Pittman®
Servo Motor Application Notes

I. Basic Motor Operation

Permanent magnet, direct current servo motors convert electrical energy into mechanical energy through the interaction of two magnetic fields. One field is produced by a permanent magnet assembly; the other field is produced by an electrical current flowing in the motor windings. These two fields result in a torque which tends to rotate the rotor. As the rotor turns, the current in the windings is commutated to produce a continuous torque output.

Fig. 1 depicts a basic d-c motor model. The back emf, V, is an induced voltage produced by the relative motion between the permanent magnet field and the winding coils. The input voltage and current, E and I, represent the input power; the torque and speed, T and ω, represent the output power.

A simple circuit analysis of Fig. 1 yields the following basic motor equation:

Eq. (1) $$E = I \times R_T + V + L\,\frac{dI}{dt} \qquad (1)$$

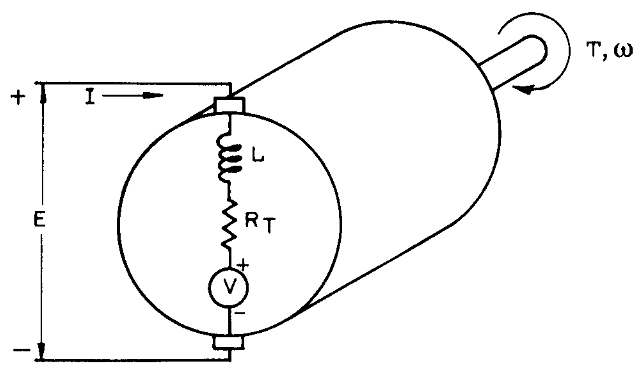


Fig. 1 Basic Motor Model

E =	applied voltage	V =	back emf voltage
I =	motor current	T =	output torque
L =	winding inductance	ω =	output speed
R_T =	resistance		

There are two important motor constants resulting from the coils of wire residing in the magnetic field produced by the magnets. The first is the motor back emf constant, K_E, which is a measure of the voltage per unit speed generated when the rotor is turning. The magnitude and polarity of K_E are functions of the shaft angular velocity, ω, and direction of rotation respectively. The back emf voltage can be expressed as the product of $K_E \times \omega$.

The second constant is the motor torque constant, K_T, which is a measure of the torque per unit current produced by the motor. In a permanent magnet d-c motor the torque is a linear function of the motor current. The torque produced by the motor is divided into two basic components: internal torque losses, T_M, and the external load torque, T_L. The motor current can be expressed as $(T_L + T_M)/K_T$.

In many applications where the motor electrical time constant is significantly less than the mechanical time constant, the $L\frac{dI}{dt}$ term in the basic motor equation, Eq. (1), can be assumed negligible. This is usually the case in iron-core motors such as those marketed by Pittman.

Incorporating the above characteristics into Eq. (1) yields the following form of the basic motor equation:

Eq. (2) $$E = \left(\frac{T_L + T_M}{K_T} \times R_T\right) + \left(K_E \times \omega\right) \qquad (2)$$

When a step voltage is applied to a motor at rest there is an initial inrush current limited only by the circuit impedance since the back emf voltage is zero. This inrush current produces a large torque which begins to accelerate the motor and the connected load.

As the angular velocity increases, the back emf voltage increases and begins to limit the motor current. The steady state speed of the motor will be that speed at which the generated back emf voltage limits the current to a value that produces a torque equal to the sum of the load and internal motor torques $\left(\text{Current} = \frac{V}{R} = \frac{E - K_E \times \omega}{R_T} = \frac{T_L + T_M}{K_T}\right)$. Any changes in the applied voltage or the motor load will alter this balance and result in speed changes which tend to restore the balance.

II. Motor Performance Curves

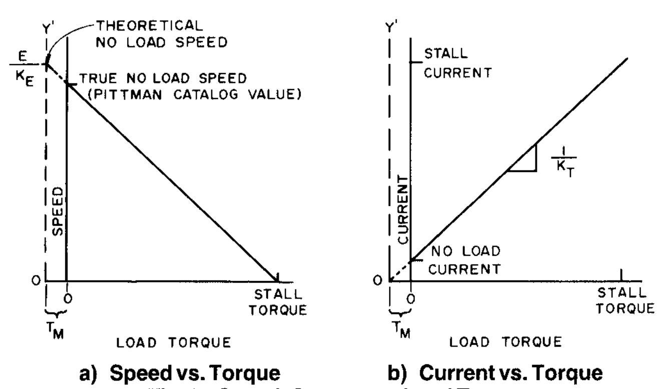
The most commonly used motor performance curves are speed, current, power, and efficiency all shown as functions of the load torque.

Both speed and current are linear functions of the load torque, T_L, as shown in Eq. (3) and Eq. (4). Both equations have the linear form $y = mx + b$ with the load torque being the independent parameter, and the current and speed being the dependent parameters.

Eq. (3) Current: $$I = \frac{1}{K_T} \times T_L + \frac{T_M}{K_T} \qquad (3)$$

Eq. (4) Speed: $$\omega = \left(\frac{-R_T}{K_T \times K_E}\right) \times T_L + \left(\frac{E}{K_E} - \frac{R_T}{K_T \times K_E} \times T_M\right) \qquad (4)$$

Fig. 2 shows characteristic performance curves for both speed and current. The projections back to the y′ axes indicate the theoretical no load values in an ideal motor which has no internal torque losses. The construction of the curves is a simple process. The no load and stall points on both graphs are connected by a straight line. The motor will operate along this line as the load torque varies.



a) Speed vs. Torque **b) Current vs. Torque**

Fig. 2. Speed, Current vs. Load Torque

Eq. (4) also indicates that the speed of the motor is a function of the applied voltage. Both no load speed and stall torque are proportional to the applied voltage (assuming T_M is small). A motor can then be operated anywhere in the first quadrant of the speed-torque plane by varying the applied voltage. This is demonstrated in Fig. 3. Pittman catalog values for no load speed and stall torque are referenced to the nominal winding voltages listed in the catalog. The current vs. torque curve is independent of the applied voltage.

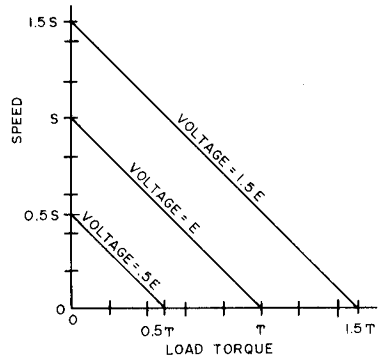
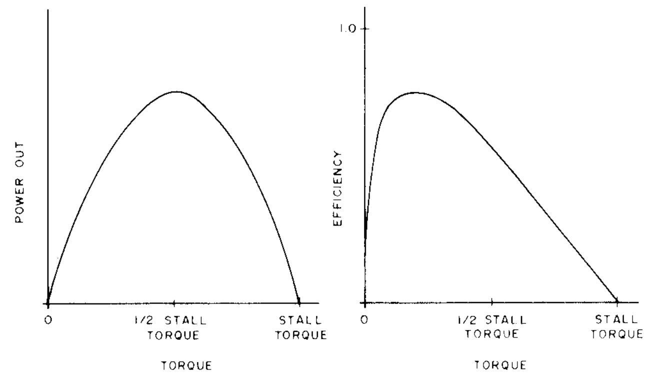


Fig. 3 Speed as a function of Voltage

Characteristic curves for power out and efficiency are shown in Fig. 4. Power out is the product of speed and torque. Input power is the product of the applied voltage and motor current. Efficiency is the ratio Power Out/Power In.



a) Power Out **b) Efficiency**

Fig.4 Power Out and Efficiency vs. Load Torque

III. Basic Motor Parameters and Tolerances

There are several fundamental motor parameters which define the motor's operating characteristics. These are listed below. The tolerances shown are standard manufacturing tolerances. Tighter tolerances for certain parameters are available upon request.

Parameter	Typical Symbols	Tolerance	Typical Units
Torque Constant	K_T, TPA	±15%	$\frac{oz \cdot in}{A}$ $\frac{N \cdot m}{A}$
Back emf Constant	K_E, BEF	±15%	$\frac{volts}{1000\ rpm}$, $\frac{volts}{rad/s}$
Terminal Resistance	R_T, RTR	±15%	ohms
Inductance	L, DUK	±10%	millihenries
Inertia	J, ERT	±10%	$oz \cdot in \cdot s^2$, $kg \cdot m^2$, $N \cdot m \cdot s^2$
Motor Torque losses	T_M	+30%	oz·in, N·m
Motor Friction	T_F, TOF	+50%	oz·in, N·m
No Load Current	I_O, INL	+30%	amperes

Pittman catalog values for terminal resistance are scaled from accumulated test data on a winding which is in the middle of the range of windings offered. This value also includes a typical brush resistance. Windings at the extremes of the range offered may not exactly conform to the scaled values (high voltage windings may have slightly lower resistance; low voltage windings may have slightly higher resistance). Contact your Pittman representative if exact values are required.

Motor torque losses are generally specified as static (breakaway) torques or dynamic (running) torques. Breakaway torque is a function of cogging (changes in magnetic circuit reluctance), brush friction, and bearing friction. These are affected by bearing type and preload, brush material and force, air gap flux density, and the magnetic circuit configuration. The Pittman catalog value for friction torque is a typical composite value. Maximum breakaway torque will depend on the motor configuration and may be 1.5 times the catalog motor friction value.

Dynamic torque losses are caused by magnetic hysteresis, eddy currents, windage, brush friction, and bearing losses. These are effected by motor speed, bearing type and preload, brush type and force, air gap flux densities, and the magnetic circuit materials.

These torque losses are generally expressed in terms of the motor no load current, INL (INL = T_M/K_T). The Pittman catalog value for no load current is a composite value. Maximum no load current may be 1.3 times the catalog value.

Several of the more commonly used derived motor parameters and their standard manufacturing tolerances are:

Parameter	Symbols	Tolerance	Derivation	Typical Units
Stall Torque	T_P, TPK	Reference	$\frac{E \times K_T}{R_T} - T_F$	oz·in, N·m
No Load Speed	So, SNL, ω_o, ωNL	±15%	$\frac{E - INL \times R_T}{K_E}$	rpm, rad/s
Stall Current	I_P, AMP	±15%	E/R_T	amperes
Motor Constant	K_M, PKO	Reference	$K_T/\sqrt{R_T}$	$\frac{oz \cdot in}{\sqrt{W}}$, $\frac{N \cdot m}{\sqrt{W}}$
Damping Constant (zero source impedance)	K_D, DPO	Reference	$\frac{K_T \times K_E}{R_T}$	oz·in/(rad/s), N·m/(rad/s)
Electrical Time Constant	τ_E, TCE	Reference	$\frac{L}{R_T}$	ms
Mechanical Time Constant	τ_M, TCM	Reference	$\frac{J \times R_T}{K_T \times K_E}$ $\frac{J}{K_D}$	ms

The motor damping constant and stall torque are functions of the total circuit impedance. When solid state drive circuits are used the dynamic resistance of the solid state devices must be included when determining damping and stall torque.

The Basic thermal parameters are:

Thermal Impedance	R_{TH}, TPR, θ_R	°C/W
Thermal Time Constant	τ_{TH}, TCT	min
Maximum Winding Temp.	TMX, θ_{MX}	°C

The Pittman catalog valves for R_{TH} and τ_{TH} are empirically derived with the rotor at stall, in free air, and without heat sinking. These conditions yield worst case values. Actual values will depend on the speed of rotation, heat sinking, and air flow over the motor.

IV. Motor Selection & Operating Considerations

Pittman® servo motors can be operated over a wide range of voltages, speeds, and loads. The major consideration in motor frame size selection is the rms load torque since the dominant portion of motor losses is usually the winding I^2R losses.

The first step in motor selection is to choose the motor type and frame size which is capable of producing the required load torque. In general the continuous torque capability of the various motor types and sizes can be calculated using Eq. (5).

Eq. (5): **(5)**

$$T_{CONT} = \sqrt{\frac{(155 - T_{amb})}{TPR} - \frac{T_M \times S}{C}} \times K \times PKO - T_M$$

where:

T_{CONT} = continuous load torque capability
155 = maximum winding temperature (155 °C)
T_{amb} = ambient temperature (°C)
TPR = motor thermal impedance (°C/W)
S = motor speed (rev/min)
C = 1352 for T_M = oz·in
9.549 for T_M = N·m
PKO = motor constant
T_M = motor friction torque
(The product of no load current and torque constant for any given winding.)
K = 0.71 for brush commutated, ferrite magnet motors
0.78 for brush commutated, rare earth magnet motors
0.79 for brushless, rare earth magnet motors
0.60 for brushless, ferrite magnet motors

Appendix A contains safe operating area curves derived using the above formula for most Pittman motor models.

After a frame size has been selected the proper winding needs to be specified. This is done by calculating the required torque constant for the selected frame size and specified load using Eq. (6).

Eq. (6): $$K_T = \frac{E}{\frac{T_L + T_M}{(PKO)^2} + \frac{S}{K}} \qquad (6)$$

where:

K_T = required torque constant
E = supply voltage
T_L = load torque
T_M = motor friction
PKO = motor constant
S = load speed
K = 1352 for T_L = oz·in, S = rpm, PKO = oz·in/$\sqrt{W}$
9.549 for T_L = N·m, S = rpm, PKO = N·m/$\sqrt{W}$
1.000 for T_L = N·m, S = rad/s, PKO = N·m/$\sqrt{W}$

The winding with a torque constant closest to the value calculated by Eq. (6) is chosen. The catalog does not represent an exhaustive winding list. Consult your Pittman representative for assistance if needed. The winding choice can be checked by inserting the load values into Eq. (2) and verifying that the calculated required voltage is consistent with your supply voltage. Use the K_T, R_T, and K_E values for the winding you have chosen. WHEN PERFORMING THE ABOVE CALCULATIONS BE CERTAIN TO CONSISTENTLY USE UNITS WHICH RATIONALIZE.

When choosing a gearmotor the same procedure is followed using a load speed and torque translated back through the gearbox to the motor shaft as shown in equations (7) and (8). Gear ratios and efficiencies are listed in the Pittman gearmotor catalog.

Eq. (7) $$S_{Motor} = S_{Output} \times \text{Ratio} \qquad (7)$$

E. (8) $$T_{Motor} = \frac{T_{Output}}{\text{Ratio} \times \text{Efficiency}} \qquad (8)$$

A gear ratio should be chosen that will result in a translated motor speed and torque consistent with the motor's capability.

V. Thermal Considerations

Power losses in the motor are dissipated as heat which causes the motor temperature to rise. The thermal impedance (ultimate temperature rise per watt) is a measure of the winding temperature rise, relative to the ambient temperature, per watt of power dissipated in the armature.

Armature power dissipation can be closely estimated using Eq. (9).

Eq. (9): $$\text{Power Loss} = I^2 R_T + \frac{T_M \cdot \omega}{K} \qquad (9)$$

where:

I = motor current
R_T = motor resistance
T_M = internal motor losses
ω = motor speed
k = rationalizing constant
= 1352 for T_M = oz·in, ω = rpm
= 141.6 for T_M = oz·in, ω = rad/s
= 9.549 for T_M = N·m, ω = rpm
= 1.000 for T_M = N·m, ω = rad/s

The armature temperature rise can then be calculated using the relationship in Eq. (10).

Eq. (10): $$\Delta T = \text{Power Loss} \times \text{TPR} \qquad (10)$$

where: TPR = motor thermal impedance (deg/W)

Power Loss = value from Eq.(9)

The catalog value of thermal impedance is determined by the change in resistance method with the motor at stall, in free air, and without heat sinking. This yields a worst case value. Motor rotation, heat sinking, and air flow over the motor will improve the heat transfer and will result in a lower thermal impedance. The actual value of thermal impedance will depend on the characteristics of each application.

The motor resistance, torque constant, and back emf constant are functions of temperature. As the motor temperature increases each of the three parameters will change in a manner which degrades motor performance and increases the power losses. If a constant torque output is maintained the motor may reach a point of "thermal runaway" and burn out even if initial calculations showed an acceptable temperature rise (using values of R_T, K_T at ambient temperature).

A thermal runaway condition is usually encountered when the I^2R losses form a significant part of the total power losses. Fig. 5 shows the actual temperature rise as a function of the calculated temperature rise (using R_T, K_T at ambient) when a constant torque output is maintained in an application where the I^2R losses are dominant.

The actual resistance, torque constant, and back emf constant values for a given temperature change can be calculated using equations (11) and (12).

For Resistance

Eq. (11): $$R_2 = R_1 \times \frac{234.5 + T_2}{234.5 + T_1} \text{ (for copper wire)} \qquad (11)$$

Actual Temperature Rise vs. Calculated Temperature Rise Given Constant Torque Output and Assuming I^2R Losses are Dominant

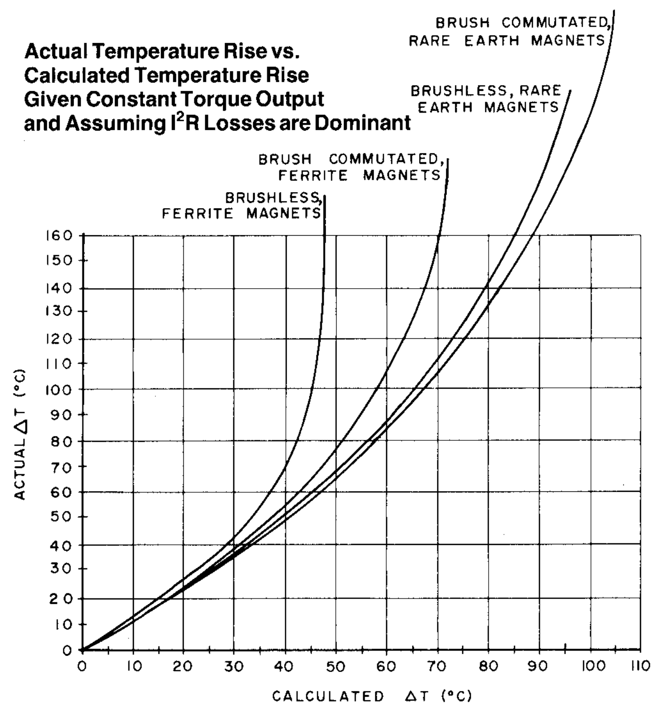


Fig. 5

For Torque/Back emf Constants

Eq. (12): $K_2 = K_1 (1 + C[T_2 - T_1])$ **(12)**

where:

R_x = resistance at a given temperature
T_x = temperature (°C)
K_x = torque/back emf constants at a given temperature
C = −0.002 for ferrite magnets
= −0.00045 for rare earth magnets (PITMO® brush motors)
= −0.00025 for rare earth magnets (ELCOM® brushless motors)

The temperature rise of the magnet may be different from the calculated temperature rise in the winding. This must be considered when using Eq. (12). Empirical data for Pittman motors shows the following:

Eq. (13): ΔT magnet = C × ΔT winding **(13)**

where:

C = 0.5 for brush commutated, ferrite magnet motors
= 0.7 for brush commutated, rare earth magnet motors
= 1.0 for brushless motors

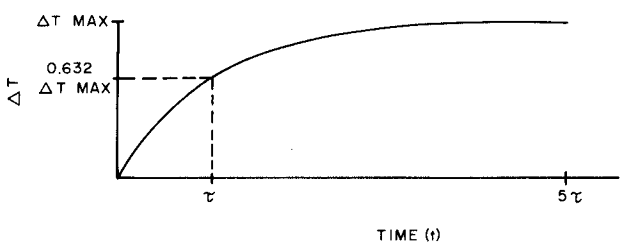


Fig. 6 Winding Temperature Rise As A Function Of Time.

Eq. (14): $\Delta T(t) = \Delta T_{max} (1 - e^{\frac{-t}{\tau}})$ **(14)**

where:

t = time τ = thermal time constant

VI. Methods of Parameter Measurement

1. Terminal Resistance

A. Brush Commutated Motors

The resistance of brush commutated motors cannot be accurately measured with a conventional ohmmeter because the low voltage and current output of such devices will not break down the normal film which is present on the commutator surface. The resistance should be measured by locking the rotor shaft, applying a d-c voltage sufficient to drive a current of several hundred milliamps through the motor, and calculating R = E/I.

The values for several different shaft angular positions should be averaged to obtain the final result.

B. Brushless Motors

Brushless motor resistance can be measured with a conventional ohmmeter by probing at the proper coil termination points.

2. Back emf Constant

A. Brush Commutated Motors

Motor back emf is determined by measuring the d-c voltage generated at the motor terminals when the shaft is driven at a constant speed (generally 1000 – 3000 rpm).

$$\frac{\text{Volts}}{1000\text{ rpm}} = \frac{\text{Volts generated} \times 1000}{\text{rpm}}$$

$$\frac{\text{Volts}}{\text{rad/s}} = \frac{\text{Volts generated} \times 9.549}{\text{rpm}}$$

B. Brushless Motors

Since the Pittman ELCOM® series brushless motors generate a sinusoidal back emf voltage, the motor back emf constant must be derived from the measured voltage, and is a function of the motor winding configuration. Table 1 is a list of conversion constants for converting the generated voltage to the desired motor parameter.

The generated voltage should be measured at the indicated coil termination points with the motor disconnected from its drive circuit.

Table 1

Chart of Conversion Constants to Derive Torque Constant and Back EMF Constant from Generated Back EMF Voltage

Generated Voltage		Derived Parameter: Torque Constant K_T (oz·in/A)	Torque Constant K_T (N·m/A)	Back emf Constant K_E (V/1000 rpm)	Back emf Constant K_E (V/(rad/s))
4-Phase ELCOM®	V_{p-p}	608.75/rpm	4.2987/rpm	450.16/rpm	0.45016/rad/s
	V_{rms}	1721.8/rpm	12.159/rpm	1273.2/rpm	1.2732/rad/s
3-Phase Full Wave or 6-Phase ELCOM®	V_{p-p}	645.67/rpm	4.5595/rpm	477.46/rpm	0.47746/rad/s
	V_{rms}	1826.2/rpm	12.896/rpm	1350.5/rpm	1.3505/rad/s

To calculate the desired parameter multiply the generated voltage by the appropriate conversion constant listed in the chart.

3. **Torque Constant**
The torque constant is a measure of the torque per unit current produced by the motor. It can easily be determined by measuring the motor current and torque at two different points and calculating:

$$\text{Torque Constant} = K_T = \frac{T_2 - T_1}{I_2 - I_1}$$

The torque constant is also related to the back emf constant and can be determined by using the following relationships:

$$K_T\left(\frac{oz \cdot in}{A}\right) = \frac{volts}{1000\,rpm} \times 1.352 = \frac{volts}{rad/s} \times 141.6$$

$$K_T\left(\frac{N \cdot m}{A}\right) = \frac{volts}{1000\,rpm} \times 9.549 \times 10^{-3} = \frac{volts}{rad/s} \times 1.000$$

Table 1 also contains the conversion constants necessary to convert the brushless motor sinusoidal voltage measurements to the torque constant value.

4. **Starting Friction**
Starting friction is most accurately measured by using a constant current source to determine the maximum current required to establish continuous rotation and multiplying the result by the motor torque constant ($T = I \times K_T$). When measuring brushless motor starting friction the current supplied to the switching electronics (i.e. transistor base drive) must be subtracted from the measured current to obtain a correct result.

5. **Dynamic Friction**
Dynamic friction can be determined by multiplying the no load current and the torque constant at a given speed. Dynamic friction will contain a speed dependent component and a constant component. The switching electronics drive current for brushless motors should be subtracted as mentioned under STARTING FRICTION.

6. **Inductance**
The motor inductance is measured at 1 kH_Z with an inductance bridge by probing at the motor terminals or coil termination points. The values for several different shaft angular positions should be averaged to obtain the final result.

7. **Stall Torque**
The stall torque is determined by fitting a straight line through several speed and torque data points taken between no load and 3/4 stall. The x-intercept of the fitted line is the stall torque. Under certain conditions and with certain drive circuits the speed torque curve may deviate from a straight line and droop near the stall point producing a true stall torque less than the projected stall torque. This only needs to be considered if the motor is to be operated near its stall point. However, operation in the stall region is generally not recommended.

VII. Pitmo® Brush Commutated D-C Servo Motor Characteristics

1. Motor torque is a linear function of motor current.
2. Motor speed is a linear function of load torque when operated at a constant voltage.
3. The no load speed and stall torque are directly proportional to the applied voltage.
4. The motor direction of rotation is reversible by reversing the power supply polarity.
5. The motors are capable of operating over a wide range of voltage, speed, and torque.
6. Either ferrite or rare earth cobalt magnets are used. These magnets are not easily demagnetized and may be subjected to open circuit conditions, high current pulses, and plug reversing at rated voltage without suffering demagnetization.

VIII. ELCOM® Brushless D-C Servo Motor Characteristics

1. Motor torque is a linear function of motor current.
2. Motor speed is a linear function of load torque when operated at a constant voltage. (This may be affected in the stall region by drive electronics).
3. The no load speed and stall torque are directly proportional to the applied voltage. (This may be affected in the stall region by drive electronics).
4. The motor direction of rotation is reversible by a logic signal control of an H-bridge drive.
5. The motors are capable of being operated over a wide range of voltage, speed, and torque.
6. The motors exhibit extremely low friction and magnetic cogging.
7. The motors have very low thermal impedance and high power capability.
8. Long operational lifetimes are achieved since no brushes are used for current commutation. The limiting factor on motor life is the ball bearings in the motor.
9. Motor generated EMI is much less than that generated in brush commutated motors.

Application Examples

A. Continuous Duty, Single Point Load

1. The following operating conditions are defined:

Load Torque (T_L)	:	3 oz·in
Speed (S)	:	3000 rpm
Voltage (E)	:	12 VDC
Ambient Temp (T_{amb})	:	25 °C

2. A choice of the Model # 9433 frame size is made based on the continuous torque capability of that motor Eq. (5) (reference catalog Bulletin 9030, Motor Size Constants and Winding Constants for 9433, 9533).

   Motor Constant (catalog item 2), PKO = 2.66 oz·in/$\sqrt{\text{watt}}$
   Thermal Impedance (catalog item 14), TPR = 19.1 °C/W
   Motor Friction (catalog item 18 × item 22), T_M = 0.67 oz·in

   Rationalization Constant (see Eq. (5)), C = 1352
   Thermal Derating Factor (see Eq. (5)), K = 0.71

   The continuous torque capability of the Model 9433 motor is found by using Eq. (5) and the above data.

$$T_{CONT} = \sqrt{\frac{155 - T_{amb}}{TPR} - \frac{T_M \times S}{C}} \times PKO \times K - T_M$$

$$= \sqrt{\frac{155-25}{19.1} - \frac{0.67 \times 3000}{1352}} \times 2.66 \times 0.71 - 0.67$$

$T_{CONT} = 3.7$ oz·in

Since the load torque (3 oz · in) is less than the continuous torque capability of the motor (3.7 oz · in) the choice is valid. Actually, the calculation of continuous torque capability is quite conservative since the thermal impedance (TPR) value used was the catalog worst case value. The same choice could have been made by using the safe operating area curves in Appendix A.

3. A winding choice is made by calculating the desired torque constant for the Model #9433 frame size using Eq. (6) and the above data.

$$K_T = \frac{E}{\frac{T_L + T_M}{PKO^2} + \frac{S}{K}} = \frac{12}{\frac{3 + 0.67}{2.66^2} + \frac{3000}{1352}}$$

$K_T = 4.38$ oz · in/A

   Winding #2 is chosen because the torque constant for that winding (catalog item 18, K_T = 4.20 oz · in/A) is closest to the calculated value.

4. The motor choice is checked by calculating the required voltage for the given load using Eq. (2).

$$E = \frac{T_L + T_M}{K_T} \times R_T + K_E \times \omega$$

T_M = motor losses = INL × K_T
= 0.159 × 4.20 = 0.67 oz · in

T_L = load torque = 3.0 oz · in

K_T = torque constant = 4.20 oz · in/A

R_T = terminal resistance = 2.48 ohms

K_E = back emf constant
$= 0.0297 \frac{V}{rad/s} = 3.11 \frac{V}{1000\,rpm}$

ω = motor speed
= 3000 rpm = 314.2 rad/s

$$E = \frac{3.0 + 0.67}{4.20} \times 2.48 + 3.11 \times \frac{3000}{1000} = 11.5 \text{ Volts}$$

This is consistent with the supply voltage of 12 VDC. The nominal motor current will be

$$I = \frac{T_L + T_M}{K_T}$$

$$I = \frac{3.0 + 0.67}{4.20} = 0.874 \text{ A}$$

5. Power and efficiency calculations.

Power input = E × I = 12 × 0.874 = 10.5W

$$\text{Power output} = \frac{T \times S}{1352} = \frac{3 \times 3000}{1352} = 6.7W$$

$$\text{Efficiency} = \frac{P_{OUT}}{P_{IN}} = 6.7/10.5 = 0.64$$

6. Thermal Considerations

   Estimating the ultimate temperature rise of the winding can be an involved process. For example, when the continuous torque capability of the motor in this example was calculated the catalog value of thermal impedance (19.1 °C/W) was used. Depending on the application, the thermal impedance may be in the range of 9 to 19 °C/W. Also, the change in resistance and torque constant due to the temperature rise will increase the power losses in the motor.

   If an estimate of the ultimate temperature rise is needed the following procedure should yield an acceptable result. Refer to the previous motor parameters and operating conditions.

   a) Calculate the $T_M \times \omega$ power losses [last term in Eq. (9)].

$$P_{L_1} = \frac{T_M \times \omega}{K} = \frac{0.67 \times 3000}{1352} = 1.49W$$

   b) Calculate the temperature rise due to the above $T_M \times \omega$ losses using Eq. (10). For this example assume an actual thermal impedance of 14 °C/W.

ΔT_1 = power loss × TPR = P_{L1} × TPR

$\Delta T_1 = 1.49 \times 14 = 20.9$°C

   c) Calculate the resistance, R_T, and torque constant, K_T, values at ΔT, using Eq. (11), Eq. (12) and Eq. (13).

$$\text{Eq. (11)}: R_{T_1} = R_T \frac{234.5 + T_2}{234.5 + T_1} = \frac{2.48\,(234.5 + (25 + 20.9))}{234.5 + 25} = 2.68\Omega$$

$$\text{Eq. (13)}: \Delta T_{magnet} = C \times \Delta T_{wdg} = 0.5 \times 20.9 = 10.45\,°C$$

$$\text{Eq. (12)}: K_{T_1} = K_T(1 + C \times \Delta T_{mag}) = 4.20(1 - 0.002 \times 10.45) = 4.11 \text{ oz·in/A}$$

   d) Calculate the I^2R losses using R_{T_1} and K_{T_1} from step "c".

$$I^2R = \left(\frac{T_L + T_M}{K_{T_1}}\right)^2 \times R_{T_1} = \left(\frac{3 + 0.67}{4.11}\right)^2 \times 2.68 = 2.14W$$

   e) Calculate the temperature rise due to the I^2R losses.

$$\Delta T_2 = I^2R \times TPR = 2.14 \times 14 = 30\,°C$$

f) Use Fig. 5 to find the actual ΔT due to I^2R losses based on the value calculated in "e".

$$\Delta T_2 = 30°C$$
using Fig. 5, actual $\Delta T = \Delta T_3 = 37°C$

g) Find the estimated ultimate ΔT by adding ΔT_1, and ΔT_3.

$$\Delta T_{ult} = \Delta T_1 + \Delta T_3 = 20.9 + 37$$

$$\Delta T_{ult} = 57.9°C$$

The final values R_T, K_T, K_E, voltage, and current can now be calculated using Eq. (11) and Eq. (12).

$$R_{T_f} = 2.48 \times \left(\frac{234.5 + 25 + 57.9}{234.5 + 25}\right) = 3.03\Omega$$

$$K_{T_f} = 4.20 \times \left(1 - 0.002 \times \frac{57.9}{2}\right) = 3.96 \text{ oz} \cdot \text{in/A}$$

$$K_{E_f} = 3.11 \times \left(1 - 0.002 \times \frac{57.9}{2}\right) = 2.93 \text{ V/1000 rpm}$$

$$V = \frac{3 + 0.67}{3.96} 3.03 + \frac{2.93 \times 3000}{1000} = 11.6 \text{ Volts}$$

$$I = \frac{3 + 0.67}{3.96} = 0.93\text{A}$$

B. Incremental Motion Example

A certain application has the following characteristics and requirements:

Load friction torque (T_L)	= 0.10 N·m
Load inertia (J_L)	= 40×10^{-6} kg·m^2
Power supply (E)	= 24 VDC
Ambient temp (T_{amb})	= 25 °C

The load will be driven under closed loop control to obtain the following velocity profile:

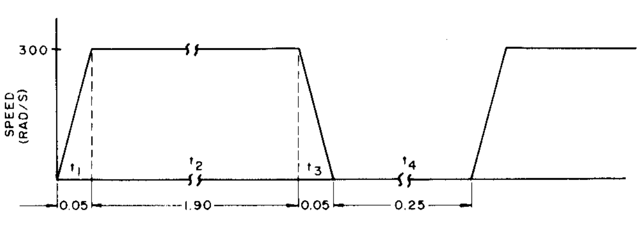


Fig. 7

Motor selection for incremental motion applications is not as straight forward as single point applications since the motor inertia must be known before calculating the rms torque. Experience would lead one to consider a motor in the 2-2⅛ inch diameter range for this example.

An initial selection of a Model #14203 is made.

Motor constant (PKO) = 55.6×10^{-3} N·m/$\sqrt{W}$
Motor inertia (J_M) = 21.2×10^{-6} kg·m^2
Motor friction (T_M) = 10.9×10^{-3} N·m
Thermal Impedance (TPR) = 8.1°C/W

T_M is found by taking the product of the torque constant and no load current for any winding.

1. The acceleration for periods t_1, t_3 is calculated.

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{300}{0.05} = 6000 \text{ rad/s}^2$$

2. The required torque for each time period is calculated.

a) Period t_1

Torque = (Inertia) (Accel) + Friction

$$T_1 = (J_L + J_M)\alpha + T_L + T_M$$

$$T_1 = (40 \times 10^{-6} + 21.2 \times 10^{-6}) \times 6000 + 0.10 + 10.9 \times 10^{-3}$$

$$T_1 = 0.478 \text{ N·m}$$

b) Period t_2

Torque = Friction

$$T_2 = (T_L + T_M) = 0.10 + 10.9 \times 10^{-3}$$

$$T_2 = 0.111 \text{ N·m}$$

c) Period t_3

Torque = Inertia (Accel) − Friction

$$T_3 = (J_L + J_M)\alpha - T_L - T_M$$

$$T_3 = (40 \times 10^{-6} + 21.2 \times 10^{-6}) \times 6000 - 0.10 - 10.9 \times 10^{-3}$$

$$T_3 = 0.256 \text{ N·m}$$

d) Period t_4

$$T_4 = 0$$

3. The rms torque is calculated.

$$T_{rms} = \sqrt{\frac{\Sigma(T_i^2 \times t_i)}{\Sigma(t_i)}}$$

where T_i = torque required for each period
t_i = time of each period

$$T_{rms} = \sqrt{\frac{(0.478)^2(0.05) + (0.111)^2(1.9) + (0.256)^2(0.05) + 0(0.25)}{0.05 + 1.9 + 0.05 + 0.25}}$$

$$T_{rms} = 0.130 \text{ N·m}$$

4. The required torque capability is checked against the continuous torque capability of the chosen motor Model #14203. For the speed in the $T_M \times \omega$ term use a weighted average of the speed during the cycle.

$$\omega_{avg} = \frac{\frac{300}{2}(0.05) + 300(1.9) + \frac{300}{2}(0.05)}{0.05 + 1.9 + 0.05 + 0.25} = 260 \text{ rad/s}$$

$$T_{cont} = \sqrt{\frac{155 - 25}{8.1} - 10.9 \times 10^{-3} \times 260} \times 55.6 \times 10^{-3} \times 0.71$$

$$T_{cont} = 0.144 \text{ N·m}$$

This shows that the motor choice is valid. The motor losses were not subtracted from the T_{cont} value as shown in the last term of Eq.(5). This is because those losses were included in the rms torque calculations.

5. A winding is chosen based on the speed and load combination which will require the greatest voltage. This will occur at the end of period t_1 (speed = 300 rad/s, torque = 0.481 N·m). Eq.(6) is used.

$$K_T = \frac{E}{\frac{T_L+T_M}{PKO^2} + \frac{S}{K}} = \frac{24}{\frac{0.478+10.9\times10^{-3}}{(55.6\times10^{-3})^2} + \frac{300}{1}}$$

$K_T = 52.4\times10^{-3}$ N·m/A

Winding #2 is chosen since it has a torque constant closest to the calculated value.

$K_T = 52.3\times10^{-3}$ N·m/A

$K_E = 52.3\times10^{-3}$ V/(rad/s)

$R_T = 0.877$ Ohms

6. The required current and maximum voltage is calculated for each time period.

a) Period t_1

$$I = \frac{T_1}{K_T} = \frac{0.478}{52.3\times10^{-3}} = 9.1\text{ A}$$

$$E = I\times R_T + K_E\times\omega = (9.1)(0.877) + (52.3\times10^{-3})(300)$$

$E = 23.6$ V

b) Period t_2

$$I = \frac{T_2}{K_T} = \frac{0.111}{52.3\times10^{-3}} = 2.1\text{ A}$$

$$E = I\times R_T + K_E\times\omega = (2.1)(0.877) + (52.3\times10^{-3})(300)$$

$E = 17.5$ V

c) Period t_3

$$I = \frac{T_3}{K_T} = \frac{0.256}{52.3\times10^{-3}} = 4.9\text{ A}$$

$$E = I\times R_T + K_E\times\omega = (4.9)(0.877) + (52.3\times10^{-3})(300)$$

$E = 20.0$ V

The required voltages for each period are within the power supply capability of 24 VDC.

7. Thermal Calculations
The power dissipation and resulting temperature rise for an incremental motion example is much more involved than that of a constant load example. If a motor is chosen consistent with the rms torque requirements, then the temperature rise should not present a problem. The general procedure in example A can be used to calculate the temperature rise if the various power dissipation components are calculated for each cycle and time weighted averages of the components are used to obtain the final result.

C. ELCOM® Brushless Motor Application Example

Choosing a Pittman ELCOM® brushless motor for an application is very similar to the process followed in choosing a brush commutated motor. Although the brushless motor catalog does not list No Load Speed, Stall Torque, and Rated Voltage values, note that these parameters were not used in selecting a brush commutated motor. All the basic motor parameters necessary to properly select a brushless motor are included in the catalog specifications.

1. Assume the following requirements are given for an application for which a brushless motor is desired:

Load Speed	= 500 rad/s
	= (0.5 krad/s)
Load Torque	= 300 mN·m
	= (0.300 N·m)
Supply Voltage	= 70 Volts

A 0.03 m^2 (300 cm^2) heat sink is provided.

A Pittman Darlington transistor H-bridge drive board will be used to drive the motor.

2. A quick selection of the motor frame size can be made by referring to the Safe Operating Area curves in the motor data sheets. The 3100 and 4100 frame sizes can quickly be eliminated because the 300 mN·m torque load is well beyond the specified operating range of those motors (refer to Appendix A). The 5100 frame size Safe Operating Area Curve indicates that the Model 5113 can safely operate at 300 mN·m, 500 rad/s if a heat sink of at least 0.025 m^2 is provided. This requirement is met since the application specified a 0.03 m^2 heat sink. Actually, mounting the motor to any sizeable metal surface would have provided the minimum required heat sink area.

3. The next step is to choose an appropriate winding within the desired 5113 frame size. A slight modification has to be made to the equation used for winding selection (Eq. (6), Pg. 4) to compensate for the voltage drop across the solid state electronic drive devices. The corrected equation to calculate the motor torque constant is:

$$K_T = \frac{E - V_D}{\frac{T_L+T_M}{(K_M)^2} + \frac{S}{K}}$$

Where:

K_T = required motor torque constant
E = supply voltage
V_D = transistor or MOSFET voltage drop
T_L = load torque
T_M = motor friction torque
K_M = motor constant
S = load speed
K = rationalization constant
= 1352 for oz·in, rpm units,
= 9.549 for N·m, rpm units,
= 1.000 for N·m, rad/s units.

The voltage drop, V_D, is determined by the application and the type of solid state devices used. Typical values for Pittman H-Bridge drive boards are:

Darlington Transistor, $V_D = 5.0$ v.
MOSFET Transistor, $V_D = 1.0$ v.

Motor friction torque, T_M, is a combination of the static and dynamic torque losses in the motor. For brushless motors T_M is the sum of the friction torque, T_F (Item 7 in the catalog), and the viscous losses.

Friction Torq. = T_F (Catalog Item 7)

Viscous losses = D_F (Catalog Item 6) × motor speed.

For this example the motor friction torque would be:

$$T_M = 4.0 \times 10^{-3} + (17 \times 10^{-6})(500 \text{ rad/s})$$

$$T_M = 12.5 \times 10^{-3} \text{ N·m}$$

(ref. Bulletin 5000, Items 6, 7 for Model 5113)

The motor constant is listed in the motor catalog as Item 2. For the ELCOM® Model 5113 the motor constant is 53.2×10^{-3} N·m/$\sqrt{W}$.

The rationalization constant, K, will be 1.0 in this example because the units used are N·m and rad/s.

The appropriate winding torque constant is chosen by filling these values into the above equation.

$$K_T = \frac{70-5}{\frac{0.300 + 12.5 \times 10^{-3}}{(53.2 \times 10^{-3})^2} + \frac{500}{1}}$$

$$K_T = 106 \times 10^{-3} \text{ N·m/A}$$

Winding 60T28 of Model 5113 has a torque constant of 104×10^{-3} N·m/A and should be chosen for this application since it most nearly matches the calculated value.

4. Check the motor choice and the required current.

   The current required is calculated by dividing the load and motor friction torques by the torque constant.

$$I = \frac{T_L + T_M}{K_T} = \frac{0.300 + 12.5 \times 10^{-3}}{104 \times 10^{-3}}$$

$$I = 3.0 \text{ A}$$

The required voltage is checked using Eq. (2), pg. 1, and adding the voltage drop V_D. The $T_L + T_M/K_T$ term is the current which was calculated above. The resistance, R, and the back EMF constant, K_E, are obtained from the catalog (Items 17 and 16 respectively for 5113 60t28). ω is the motor speed.

$$E = I \times R + K_E \times \omega + V_D$$

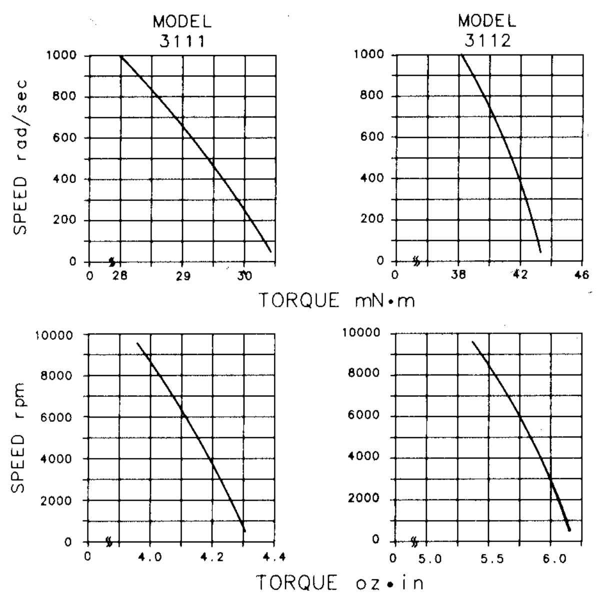
$$= 3.0 \times 3.83 + (104 \times 10^{-3}) \times 500 + 5.0$$

$$E = 68.5 \text{v.}$$

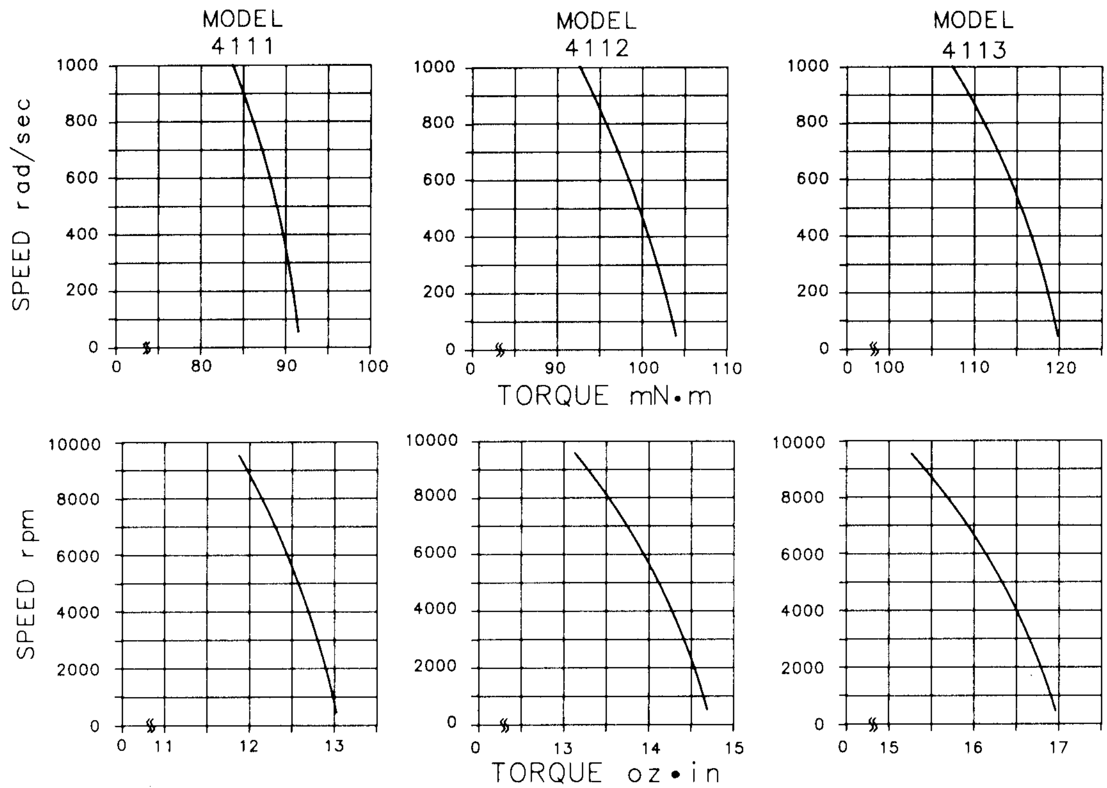
This compares favorably with the 70v supply specified.

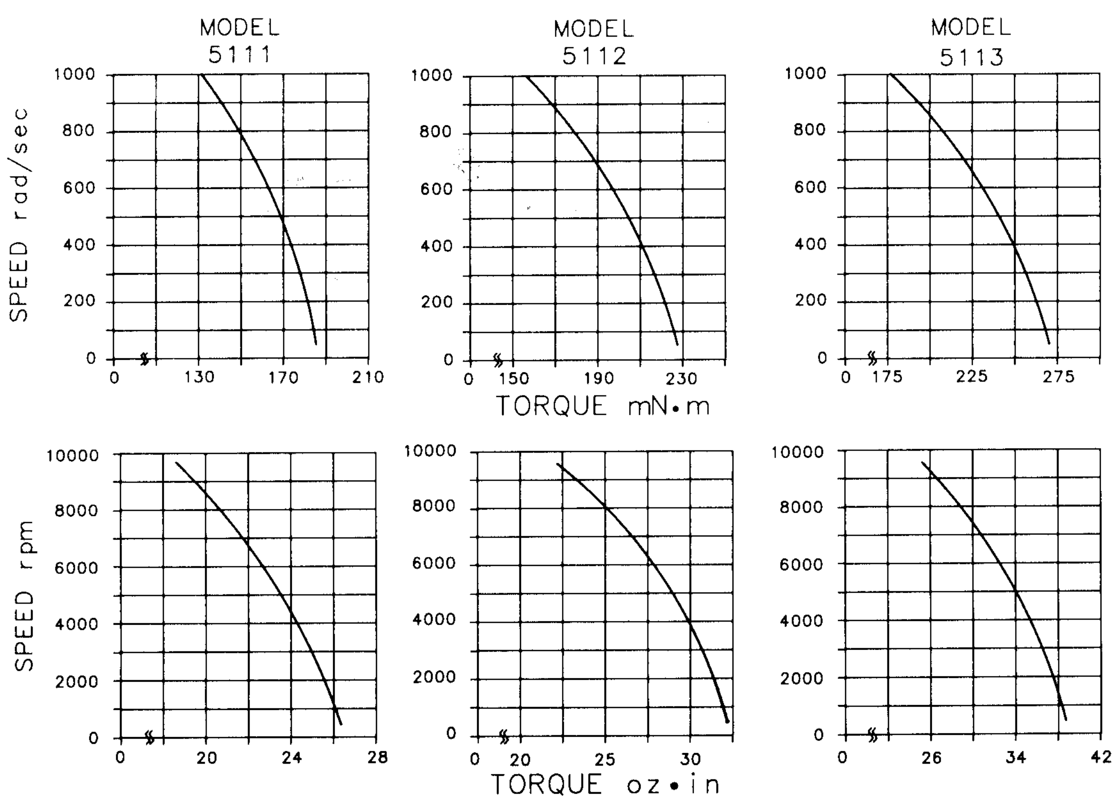
APPENDIX A
SAFE OPERATING AREA CURVES

ELCOM® BRUSHLESS MOTORS

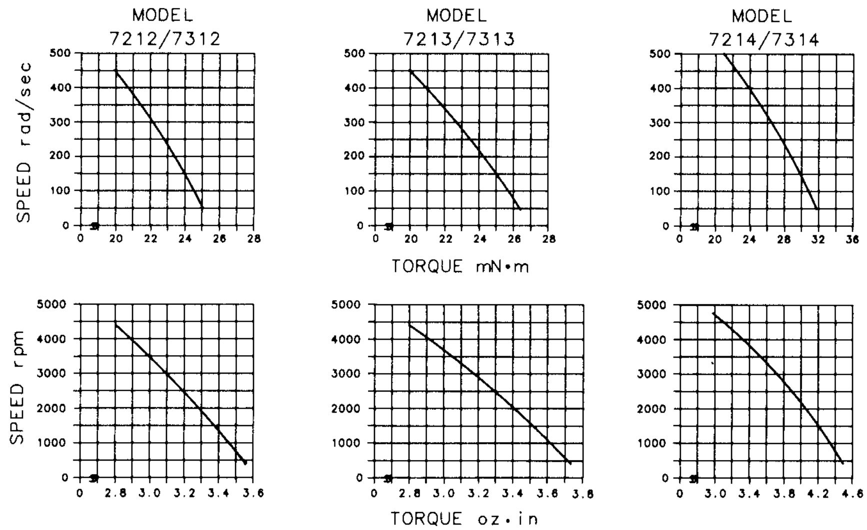


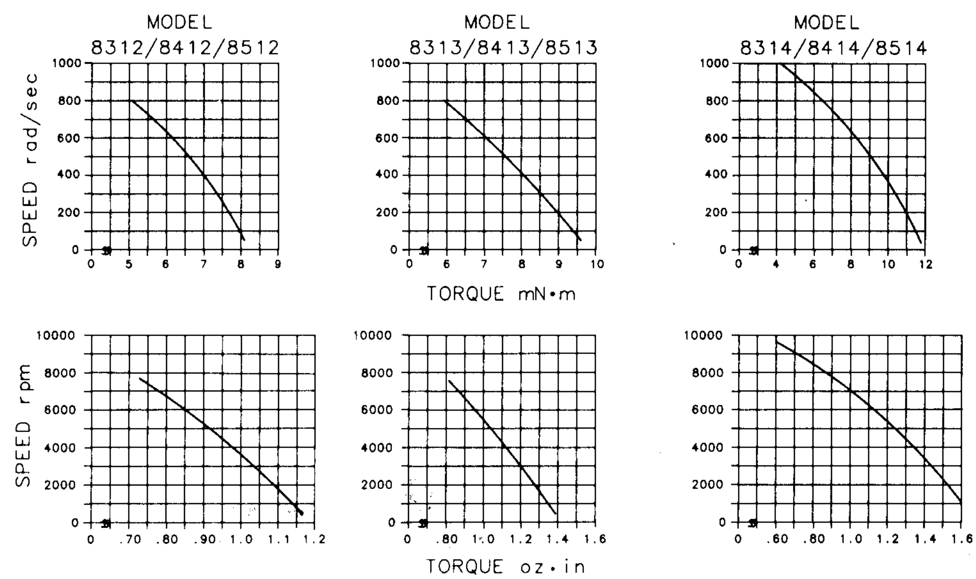
ELCOM® BRUSHLESS MOTORS

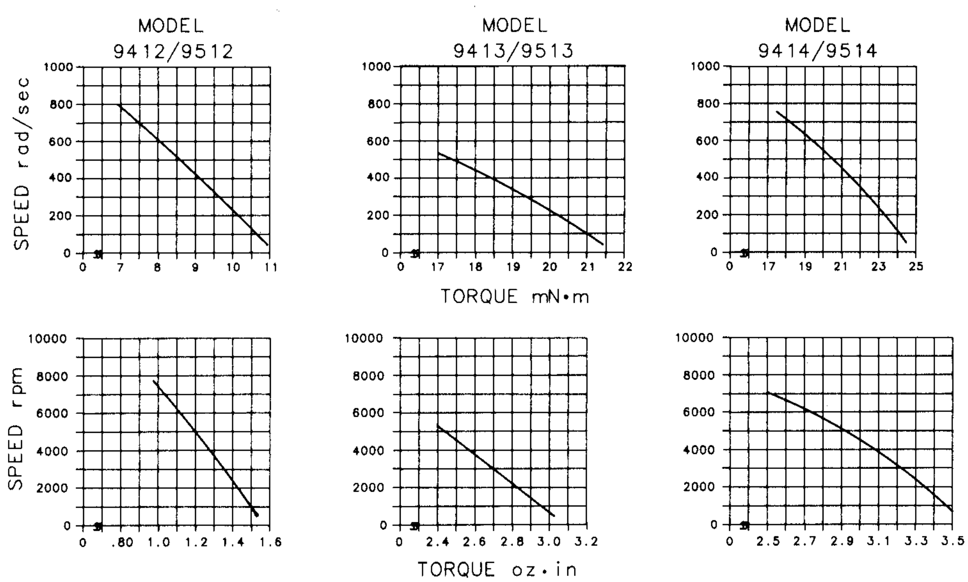




PITMO® BRUSH-COMMUTATED MOTORS







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