	**1**
Limitations of t.r.f. receiver	3	14	1	51
Linear broadside array	3	16	3	11
Lines of force, electric	**1**	**4**	**1**	**7**
magnetic	**1**	**2**	**1**	**5**
Lissajous figures	3	18	3	18
Load line, a.c.	2	10	1	14
pentode	2	8	3	21
triode	2	8	2	31
Local oscillator	3	14	2	31
Logic circuits	3	20	2	35
Loose coupling (transformer)	**1**	**7**	**1**	**11**
Losses, aerial	3	16	2	8
copper	**1**	**7**	**2**	**17**
dielectric	**1**	**4**	**2**	**5**
flux leakage	**1**	**7**	**2**	**17**
generator	**1**	**3**	**1**	**36**
iron	**1**	**7**	**2**	**17**
motor	**1**	**3**	**2**	**20**
transformer	**1**	**7**	**2**	**17**

	BOOK	SECT.	CHAP.	PARA.
Low frequency aerials	3	16	5	2
Low-pass filter	3	15	1	18
Low tension (l.t.)	2	9	1	1
Lumens	2	8	6	5
M				
Magic eye tuning indicator	3	14	2	69
Magnetic—amplifier	2	10	4	1
—circuit	**1**	**2**	**1**	**15**
—c.r.t.	2	8	5	29
—deflection	2	8	5	33
—effect (of current)	**1**	**2**	**1**	**10**
—field,	**1**	**2**	**1**	**4**
energy stored in	**1**	**2**	**2**	**24**
—field strength	**1**	**2**	**1**	**17**
—flux	**1**	**2**	**1**	**7**
—flux density	**1**	**2**	**1**	**8**
—focusing	2	8	5	30
—materials	**1**	**2**	**1**	**29**
—saturation	**1**	**2**	**1**	**32**
—space constant	**1**	**2**	**1**	**19**
—storage	3	20	2	27
—storms	3	17	1	23
Magnetism	**1**	**2**	**1**	**1**
Magnetizing—current	**1**	**7**	**2**	**4**
—force	**1**	**2**	**1**	**17**
Magnets	**1**	**2**	**1**	**1**
Magnetomotive force (m.m.f.)	**1**	**2**	**1**	**16**
Magnetostriction—	**1**	**2**	**1**	**27**
—delay line	3	20	2	33
Magnification, circuit	**1**	**5**	**2**	**51**
Magnitude	**1**	**5**	**2**	**18**
Magslip	3	19	1	35
Marconi quarter-wave aerial	3	16	2	19
Master oscillator	3	13	1	9
Matching, aerial	3	16	2	12
a.f. amplifier	2	10	2	8
balanced to unbalanced	3	15	4	17
delta	3	16	2	14
transformer	**1**	**7**	**2**	**16**
Matching—stubs	3	15	4	13
—transformer, quarter-wave	3	15	4	13
Matrix storage system	3	20	2	29
Matter	**1**	**1**	**1**	**8**
Maximum—power transfer	**1**	**1**	**2**	**21**
—usable frequency	3	17	1	14
Maxwell's circulating currents	**1**	**1**	**2**	**31**
Mean value	**1**	**5**	**1**	**3**
Measurement of—current	3	18	1	2
—field strength	3	18	1	21
—frequency	3	18	1	7
—frequency response	3	18	2	8
—modulation	3	18	4	2
—phase	3	18	3	21
—power	3	18	5	2
—standing waves	3	18	5	13
—voltage	3	18	1	3
—waveforms	3	18	3	1
Measuring instruments	**1**	**6**	**1**	**1**
Medium frequency aerials	3	16	5	5
Megger	**1**	**6**	**2**	**19**
Meissner oscillator	2	12	1	15
Mercury-arc rectifier	2	9	3	33
Mercury-vapour diode	2	8	4	4
Metal—film resistors	**1**	**1**	**3**	**9**
—rectifier	2	9	3	28
—to semiconductor junction	2	8	7	9

	BOOK	SECT.	CHAP.	PARA.
Meters,	**1**	**6**	**1**	**1**
decibel	**1**	**6**	**3**	**16**
field strength	3	18	1	21
frequency	3	18	1	10
r.f. power	3	18	5	5
Methods of biasing	2	8	2	23
Mica capacitors	**1**	**4**	**2**	**9**
Microphone	2	10	1	6
Microwave aerials	3	16	5	22
Miller—effect	2	11	1	5
—feedback	2	12	1	40
—integrator	3	20	1	47
M.K.S. units	**1**	**1**	**1**	**5**
Modulation—	3	13	1	18
—factor	3	13	1	23
—measurement	3	18	4	2
Modulator, anode	3	13	1	25
ferrite	3	18	2	16
Molecules	**1**	**1**	**1**	**8**
Monitor, frequency	3	18	1	18
M.O.-P.A. transmitter	3	13	1	9
Morse telegraphy	3	13	1	13
Mosaic telegraphy	3	13	1	13
Motor, commutator	**1**	**5**	**4**	**38**
d.c.	**1**	**3**	**2**	**9**
induction	**1**	**5**	**4**	**26**
losses in	**1**	**3**	**2**	**20**
speed of	**1**	**3**	**2**	**22**
starters for	**1**	**3**	**2**	**24**
synchronous	**1**	**5**	**4**	**20**
Motor controlled tapped transformer	**1**	**7**	**2**	**40**
Moving coil instrument	**1**	**6**	**1**	**5**
Moving iron instrument	**1**	**6**	**1**	**15**
M-type transmission	3	19	1	8
Multimeters	**1**	**6**	**2**	**4**
Multiple-hop propagation	3	17	1	17
Multiplication (analogue)	3	20	1	38
Multiplicative frequency changer	3	14	2	35
Multiplier, electron	2	8	6	7
frequency	2	11	2	26
voltmeter	**1**	**6**	**1**	**10**
Multi-section filters	3	15	1	32
Multi-stage r.f. amplifiers	2	11	1	14
Multi-unit valves	2	8	3	34
Multivibrator	2	12	3	7
Mutual characteristic, pentode	2	8	3	18
triode	2	8	2	9
Mutual conductance	2	8	2	13
Mutual inductance	**1**	**2**	**2**	**13**
Mutual inductive coupling	**1**	**7**	**1**	**3**

N

	BOOK	SECT.	CHAP.	PARA.
Narrow-band amplifiers	2	11	1	1
Negative feedback—	2	10	3	3
—in computers	3	20	1	20
—in transistors	2	10	3	44
Negative resistance—	2	8	3	8
—oscillators	2	12	1	37
Neper	**1**	**6**	**3**	**17**
Neutralization	2	11	2	6
Neutron	**1**	**1**	**1**	**9**
Newton	**1**	**1**	**1**	**6**
Noise, receiver	3	14	2	26
valve	2	8	3	46
Non-inductive winding	**1**	**2**	**4**	**17**
Non-linear—device	**1**	**1**	**2**	**4**
—distortion	2	10	1	23
NOR gate	3	20	2	42
North-south rule	**1**	**2**	**1**	**13**
NOT-AND gate	3	20	2	44
NOT-EQUIVALENT circuit	3	20	2	47
NOT gate	3	20	2	46
N-P-N transistor	2	8	7	23
N-type semiconductor	2	8	7	7
Number period, computer	3	20	2	22

O

	BOOK	SECT.	CHAP.	PARA.
Octode valve	2	8	3	32
Ohm	**1**	**1**	**2**	**5**
Ohmmeter	**1**	**6**	**2**	**3**
Ohm's law	**1**	**1**	**2**	**2**
Ohms-per-volt rating	**1**	**6**	**1**	**11**
On-off keying	3	13	1	15
Open-circuited transmission line	3	15	3	12
Open loop control system	3	19	2	12
Open wire feeder	3	15	3	32
Operating conditions, oscillator	2	12	1	21
Operator "j"	**1**	**5**	**5**	**1**
Optimum working frequency	3	17	1	14
OR gate	3	20	2	43
Oscillator, beat frequency	3	14	1	46
Colpitts	2	12	1	19
crystal	2	12	2	10
dynatron	2	12	1	38
electron-coupled	2	12	1	30
Franklin	2	12	1	31
Hartley	2	12	1	16
local	3	14	2	31
master	3	13	1	9
Meissner	2	12	1	15
negative resistance	2	12	1	37
phase-shift	2	12	3	2
Pierce	2	12	2	13
push-pull	2	12	1	20
RC	2	12	3	2
relaxation	2	12	3	7
squegging	2	12	1	23
transistor	2	12	1	36
transitron	2	12	1	39
tuned anode	2	12	1	12
tuned anode-crystal grid	2	12	2	10
tuned anode-tuned grid	2	12	1	40
tuned grid	2	12	1	14
variable frequency	2	12	1	32
v.h.f.	2	12	1	33
Wien bridge	2	12	3	6
Oscilloscope	3	18	3	2
Out-of-phase current	**1**	**5**	**2**	**35**
Output devices, computer	3	20	2	75
Overload relay	2	9	3	26
Oxide-coated emitter	2	8	1	13

P

	BOOK	SECT.	CHAP.	PARA.
Padding	3	14	2	21
Paper tape	3	20	2	74

	BOOK	SECT.	CHAP.	PARA.
Paper type capacitor	**1**	**4**	**2**	**8**
Parallel—connection of valves	2	10	2	34
—negative feedback	2	10	3	27
—resonance	**1**	**5**	**3**	**11**
—tuned circuit	**1**	**5**	**3**	**7**
Para-magnetism	**1**	**2**	**1**	**30**
Parasitic—aerial	3	16	3	2
—oscillations	2	10	3	38
Partition noise	2	8	3	48
Peak—inverse voltage	2	9	3	18
—value	**1**	**5**	**1**	**3**
Pentagrid frequency changer	3	14	2	38
Pentode valve,	2	8	3	17
variable-mu	2	8	3	24
Percentage modulation measurement	3	18	4	3
Period (of sine wave)	**1**	**5**	**1**	**2**
Permanent magnets	**1**	**2**	**1**	**1**
Permeability	**1**	**2**	**1**	**18**
Permittivity	**1**	**4**	**1**	**21**
Phase—difference	**1**	**5**	**1**	**11**
—distortion	2	10	1	23
—measurement	3	18	3	21
—modulation	3	13	1	18
Phase-advance networks	3	19	2	39
Phase-change coefficient (of line)	3	15	3	30
Phase-sensitive rectifier	3	19	2	54
Phase-shift oscillator	2	12	3	2
Phase-shifting—transformer	**1**	**7**	**2**	**37**
—by resolver synchro	3	19	1	57
Phase-splitter	2	10	2	32
Phasing loop, half-wave	3	15	4	19
Photocells	2	8	6	1
Photo-electric emission	2	8	1	6
Photodiode	2	8	7	33
Photometer	2	8	6	5
Phototransistor	2	8	7	34
Pierce oscillator	2	12	2	13
Piezo-electric effect	2	12	2	1
Pi-filter	3	15	1	13
Plane of polarization	3	16	1	4
Plane wave	3	16	1	12
P-N junctions	2	8	7	14
P-N-P transistor	2	8	7	23
Point contact—rectifier	2	8	7	13
—transistor	2	8	7	19
Polarization—in batteries	2	9	1	8
—of e.m. wave	3	17	1	7
Polyphase a.c.	**1**	**5**	**4**	**1**
Polystyrene capacitors	**1**	**4**	**2**	**12**
Position control systems	3	19	2	14
Positive feedback	2	10	3	1
Post-deflection accelerator	2	8	5	20
Potential, ionization	2	8	4	3
striking	2	8	4	16
Potential—difference	**1**	**1**	**1**	**27**
—divider	**1**	**1**	**2**	**23**
—energy	**1**	**1**	**1**	**4**
—gradient	**1**	**4**	**1**	**4**
Potentiometer	**1**	**1**	**3**	**10**
Power,	**1**	**1**	**1**	**33**
absolute	**1**	**6**	**3**	**13**
apparent	**1**	**5**	**2**	**36**
maximum transfer of	**1**	**1**	**2**	**21**
true	**1**	**5**	**2**	**36**
Power—amplifier, a.f.	2	10	2	1
r.f.	2	11	2	1
—factor	**1**	**5**	**2**	**36**

	BOOK	SECT.	CHAP.	PARA.
Power—(*continued*)				
—in a.c. circuits	**1**	**5**	**2**	**33**
—in three-phase circuits	**1**	**5**	**4**	**16**
—loss (in components)	**1**	**5**	**2**	**40**
—measurement	3	18	5	2
—ratio (decibels)	**1**	**6**	**3**	**6**
—supplies, c.r.t.	2	8	5	22
electrochemical	2	9	1	1
electronic	2	9	3	1
mechanically-derived	2	9	2	1
—transformers	**1**	**7**	**2**	**23**
Primary—cell	2	9	1	6
—no-load current	**1**	**7**	**2**	**9**
Printing telegraphy	3	13	1	13
Propagation,	3	17	1	1
multiple-hop	3	17	1	17
scatter	3	17	1	28
tropospheric	3	17	1	25
velocity of (in line)	3	15	3	26
Propagation—coefficient (of line)	3	15	3	27
—in ionosphere	3	17	1	10
Proton	**1**	**1**	**1**	**9**
Proximity effect	**1**	**2**	**4**	**16**
P-type semiconductor	2	8	7	8
Pulsating current	**1**	**1**	**1**	**23**
Pulse—modulation	3	13	1	21
—transformer	**1**	**7**	**2**	**38**
Punched—cards	3	20	2	74
—paper tape	3	20	2	74
Push-pull—amplifiers	2	10	2	19
—connection	2	10	2	16
—input circuits	2	10	2	31
—oscillator	2	12	1	20
Push-push doubler	2	11	2	28
Pythagoras' theorem	**1**	**5**	**2**	**18**

Q

	BOOK	SECT.	CHAP.	PARA.
Q-factor—	**1**	**5**	**2**	**49**
—of components	**1**	**5**	**2**	**54**
Quarter-wave—aerial	3	16	2	19
—stub	3	15	4	12
—transformer	3	15	4	13
Quartz crystal	2	12	2	1

R

	BOOK	SECT.	CHAP.	PARA.
Radian	**1**	**5**	**1**	**9**
Radiation (from aerials)	3	16	1	7
Radio frequency bands	2	11	1	2
Ratiometer instrument	**1**	**6**	**2**	**14**
RC—circuits, parallel	**1**	**5**	**3**	**6**
series	**1**	**5**	**2**	**22**
—coupled amplifier	2	10	1	8
—filter	3	15	1	4
—oscillator	2	12	3	2
Reactance, capacitive	**1**	**5**	**2**	**13**
inductive	**1**	**5**	**2**	**7**
Reactance sketch	**1**	**5**	**2**	**8**
Reaction, armature, generator	**1**	**3**	**1**	**22**
motor	**1**	**3**	**2**	**13**
Reaction type wavemeter	3	18	1	9
Reactive—matching stubs	3	15	4	14
—sparking	**1**	**3**	**1**	**19**
Receiver, superhet	3	14	2	73
t.r.f.	3	14	1	40

	BOOK	SECT.	CHAP.	PARA.
Receiver—noise	3	14	2	26
—output measurement	3	18	5	3
—tuning	3	14	1	7
Reception—of a.m. signal	3	14	2	50
—of c.w. signal	3	14	2	47
Rectifier, copper-oxide	2	9	3	29
full-wave bridge	2	9	3	19
gas-filled diode	2	9	3	25
hard-vacuum—full wave	2	9	3	4
—half wave	2	9	3	3
instrument	**1**	**6**	**1**	**14**
mercury-arc	2	9	3	33
mercury-vapour	2	9	3	25
phase-sensitive	3	19	2	54
plate-type (metal)	2	9	3	28
point contact	2	8	7	13
selenium	2	9	3	30
three-phase	2	9	3	32
Reflected impedance	**1**	**7**	**1**	**5**
Reflection—of e.m. waves	3	16	2	24
—in transmission line	3	15	2	12
Reflectometer	3	18	5	19
Reflector	3	16	3	3
Registers, computer	3	20	2	55
Regulation, rectifier	2	9	3	21
Regulator, carbon pile	2	9	2	5
Rejector circuit	**1**	**5**	**3**	**13**
Relaxation oscillator	2	12	3	7
Relay,	**1**	**2**	**1**	**24**
overload	2	9	3	26
time-delay	2	9	3	26
Reluctance	**1**	**2**	**1**	**21**
Remanence	**1**	**2**	**1**	**36**
Remote indication, a.c.	3	19	1	19
d.c.	3	19	1	3
Remote position control servo	3	19	2	14
Resistance,	**1**	**1**	**2**	**5**
anode slope	2	8	2	12
dynamic	**1**	**5**	**2**	**12**
high frequency	**1**	**2**	**4**	**14**
internal	**1**	**1**	**2**	**18**
negative	2	8	3	8
temperature coefficient of	**1**	**1**	**2**	**9**
Resistance—in a.c. circuits	**1**	**5**	**2**	**3**
Resistivity	**1**	**1**	**2**	**8**
Resistor colour code	**1**	**1**	**3**	**6**
Resistors, types of	**1**	**1**	**3**	**3**
Resolution, synchro	3	19	1	54
Resolver synchro—	3	19	1	22
—as a phase-shifter	3	19	1	57
—system	3	19	1	51
Resonance, parallel	**1**	**5**	**3**	**11**
series	**1**	**5**	**2**	**41**
Resonant—aerial	3	16	2	2
—cavity wavemeter	3	18	1	24
Retentivity	**1**	**2**	**1**	**37**
R.F.—choke	**1**	**2**	**4**	**10**
—gain control	3	14	1	11
—power amplifier	2	11	2	1
—power meter	3	18	5	5
—signal generator	3	18	2	9
—transformer	**1**	**7**	**1**	**8**
—voltage amplifier	2	11	1	1
Rheostat	**1**	**1**	**3**	**10**
Rhombic aerial	3	16	4	8
Ripple factor	2	9	3	10

	BOOK	SECT.	CHAP.	PARA.
RL circuit, parallel	**1**	**5**	**3**	**4**
series	**1**	**5**	**2**	**16**
Root-mean-square values	**1**	**5**	**1**	**3**
Rotary—converters	**1**	**3**	**2**	**38**
—inverters	2	9	2	3
—transformers	2	9	2	4
Rotating magnetic field	**1**	**5**	**4**	**17**
Rotor,	**1**	**5**	**4**	**8**
squirrel-cage	**1**	**5**	**4**	**26**
R-pot	3	20	1	13
S				
Saturable reactor	**1**	**7**	**3**	**2**
Saturated core transformer	**1**	**7**	**2**	**41**
Saturation, magnetic	**1**	**2**	**1**	**32**
valve	2	8	1	24
Scalar quantity	**1**	**5**	**1**	**14**
Scatter propagation	3	17	1	28
Scott-connected transformer	**1**	**7**	**2**	**35**
Screen, c.r.t.	2	8	5	10
Screen grid	2	8	3	3
Screening, electric	**1**	**4**	**1**	**33**
magnetic	**1**	**7**	**1**	**22**
Second channel interference	3	14	2	13
Secondary emission,	2	8	1	6
effects of	2	8	3	6
See-saw—amplifier	3	20	1	33
—summing amplifier	3	20	1	36
Selective fading	3	17	1	19
Selectivity, receiver	3	14	1	3
tuned circuit	**1**	**5**	**3**	**18**
Selenium rectifier	2	9	3	30
Self-exciting transmitter	3	13	1	6
Self-inductance	**1**	**2**	**2**	**7**
Semiconductors	2	8	7	1
Sensitivity,	3	14	1	3
deflection	2	8	5	18
Sequential operation, computer	3	20	2	64
Serial operation, computer	3	20	2	56
Series—negative feedback	2	10	3	26
—resonance	**1**	**5**	**2**	**41**
—tuned circuit	**1**	**5**	**2**	**42**
Servomechanisms,	3	19	2	1
power requirements of	3	19	2	49
remote position control	3	19	2	14
response and stability of	3	19	2	20
velocity	3	19	2	32
Shell-type transformer	**1**	**7**	**2**	**20**
S.H.F.—aerials	3	16	5	22
—power measurement	3	18	5	8
Shift—control (c.r.t.)	2	8	5	23
—registers (computer)	3	20	2	59
Short-circuited transmission line	3	15	3	8
Short transmission lines	3	15	4	11
Shot effect (valves)	2	8	3	47
Shunt, ammeter	**1**	**6**	**1**	**12**
Shunt-fed diode detector	3	14	1	26
Sidebands	3	13	1	24
Signal generators	3	18	2	1
Signal-to-noise ratio	3	14	1	3
Simple a.g.c.	3	14	2	56
Simultaneous operation, computer	3	20	2	64
Sine curve	**1**	**5**	**1**	**8**
Sine-cosine potentiometers	3	20	1	54

	BOOK	SECT.	CHAP.	PARA.
Single phase—commutator motor	1	5	4	38
—induction motor	1	5	4	37
Sinusoidal waveform	1	5	1	2
Skin effect	1	2	4	13
Skip distance	3	17	1	15
Sky wave	3	17	1	2
Slip speed	1	5	4	29
Slope resistance	2	8	2	12
Slot aerials	3	16	2	35
Smoothing circuits	2	9	3	6
Soft valves	2	8	4	1
Solar flares	3	17	1	21
Solenoid	1	2	1	11
Sound	2	10	1	4
Space—charge	2	8	1	22
—wave	3	17	1	2
Specific—gravity	2	9	1	19
—resistance	1	1	2	8
Speed, control of (in servo)	3	19	2	6
d.c. motor	1	3	2	22
slip	1	5	4	29
synchronous	1	5	4	11
Square-law scale	1	6	1	17
Square waveform	1	5	1	2
Squegging oscillator	2	12	1	23
Squirrel-cage rotor	1	5	4	26
Stability, frequency	2	12	1	24
servomechanism	3	19	2	20
Stabilizer, current	2	9	3	24
voltage	2	9	3	22
Stabilivolt	2	8	4	18
Stacked arrays	3	16	3	17
Stage gain	2	10	1	11
Staggered tuning	2	11	1	20
Standing wave—aerial	3	16	2	2
—measurement	3	18	5	12
—dipole	3	16	2	3
—on line	3	15	3	8
—ratio	3	15	3	22
Star-delta—starter	1	5	4	30
—transformation	1	5	4	15
Star-point adding circuit	3	20	1	30
Static characteristics, triode	2	8	2	8
Stator	1	5	4	8
'Stig' control, c.r.t.	2	8	5	28
Storage devices	3	20	2	25
Striking potential	2	8	4	16
Strip feeder	3	15	3	35
Stubs, matching	3	15	4	14
quarter-wave	3	15	4	12
Sulphation	2	9	1	25
Sunspots	3	17	1	20
Superconductivity	3	20	2	32
Superheterodyne—principle	3	14	2	4
—receiver	3	14	2	73
—receiver alignment	3	18	2	20
Superposition theorem	1	1	2	32
Super-refraction	3	17	1	30
Suppressed aerials	3	16	5	15
Suppressor grid	2	8	3	17
Surface wave	3	17	1	2
Susceptance	1	5	3	2
Swinging choke	2	9	3	15
Synchronization, computer	3	20	2	56
M-type transmission	3	19	1	16
timebase	3	18	3	7
Synchronous motor	1	5	4	20

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Synchros,	3	19	1	21
control	3	19	1	45
resolver	3	19	1	51
torque	3	19	1	26

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Tachometer generator, a.c.	3	19	2	54
d.c.	3	19	2	10
Tank circuit	2	11	2	21
Telegraphy	3	13	1	13
Telephone receiver	2	10	1	7
Telephony	3	13	1	13
Temperature coefficient, resistance	1	1	2	9
Temperature-limited valve	2	8	1	25
Ten-hour rate	2	9	1	19
Termination, transmission line	3	15	3	5
Testmeters	1	6	2	9
Tetrode valves	2	8	3	2
T-filter	3	15	1	13
Thermionic emission	2	8	1	6
Thermistor bridge	3	18	5	10
Thermo-junction meter	1	6	1	24
Thoriated-tungsten emitter	2	8	1	12
Three-phase—connection	1	5	4	12
—generator	1	5	4	5
—rectifier	2	9	3	32
—transformer	1	7	2	33
Three-point tracking	3	14	2	23
Thyratron—	2	8	4	11
—timebase	3	18	3	8
Tight coupling (transformers)	1	7	1	12
Timebase—	2	8	5	24
—generators	3	18	3	4
Time constant, capacitive	1	4	3	16
inductive	1	2	3	16
Time-delay relay	2	9	3	26
Time-sharing, computer	3	20	2	64
Torque, motor	1	3	2	15
Torque—differential synchro	3	19	1	37
—synchro	3	19	1	22
Tracking (and ganging)	3	14	2	19
Transducer	3	19	1	2
Transductors—	1	7	3	1
—in cascade	2	10	4	8
—in push-pull	2	10	4	9
Transformation, impedance	1	7	2	15
Transformation ratio	1	7	2	6
Transformer, audio frequency	1	7	2	25
constant voltage	1	7	2	39
instrument	1	7	2	36
intervalve	1	7	2	29
iron-cored	1	7	2	1
load conditions in	1	7	2	10
losses in	1	7	2	17
matching, quarter-wave	3	15	4	13
motor controlled tapped	1	7	2	40
phase-shifting	1	7	2	37
power	1	7	2	23
pulse	1	7	2	38
r.f.	1	7	1	8
r.f. power	1	7	1	18
rotary	2	9	2	4
saturated core	1	7	2	41
Scott-connected	1	7	2	35
three-phase	1	7	2	33

	BOOK	SECT.	CHAP.	PARA.
Transformer-coupled—a.f. amplifier	2	10	1	18
—r.f. amplifier	2	11	1	13
Transient velocity damping	3	19	2	42
Transistor, junction	2	8	7	23
point contact	2	8	7	19
Transistor—bistable circuit	3	20	2	26
—oscillators	2	12	1	36
—power amplifiers	2	10	2	35
—superhet receiver	3	14	2	75
—t.r.f. receiver	3	14	1	49
—voltage amplifiers	2	10	1	26
Transit time	2	8	3	38
Transitron oscillator	2	12	1	39
Transmission line, electrical length of	3	15	4	4
finite	3	15	3	1
infinite	3	15	2	3
termination of	3	15	3	5
Transmission line—techniques	3	15	4	2
—terminated in open circuit	3	15	3	12
—terminated in short circuit	3	15	3	8
—terminated in Z_0	3	15	3	7
Transmitter, communication	3	13	1	5
desynn	3	19	1	4
M-type	3	19	1	9
synchro	3	19	1	25
Transmitter—output measurements	3	18	5	4
Trapezium distortion	2	8	5	26
Travelling wave aerial	3	16	4	1
T.R.F. receiver,	3	14	1	40
transistor	3	14	1	49
Triangular waveform	**1**	**5**	**1**	**2**
Trigatron	2	8	4	19
Trimmer capacitors	**1**	**4**	**2**	**22**
Triode,	2	8	2	1
cold-cathode	2	8	4	19
grounded-grid	2	11	1	21
hot-cathode gas-filled	2	8	4	11
inter-electrode capacitances of	2	8	2	37
Triode hexode—	2	8	3	33
—operation	3	14	2	37
Tropospheric propagation	3	17	1	25
True power	**1**	**5**	**2**	**36**
Tuned anode oscillator	2	12	1	12
Tuned anode-crystal grid oscillator	2	12	2	11
Tuned anode-tuned grid oscillator	2	12	1	40
Tuned circuit, parallel	**1**	**5**	**3**	**7**
series	**1**	**5**	**2**	**42**
Tuned grid oscillator	2	12	1	14
Tuned voltage amplifiers	2	11	1	1
Tungsten emitter	2	8	1	11
Tuning, aerial	3	16	2	22
superhet receiver	3	14	2	7
t.r.f. receiver	3	14	1	7
Tuning indicators	3	14	2	68
Turnstile aerial	3	16	5	18
Twin wire feeder	3	15	3	34
Two-phase generator	**1**	**5**	**4**	**4**
Two-point tracking	3	14	2	21

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U.H.F.—aerials	3	16	5	16
—power measurements	3	18	5	8
Unipole aerial	3	16	5	8

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	BOOK	SECT.	CHAP.	PARA.
Valency electron	2	8	7	3
Value, average	**1**	**5**	**1**	**3**
r.m.s.	**1**	**5**	**1**	**3**
Valve, beam tetrode	2	8	3	13
cold-cathode diode	2	8	4	16
cold-cathode triode	2	8	4	19
critical distance tetrode	2	8	3	11
diode	2	8	1	18
gas-filled	2	8	4	2
heptode	2	8	3	30
hexode	2	8	3	31
hot-cathode gas-filled diode	2	8	4	4
hot-cathode gas-filled triode	2	8	4	11
multi-unit	2	8	3	34
octode	2	8	3	32
pentode	2	8	3	17
soft	2	8	4	1
tetrode	2	8	3	2
thyratron	2	8	4	11
triode	2	8	2	1
triode-hexode	2	8	3	33
variable-mu pentode	2	8	3	24
Valve—constants	2	8	2	11
—noise	2	8	3	46
—voltmeter	3	18	1	4
Valves—in parallel	2	10	2	34
—in push-pull	2	10	2	16
Variable—capacitor	**1**	**4**	**2**	**16**
—frequency oscillator	2	12	1	32
—mu pentode	2	8	3	24
—resistor	**1**	**1**	**3**	**10**
Variac	**1**	**7**	**2**	**32**
Vectors,	**1**	**5**	**1**	**13**
resolution of	**1**	**5**	**1**	**23**
Velocity,	**1**	**1**	**1**	**2**
angular	**1**	**5**	**1**	**9**
Velocity—feedback damping	3	19	2	26
—lag	3	19	2	32
—of propagation (line)	3	15	3	26
—servos	3	19	2	32
Velodyne integrator	3	20	1	50
Vertical polar diagrams	3	16	2	18
Vertically polarized waves	3	16	1	14
V.H.F.—aerials	3	16	5	16
—oscillators	2	12	1	33
Vibrator	2	9	2	11
Video frequency amplifier	2	10	1	24
Viscous damping	3	19	2	23
Volt	**1**	**1**	**1**	**28**
Voltage, striking	2	8	4	16
Voltage—amplifiers, a.f.	2	10	1	1
r.f.	2	11	1	1
—doubler	2	9	3	20
—measurement	3	18	1	3
—negative feedback	2	10	3	13
—stabilizer	2	9	3	22

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Voltmeter,	1	6	1	1
electrostatic	1	6	1	26
Volume control	3	14	1	11
W				
Ward-Leonard system	3	19	2	6
Watt	1	1	1	33
Wattless current	1	5	2	34
Wattmeter	1	6	3	2
Waveform, distortion of	1	7	2	26
types of	1	5	1	2
Wavelength (and frequency)	3	13	1	2
Wavemeters	3	18	1	8
Waves, electromagnetic	3	16	1	9
standing (on line)	3	15	3	8
Weber	1	2	1	7
Wheatstone's bridge—	1	1	2	27
—remote indicator	3	19	1	17
Wide-band—amplifiers	2	10	1	2
—r.f. transformers	1	7	1	17
Wien bridge oscillator	2	12	3	6
Wireless communication	3	14	1	1
Wire-wound resistor	1	1	3	7
Word length (computer)	3	20	2	22
Work	1	1	1	3
Work function	2	8	1	5
Wound induction motor	1	5	4	36
Y				
Yagi aerial array	3	16	3	4
Z				
Zeppelin aerial	3	16	5	10