

SLOTTED LINE MEASUREMENTS

BASED ON THE GR 874-LBA SLOTTED LINE DOCUMENTATION

(Version 1.1, April 21, 2004)
(Originated in March 11, 2003)

SECTION 1

GENERAL DESCRIPTION

One of the important basic measuring instruments used at ultra-high frequencies is the slotted line. With it, the standing-wave pattern of the electric field in coaxial transmission line of known characteristic impedance can be accurately determined. From the knowledge of the standing-wave pattern several characteristics of the circuit connected to the load end of the slotted line can be obtained. For instance, the degree of mismatch between the load and the transmission line can be calculated from the ratio of the amplitude of the maximum of the wave to the amplitude of the minimum

of the wave. This is called the voltage standing-wave ratio, VSWR. The load impedance can be calculated from the standing wave ratio and the position of a minimum point on the line with respect to the load. The wavelength of the exciting wave can be measured by obtaining the distance between minima, preferably with a lossless load to obtain the great resolution, as successive minima or maxima are spaced by half wavelengths. The properties outlined above make the slotted line valuable for many different types of measurements on antennas, components, coaxial elements, and networks.

SECTION 2

THEORY

2.1 CHARACTERISTIC IMPEDANCE AND VELOCITY OF PROPAGATION.

A transmission line has uniformly distributed inductance and capacitance, as shown in Figure 1. The

series resistance due to conductor losses and the shunt resistance due to dielectric losses are also uniformly distributed, but they will be neglected for the present. The square root of the ratio of the inductance per unit length, L , to the capacitance per

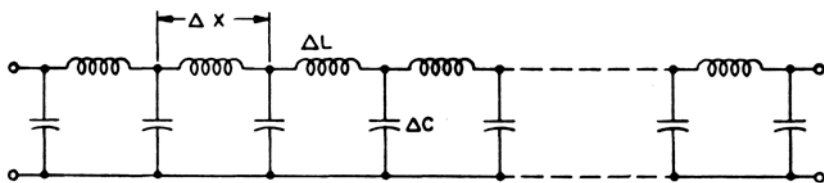


Figure 1. Circuit showing the distribution of inductance and capacitance along a transmission line.

unit length, C , is defined as the characteristic impedance, Z_0 , of the line.

$$Z_0 = \sqrt{\frac{L}{C}} \quad (1)$$

This is an approximation, which is valid when line losses are low. It gives satisfactory results for most practical applications at high frequencies.

In the next paragraph, transmission-line behavior will be discussed in terms of electromagnetic waves traveling along the line. The waves travel with a velocity, v , which depends on L and C in the following manner:

$$v = \frac{1}{\sqrt{LC}} \quad (2)$$

If the dielectric used in the line is air, (permeability unity), the product of L and C for any uniform line is always the same. The velocity is equal to the velocity of light, c , (3×10^{10} cm/sec). If the effective dielectric constant, ϵ_r , is greater than unity, the velocity of propagation will be the velocity of light divided by the square root of the effective dielectric constant.

$$v = \frac{c}{\sqrt{\epsilon_r}} \quad (3)$$

The relationship between frequency, f , and wavelength, λ , in the transmission line is

$$\lambda f = v \quad (4a)$$

$$f = \frac{v}{\lambda} \quad (4b)$$

$$\lambda = \frac{v}{f} \quad (4c)$$

If the dielectric is air (permeability is unity),

$$\lambda f = 3 \cdot 10^{10} \quad \text{cm/sec} \quad (4d)$$

if λ is in centimeters and f is in cycles per second (Hz).

2.2 TRAVELING AND STANDING WAVES.

The performance of a transmission line having a uniform characteristic impedance can be explained in terms of the behavior of the electromagnetic wave that travels along the line from the generator to the load, where all or a portion of it may be reflected with or without a change in phase, as shown in Figure 2a. The reflected wave travels in the opposite direction along the line, back toward the generator. The phases of these waves are retarded linearly 360° for each wavelength traveled.

The wave traveling from the generator is called the incident wave, and the wave traveling toward the

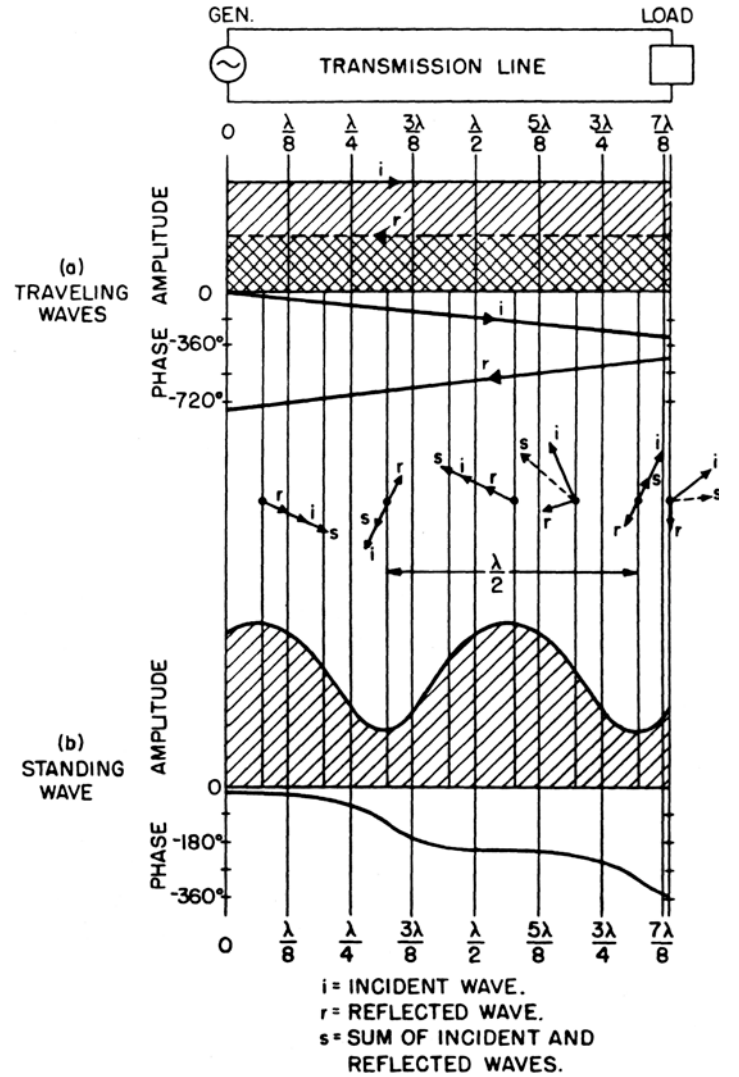


Figure 2. (a) Chart showing the variations in the amplitude and phase of incident and reflected waves along a transmission line. (b) The vector combination of the incident and reflected waves at various points along the line is illustrated and the resultant standing wave produced by the combination of the two waves is plotted.

generator is called the reflected wave. The combination of these traveling waves produces a stationary interference pattern, which is called a standing wave, as shown in Figure 2b. The maximum amplitude of the standing wave occurs when the incident and reflected waves are in phase or when they are an integral multiple of 360° out of phase. The minimum amplitude occurs when the two waves are 180° , or an odd integral multiple thereof, out of phase. The amplitude of the standing wave at other points along the line is the vector sum of incident and reflected waves. Successive minima and maxima are spaced, respectively, a half-wavelength along the line, as shown in the figure.

The magnitude and phase of the reflected wave at the load, relative to the incident wave, are functions of the load impedance. For instance, if the load impedance is the same as the characteristic impedance of the transmission line, the incident wave is totally absorbed in the load and there is no reflected wave. On the other hand, if the load is lossless, the incident wave is always completely reflected, with no change in amplitude but with a change in phase.

A traveling electromagnetic wave actually consists of two component waves: a voltage wave and a current wave. The ratio of the magnitude and phase of the incident voltage wave, E_i , to the magnitude and phase of the incident current wave, I_i , is always equal to the characteristic impedance, Z_0 . The reflected waves travel in the opposite direction from the incident waves, and consequently the ratio of the reflected voltage wave, E_r , to the reflected current wave, I_r , is $-Z_0$. Since the characteristic impedance in most cases is practically a pure resistance¹, the incident voltage and current waves are in phase with each other, and the reflected voltage and current waves are 180° out of phase.

$$\frac{E_i}{I_i} = Z_0 \quad (5a)$$

$$\frac{E_r}{I_r} = -Z_0 \quad (5b)$$

Equations (5a) and (5b) are valid at all points along the line.

The magnitude and phase of the reflected voltage wave, E_r , relative to the incident wave, E_i , at the load is called the reflection coefficient, Γ , which can be calculated from the expression

$$\Gamma = \frac{Z_x - Z_0}{Z_x + Z_0} = \frac{Y_0 - Y_x}{Y_0 + Y_x} \quad (6)$$

$$E_r = E_i \Gamma \quad \text{at the load} \quad (7b)$$

$$I_r = -I_i \Gamma \quad \text{at the load} \quad (7c)$$

where Z_x and Y_x are the complex load impedance and admittance, and Z_0 and Y_0 are the characteristic impedance and admittance of the line ($Y_0 = 1/Z_0$).

2.2 LINE IMPEDANCE.

2.2.1 VOLTAGE AND CURRENT DISTRIBUTION.

If the line is terminated in an impedance equal to the characteristic impedance of the line, there will be no reflected wave, and $\Gamma = 0$, as indicated by Equation (6). The voltage and current distributions along the line for this case are shown in Figure 3.

If the line is open-circuited at the load, the voltage wave will be completely reflected and will undergo no phase shift on reflection, as indicated by Equation (6), ($Z_x = \infty$), while the current wave will also be completely reflected but will undergo a 180° phase shift on reflection, as shown in Figure 4. If the line is short-circuited, the current and voltage roles are interchanged, and the impedance pattern is shifted $\lambda/4$ along the line. The phase shifts of the voltage and current waves on reflection always differ by 180° , as the reflected wave travels in the opposite direction from the incident wave. A current maximum, therefore, always occurs at a voltage minimum, and vice versa.

The voltage at a maximum of the standing-wave pattern is $|E_i| + |E_r|$ or $|E_i| \cdot (1 + |\Gamma|)$ and at a minimum is $|E_i| - |E_r|$ or $|E_i| \cdot (1 - |\Gamma|)$. The

where L is the inductance per unit length in henrys, C is the capacitance per unit length in farads, R is the series resistance per unit length in ohms, and G is the shunt conductance per unit length in mhos. The approximation is valid when the line losses are

low, or when $\frac{R}{L} = \frac{G}{C}$.

¹

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C} \cdot \frac{1 - j \frac{R}{\omega L}}{1 - j \frac{G}{\omega C}}} \cong \sqrt{\frac{L}{C}}$$

ratio of the maximum to minimum voltages, which is called the voltage standing wave ratio, VSWR, is

$$\text{VSWR} = \frac{E_{\max}}{E_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad (8a)$$

The standing-wave ratio is frequently expressed in decibels.

$$\text{VSWR in dB} = 20 \log_{10} \frac{E_{\max}}{E_{\min}} \quad (8b)$$

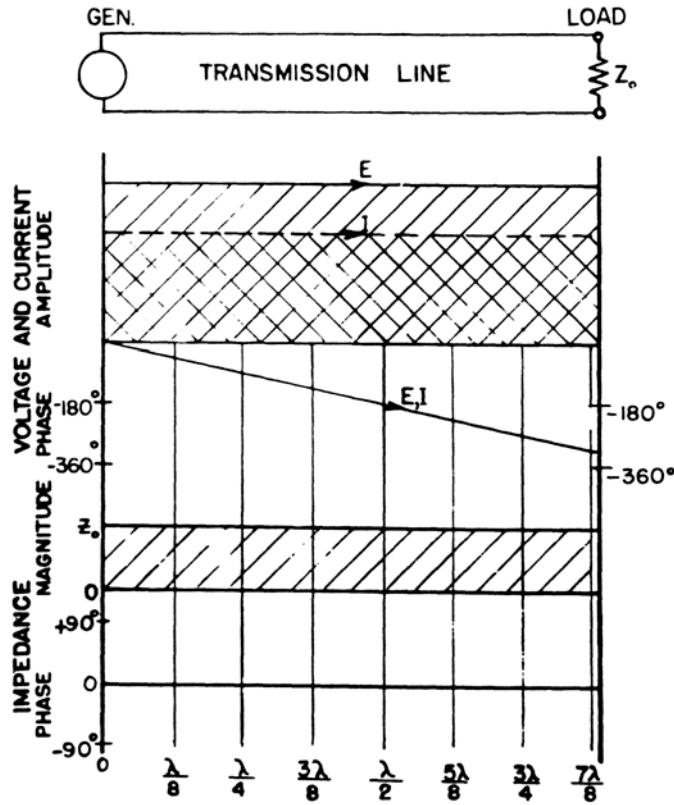


Figure 3. Chart showing voltage and current waves along a transmission line terminated in its characteristic impedance. Note the absence of reflected waves and that the impedance is constant and equal to the characteristic impedance at all points along the line.

At any point along a uniform lossless line, the impedance, Z_p , seen looking towards the load, is the ratio of the complex voltage to the complex current at that point. It varies along the line in a cyclical manner, repeating each half-wavelength of the line, as shown in Figure 4.

At a voltage maximum on the line, the incident and reflected voltage waves are in phase, and the incident and reflected current waves are 180° out of

phase with each other. Since the incident voltage and incident current waves are always in phase (assuming Z_0 is a pure resistance), the effective voltage and current at the voltage maximum are in phase and the effective impedance at that point is pure resistance. At a voltage maximum, the effective impedance is equal to the characteristic impedance multiplied by the VSWR.

$$R_{p \max} = Z_0 \cdot \text{VSWR} \quad (9a)$$

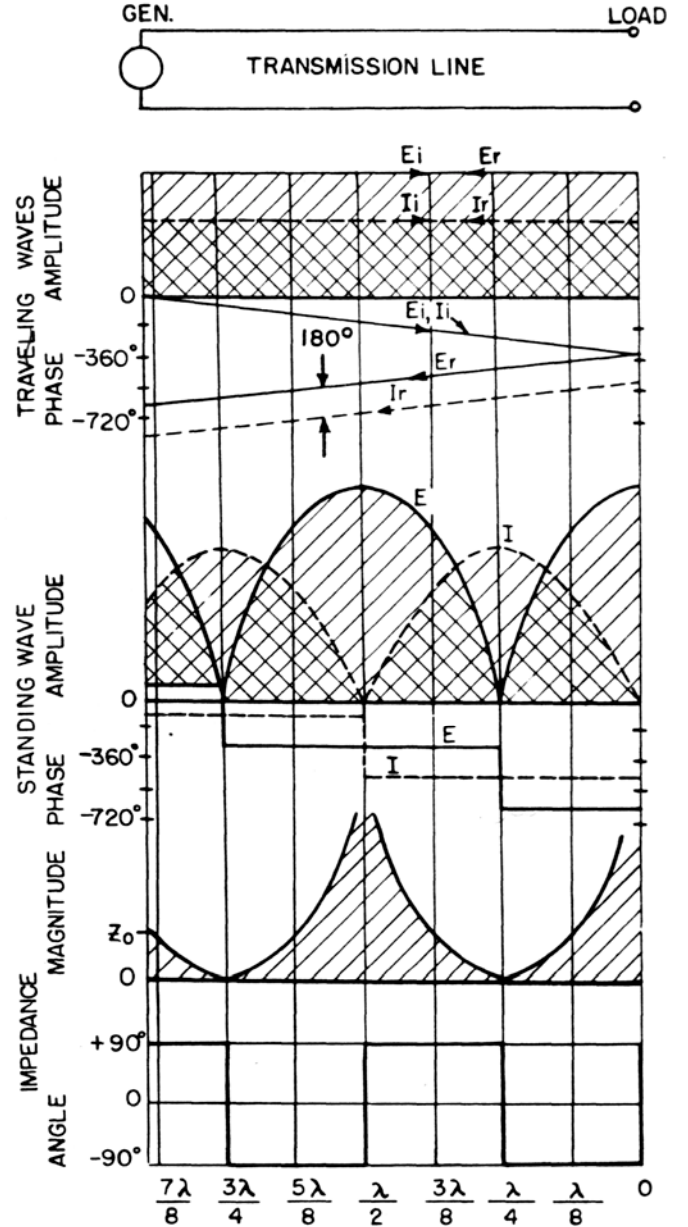


Figure 4. Chart showing voltage and current waves along a transmission line terminated in an open-circuit. Note that the minima of the voltage waves occur at the maxima of the current waves, and vice versa, and that the separation of adjacent minima for each wave is a half-wavelength. The variation in the magnitude and phase angle of impedance is also shown.

At a voltage minimum, the two voltage waves are opposing and the two current waves are aiding. Again the effective impedance is a pure resistance and is equal to the characteristic impedance of the line divided by the VSWR.

$$R_{p \min} = \frac{Z_0}{\text{VSWR}} \quad (9b)$$

The impedance, Z_p , at any point along the line is related to the load impedance by the expression

$$Z_p = Z_0 \cdot \frac{Z_x + jZ_0 \tan \theta}{Z_0 + jZ_x \tan \theta} \quad (10a)$$

$$Y_p = Y_0 \cdot \frac{Y_x + jY_0 \tan \theta}{Y_0 + jY_x \tan \theta} \quad (10b)$$

where Z_x and Y_x are the complex load impedance and admittance, Z_0 and Y_0 are the characteristic impedance and admittance of the line, and θ is the electrical length of line between the load and the point along the line at which the impedance is measured. (See Figure 5.)² The effective length, l_e , is proportional to the physical length, l , multiplied by the square root of the effective dielectric constant, ϵ_r , of the insulating material between the inner and outer conductors.

² In Figure 5, point p is shown at a voltage minimum. However, Equations (10a) and (10b) are valid for any location of point p on the line.

$$l_e = l\sqrt{\epsilon_r} \quad (11a)$$

$$\Theta = \frac{l_e}{\lambda} \text{ in wavelengths} \quad (11b)$$

$$\Theta = \frac{2\pi l}{\lambda} \sqrt{\epsilon_r} \text{ in radians} \quad (11c)$$

$$\Theta = \frac{360\pi l}{\lambda} \sqrt{\epsilon_r} \text{ in degrees} \quad (11d)$$

If l is in centimeters,

$$\Theta = 0.012 f_{\text{MHz}} l \sqrt{\epsilon_r} \text{ in degrees} \quad (11e)$$

2.3.2 DETERMINATION OF THE LOAD IMPEDANCE FROM THE IMPEDANCE AT ANOTHER POINT ON THE LINE.

The load impedance, Z_x , or admittance, Y_x , can be determined if the impedance, Z_p , at any point along a lossless line is known. The expressions relating the impedances are:

$$Z_x = Z_0 \cdot \frac{Z_p - jZ_0 \tan \theta}{Z_0 - jZ_p \tan \theta} \quad (12a)$$

$$Y_x = Y_0 \cdot \frac{Y_p - jY_0 \tan \theta}{Y_0 - jY_p \tan \theta} \quad (12b)$$

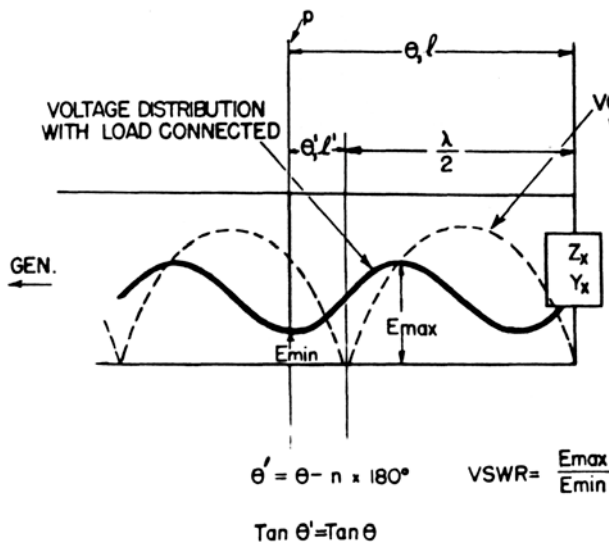


Figure 5. Voltage variation along a transmission line with a load connected and with the line short-circuited at the load.

If the line loss cannot be neglected, the equations are:

$$Z_x = Z_0 \cdot \frac{Z_p - Z_0 \tanh \gamma l}{Z_0 - Z_p \tanh \gamma l} \quad (13a)$$

$$Y_x = Y_0 \cdot \frac{Y_p - Y_0 \tanh \gamma l}{Y_0 - Y_p \tanh \gamma l} \quad (13b)$$

when $\gamma = \alpha + j\beta$, and

$$\alpha = \text{attenuation constant in nepers/m} \\ = (\text{att. in dB/100ft})/269.40$$

$$\beta = \text{phase constant in radians/m} \\ = 2\pi f \sqrt{LC} = 2\pi \sqrt{\epsilon_r}/\lambda$$

2.3.3 DETERMINATION OF THE LOAD IMPEDANCE FROM THE STANDING-WAVE PATTERN.

The load impedance can be calculated from a knowledge of the VSWR present on the line and the position of a voltage minimum with respect to the load, since the impedance at a voltage minimum is related to the VSWR as indicated by Equation (9b). The equation can be combined with Equation (12a) to obtain an expression for the load impedance in terms of the VSWR and the electrical distance, θ , between the voltage minimum and the load.

$$Z_x = \\ = Z_0 \cdot \frac{1 - j(\text{VSWR}) \tan \theta}{\text{VSWR} - j \tan \theta} = \quad (14a)$$

$$= Z_0 \cdot \frac{2(\text{VSWR}) - j(\text{VSWR}^2 - 1) \sin 2\theta}{(\text{VSWR}^2 + 1) + (\text{VSWR}^2 - 1) \cos 2\theta} \quad (14b)$$

Since in a lossless line the impedance is the same at half-wavelength intervals along the line, θ can be the electrical distance between a voltage minimum and any multiple of a half-wavelength from the load (see Figure 5). Of course, if the half-wavelength point used is on the generator side of the voltage

minimum located with the load connected, θ will be negative. The points corresponding to half-wavelength distances from the load can be determined by short-circuiting the line at the load and noting the positions of the voltage minima on the line. The minima will occur at multiples of a half-wavelength from the load.

If the VSWR is greater than $10 \tan \theta$, the following approximation of Equation (14b) gives good results:

$$R_x \cong \frac{Z_0}{\text{VSWR} \cdot \cos^2 \theta} \quad (15a)$$

$$X_x \cong -Z_0 \cdot \tan \theta \quad (15b)$$

2.3.4 SMITH CHART.

The calculation of the impedance transformation produced by a length of transmission line using the equations previously presented can be time consuming. Mr. P. H. Smith³ has devised a chart, shown in Figure 6, which simplifies these calculations. In this chart the circles whose centers lie on the resistance component axis correspond to constant values of resistance. The arcs of circles whose centers lie on an axis perpendicular to the resistance axis correspond to constant values of reactance. The chart covers all values of impedance from zero to infinity. The position of a point corresponding to any given complex impedance can be found from the intersection of the resistance and reactance coordinates corresponding to the resistive and reactive components of the unknown impedance.

As the distance from the load is increased or decreased, the impedance seen looking along the line toward a fixed unknown will travel around a circle with its center at the center of the chart. The angular movement around the circle is proportional to the electrical displacement along the line. One complete traverse of the circle will be made for each half-wavelength of travel. The radius of the circle is a function of the VSWR.

2.3.4.1 Calculation of Impedance at One Point from the Impedance at Another Point on a Line. If the impedance at one point on a line, say at a point p is known, and the impedance at another point a known

³ Smith, P. H., Electronics, Vol. 17, No. 1, pp. 130-133, 318-325, January 1944.

electrical distance away (for instance, at the load) is desired, the problem can be solved using the Smith Chart in the following manner: First, locate the point on the chart corresponding to the known impedance, as shown in Figure 6. (For example, assume that $Z_p = 20 + j25$ ohms.) Then, draw a line from the center of the chart through Z_p to the outside edge of the chart. If the point at which the impedance is desired is on the load side of the point at which the impedance is known, travel along the WAVELENGTHS TOWARD LOAD scale, from the intersection of the line previously drawn, a distance equal to the electrical distance in wavelengths between the point at which the impedance is known and the point at which it is desired. If the point at which the impedance is desired is on the generator side of the point at which the

impedance is known, use the WAVELENGTHS TOWARD GENERATOR scale. (In this example, assume that the electrical distance is 0.11 wavelength toward the load.) Next, draw a circle through Z_p with its center at the center of the chart, or lay out, on the last radial line drawn, a distance equal to the distance between Z_p and the center of the chart. The coordinates of the point found are the resistive and reactive components of the desired impedance. (In the example chosen, the impedance is $16 - j8$ ohms.)

The VSWR on the line is function of the radial distance from the point corresponding to the impedance, to the center of the chart. To find the VSWR, lay out the distance on the STANDING WAVE RATIO scale located at the bottom of the chart, and read the

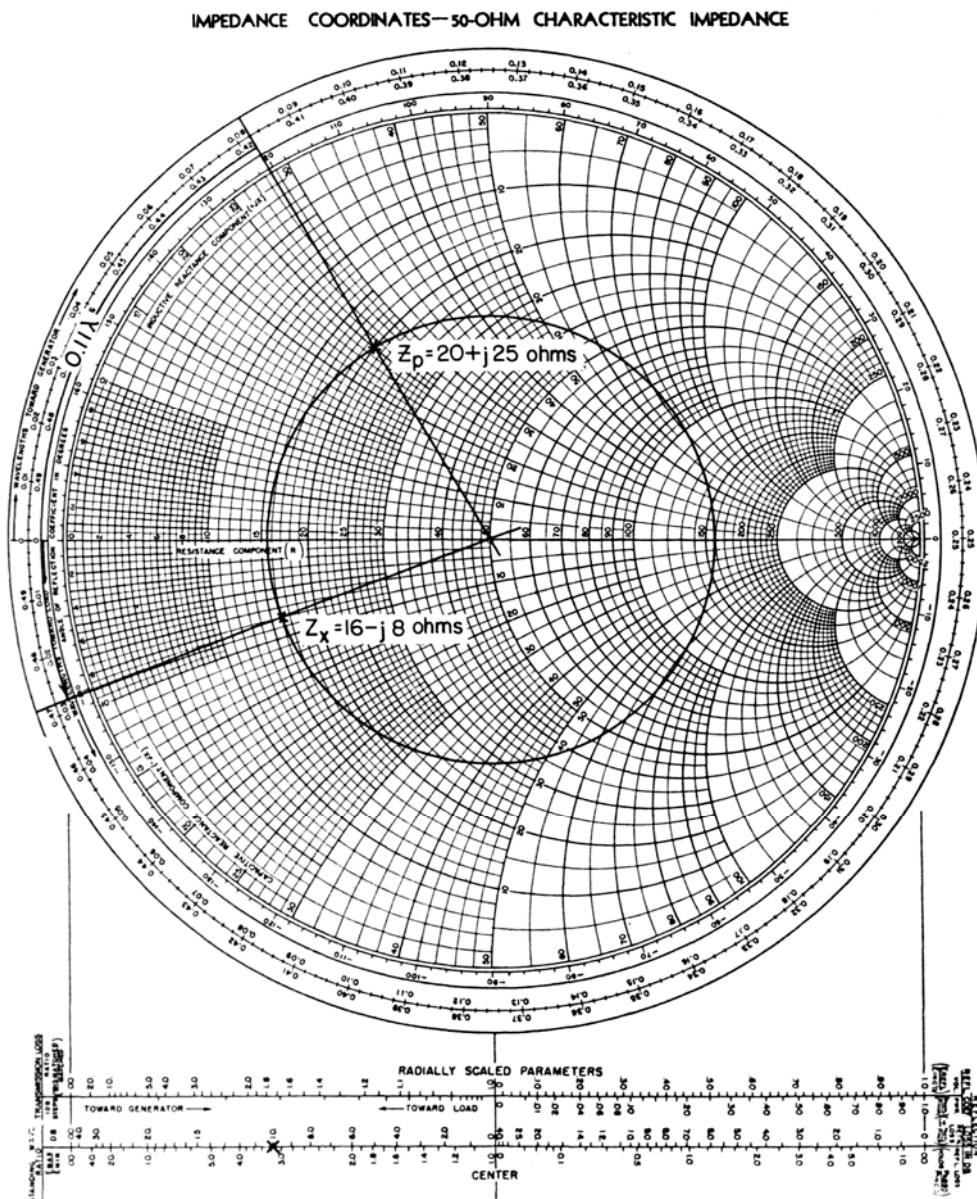


Figure 6. Illustration of the use of the Smith Chart for determining the impedance at a certain point along a line when the impedance a specified electrical distance away is known. In the example plotted, the known impedance, Z_p , is $20 + j25$ ohms and the impedance, Z_x , is desired at a point 0.11 wavelength toward the load from the point at which the impedance is known.

VSWR as a ratio, $\frac{E_{\max}}{E_{\min}}$, or in dB on the appropriate scale. (In the example of Figure 6, the VSWR is 3.2 or 10.1 dB.)

2.3.4.2 Calculation of Impedance at the Load from the VSWR and Position of a Voltage Minimum. In impedance measurements in which the voltage standing-wave pattern is measured, the impedance at a voltage minimum is a pure resistance having a magnitude of $\frac{Z_0}{\text{VSWR}}$. Plot this point on the resistance component axis and draw a circle having its center at the center of the chart drawn through the point. The impedance at any point along the transmission line

must lie on this circle. To determine the load impedance, travel around the circle from the original point an angular distance on the WAVELENGTHS TOWARD LOAD scale equal to the electrical distance, expressed as a fraction of a wavelength, between the voltage minimum and the load (or a point a half-wavelength away from the load, as explained in Paragraph 2.3.3.) If the half-wave point chosen lies on the generator side of the minimum found with the load connected, travel a round the chart in the opposite direction, using the WAVELENGTHS TOWARD GENERATOR scale. The radius of the circle can be determined directly from the VSWR, expressed as a ratio, or, if desired, in decibels by use of the scales labeled STANDING WAVE RATIO, located at the bottom of the chart.

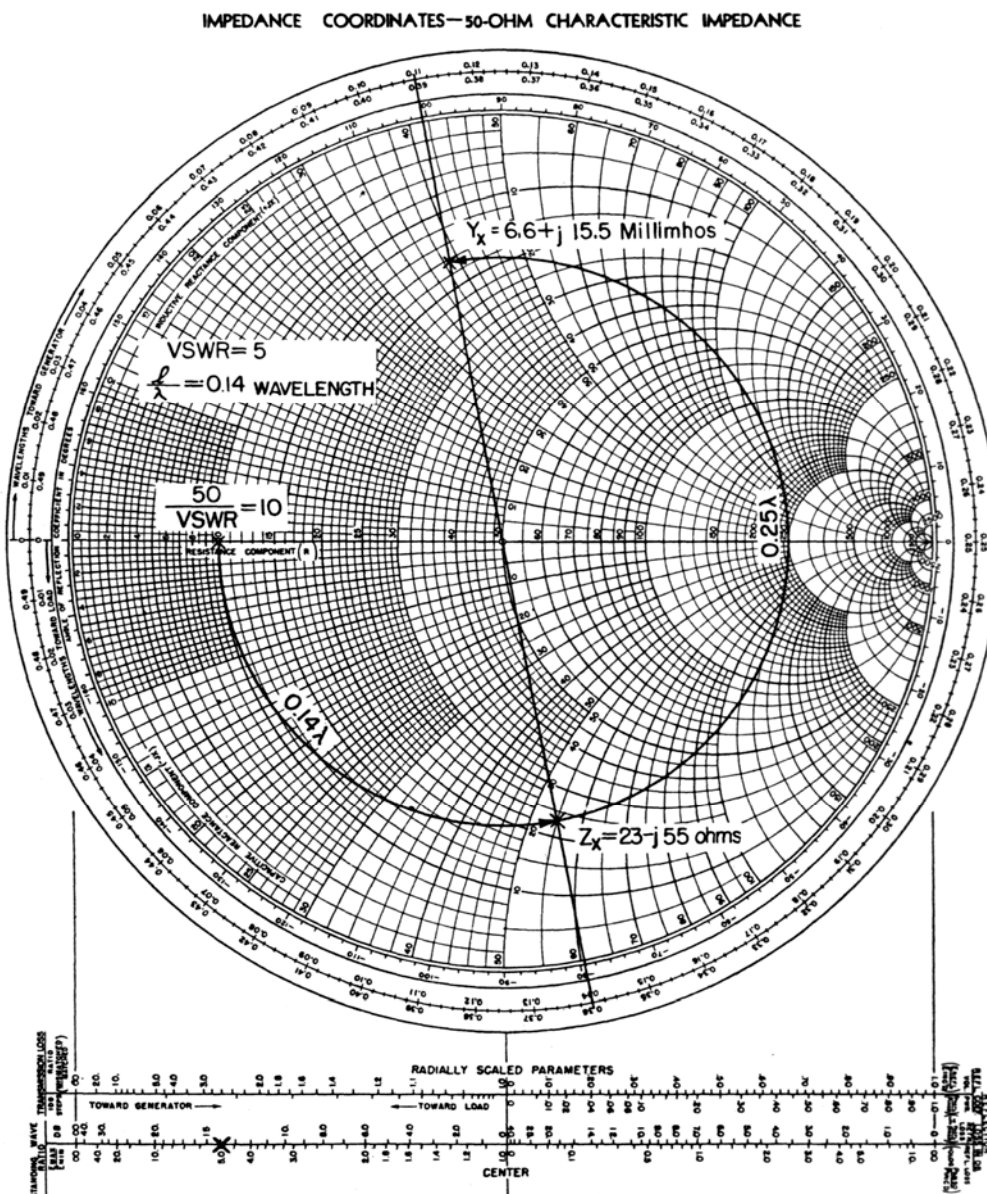


Figure 7. Example of the calculation of the unknown impedance from measurements of the VSWR and position of a voltage minimum, using a Smith Chart. The measured VSWR is 5 and the voltage minimum with the unknown connected is 0.14 wavelength from the effective position of the unknown. A method of determining the admittance of the unknown is also illustrated.

The example plotted on the chart in Figure 7 shows the procedure for determining the load impedance when the VSWR is 5 to 1, and the electrical distance between the load or a half-wavelength point and a voltage minimum is 0.14 wavelength. The unknown impedance, read from the chart, is $23 - j55$ ohms.

The Smith Chart can also be used when the line between the load and the measuring point is not lossless. The procedure for correcting for loss is outlined in Paragraph 4.6.2.

NOTE

Additional copies of the Smith Chart are available, drawn for a 50-ohm system in either impedance or admittance coordinates. The Impedance Chart,

similar to the one shown in Figure 6 but printed on transparent paper, is Form 756-Z. The Admittance Chart, similar to Figure 8, is Form 756-Y. A normalized chart, with an expanded center portion for low VSWR measurements, is also available on Form 756-NE.

2.3.4.3 Conversion from Impedance to Admittance.

The Smith Chart can also be used to obtain the transformation between impedance and admittance. Follow around the circle of constant VSWR a distance of exactly 0.25 wavelength from the impedance point. To obtain the conductance and susceptance in millimhos, simply multiply the coordinates of the newly determined point by 0.4 (see Figure 7). This conversion property is a result of the inversion of impedance every quarter-wavelength along a uniform transmis-

ADMITTANCE COORDINATES—20-MILLIMHO CHARACTERISTIC ADMITTANCE

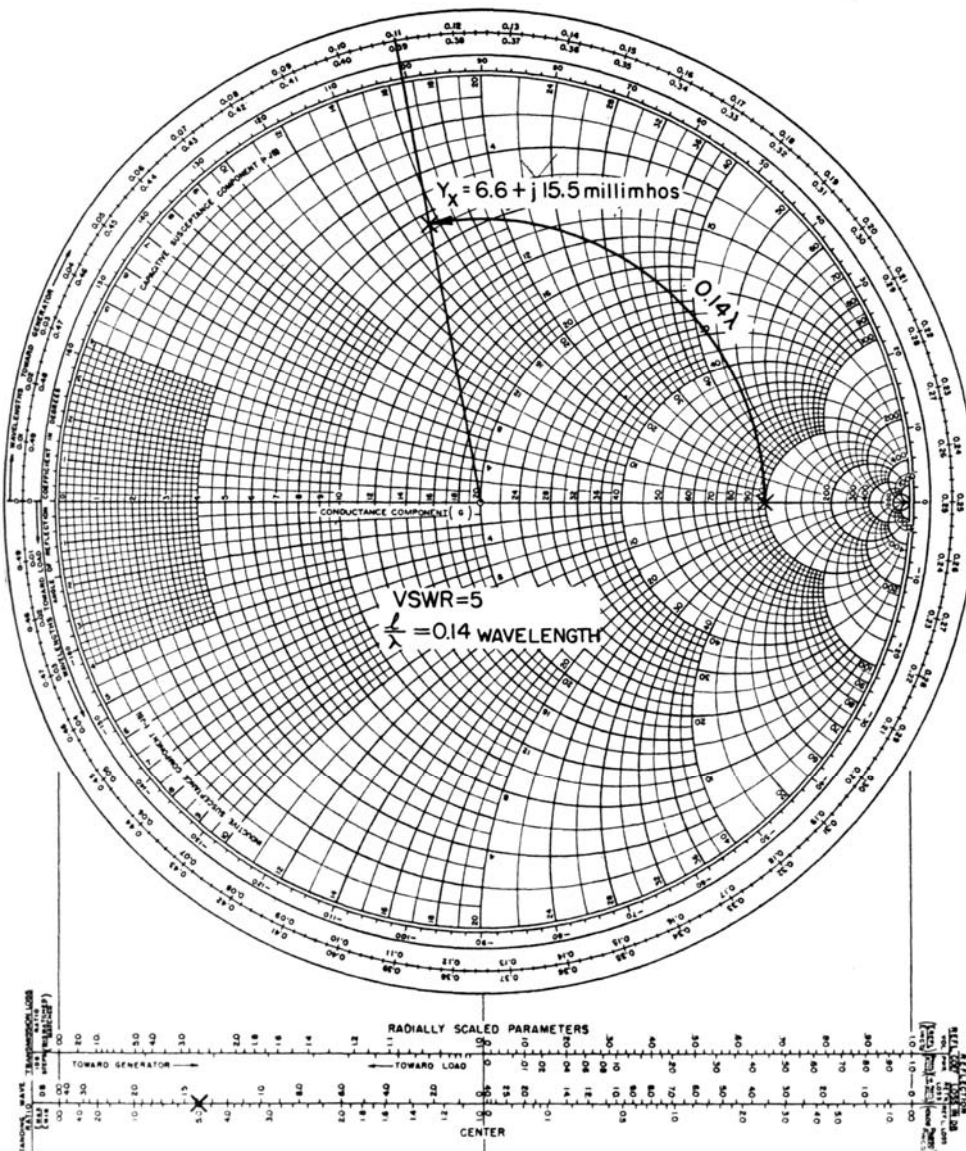


Figure 8. Example of the calculation of the unknown admittance from measurements of the VSWR and the position of a voltage minimum, using the Smith Chart drawn for admittance measurements on lines having characteristic admittances of 20 millimhos (50 ohms).

sion line. The impedances at points 1 and 2, a quarter-wavelength apart, are related by the equation

$$Z_1 = \frac{Z_0^2}{Z_2} \quad (16a)$$

or

$$Z_1 = Z_0^2 Y_2 \quad (16b)$$

2.3.4.4 Admittance Measurements using the Smith Chart. The admittance of the unknown can be obtained directly from a normalized Smith Chart, or from the chart shown in Figure 8, whose coordinates are admittance component, rather than by the procedure outline in Paragraph 2.3.4.3. When the chart shown in Figure 8 is used, the characteristic admittance, 20 millimhos, is multiplied by the measured VSWR to find the conductance at the voltage minimum. The radius of the corresponding admittance circle on the chart can be found by plotting the measured conductance directly on the conductance axis. The radius can also be found from the STANDING WAVE RATIO scale located at the bottom of the chart. The electrical distance to the load is found and laid off on the WAVE-

LENGTHS TOWARD LOAD scale, starting at 0.25 wavelength. On the VSWR circle, the coordinates of the point corresponding to the angle found on the WAVELENGTHS scale are the values of conductance and susceptance of the unknown.

The example plotted on the chart is the same as the used for the impedance example of Figure 7.

2.3.4.5 Use of Other Forms of the Smith Chart. In some forms of the Smith Chart, all components are normalized with respect to the characteristic impedance to make the chart more adaptable to all values of characteristic impedance lines. If normalized charts are used, the resistance component value used for the voltage-minimum resistance is $\frac{1}{\text{VSWR}}$ and the unknown impedance coordinates obtained must be multiplied by the characteristic impedance of the line to obtain the unknown impedance in ohms. If the admittance is desired, the coordinates that correspond to the admittance should be multiplied by the characteristic admittance.

The normalized Smith Chart is produced in a slide rule form by the Emeloid Corporation, Hillside, New Jersey.

SECTION 3

USEFUL FORMULAS

Characteristic impedance, Z_0 , of the line with loss

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\chi = \sqrt{(R + j\omega L)(G + j\omega C)}$$

The impedance, Z_p , at any point along the line with loss in distance l in relation to the load impedance Z_L

$$Z_p = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$$

The admittance, Y_p , at any point along the line with loss in distance l in relation to the load admittance Y_L

$$Y_p = Y_0 \frac{Y_L + Y_0 \tanh \gamma l}{Y_0 + Y_L \tanh \gamma l}$$

Characteristic impedance, Z_0 , of the lossless line

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$\chi = j\omega \sqrt{LC}$$

The impedance, Z_p , at any point along the lossless line in distance l in relation to the load impedance Z_L

$$Z_p = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

The admittance, Y_p , at any point along the lossless line in distance l in relation to the load admittance Y_L

$$Y_p = Y_0 \frac{Y_L + jY_0 \tan \beta l}{Y_0 + jY_L \tan \beta l}$$

The impedance, Z_{in} , at input point of the opened line with loss of length l

$$Z_{in} = Z_0 \tanh \gamma l \quad \lim_{l \rightarrow \infty} Z_{in}(l) = Z_0$$

The admittance, Y_{in} , at input point of the opened line with loss of length l

$$Y_{in} = Y_0 \tanh^{-1} \gamma l \quad \lim_{l \rightarrow \infty} Y_{in}(l) = Y_0$$

where $\gamma = \alpha + j\beta$, α = attenuation constant in nepers/m
 β = phase constant in radians/m
 $\beta = 2\pi f \sqrt{LC} = 2\pi \sqrt{\epsilon_r} / \lambda$

The impedance, Z_{in} , at input point of the opened lossless line of length l

$$Z_{in} = Z_0 \tan \beta l$$

The admittance, Y_{in} , at input point of the opened lossless line of length l

$$Y_{in} = Y_0 \tan^{-1} \beta l$$

where β = phase constant in radians/m
 $\beta = 2\pi f \sqrt{LC} = 2\pi \sqrt{\epsilon_r} / \lambda$

The load impedance Z_L calculated from the knowledge of the VSWR present on the line with impedance Z_0 and the position θ of a voltage minimum with respect to the load (see Figure 5), of the lossless line

$$Z_L = Z_0 \frac{1 - j(\text{VSWR}) \tan \theta}{\text{VSWR} - j \tan \theta} = Z_0 \frac{2(\text{VSWR}) - j[(\text{VSWR})^2 - 1] \sin 2\theta}{[(\text{VSWR})^2 + 1] + [(\text{VSWR})^2 - 1] \cos 2\theta}$$

and the VSWR and position θ back calculated from the load impedance Z_L

$$\text{VSWR} = \frac{1}{2Z_0 \text{Re}\{Z_L\}} \left(m \pm \sqrt{m^2 + 4Z_0^2 \text{Im}\{Z_L\}} \right) \quad \theta = \arctan \left[\frac{1}{2Z_0 \text{Im}\{Z_L\}} \left(m \pm \sqrt{m^2 + 4Z_0^2 \text{Im}\{Z_L\}} \right) \right]$$

where $m = Z_0^2 - |Z_L|^2$.

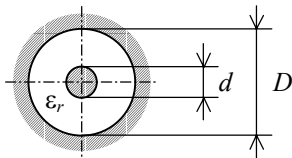
Transmission line Z_0 (Y_0) with load impedance Z_L (Y_L) and its relation to reflection coefficient Γ and VSWR

$$Z_L = \frac{1}{Y_L} = Z_0 \frac{1 + \Gamma}{1 - \Gamma} \quad Y_L = \frac{1}{Z_L} = Y_0 \frac{1 - \Gamma}{1 + \Gamma}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Y_0 - Y_L}{Y_0 + Y_L} \quad |\Gamma| = \frac{\text{VSWR} - 1}{\text{VSWR} + 1}$$

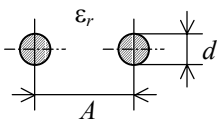
$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{|Z_L + Z_0| + |Z_L - Z_0|}{|Z_L + Z_0| - |Z_L - Z_0|} = \frac{|Y_0 + Y_L| + |Y_0 - Y_L|}{|Y_0 + Y_L| - |Y_0 - Y_L|}$$

TL1. Coaxial line impedance calculated from dimensions



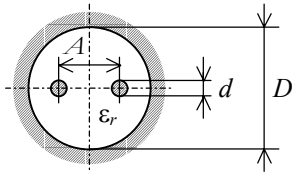
$$Z_0 = \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \log_e \left(\frac{D}{d} \right) \cong \frac{59.958}{\sqrt{\epsilon_r}} \log_e \left(\frac{D}{d} \right)$$

TL2. Two parallel lines impedance calculated from dimensions



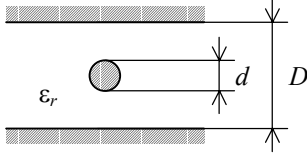
$$Z_0 = \frac{1}{\pi} \sqrt{\frac{\mu}{\epsilon}} \log_e \left(\frac{A}{d} + \sqrt{\left(\frac{A}{d} \right)^2 - 1} \right) \cong \frac{276}{\sqrt{\epsilon_r}} \log_{10} \left(\frac{2A}{d} \right) \Bigg|_{\frac{A}{d} \gg 1}$$

TL3. Two shielded parallel lines impedance calculated from dimensios



$$Z_0 = 120 \sqrt{\frac{\mu_r}{\epsilon_r}} \left[\log_e \left(\frac{2A}{d} \frac{D^2 - A^2}{D^2 + A^2} \right) - \frac{1 - \left(\frac{2A}{d} \right)^2}{\left(\frac{2A}{d} \right)^4} \left(1 - \left(\frac{2A}{D} \right)^2 \right) \right]$$

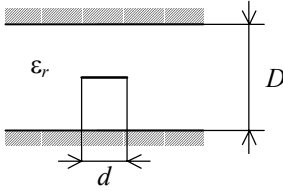
TL4. Impedance of line constructed by round wire symmetrically centered between boundless ground planes



$$\frac{60}{\sqrt{\epsilon_r}} \log_e \tan^{-1} \left(\frac{\pi d}{4D} \right) \leq Z_0 \leq \frac{60}{\sqrt{\epsilon_r}} \log_e \tanh^{-1} \left(\frac{\pi d}{4D} \right)$$

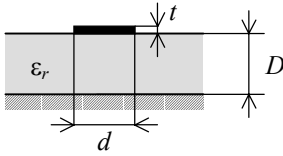
or $Z_0 \cong \frac{60}{\sqrt{\epsilon_r}} \log_e \left(\frac{\pi d}{4D} \right) \Big|_{d \ll D}$

TL5. Impedance of symmetrically centered track of zero thickness between boundless ground planes



$$Z_0 \cong \frac{\pi}{4} \sqrt{\frac{\mu}{\epsilon}} \frac{1}{\arg \sinh \exp \left(\frac{\pi d}{2D} \right) + \arg \cosh \exp \left(\frac{\pi d}{2D} \right)} \cong \frac{30\pi^2}{\sqrt{\epsilon_r} \log_e \left(4 \exp \left(\frac{\pi d}{D} \right) \right)}$$

TL6. Impedance of surface stripline over the boundless ground plane with dielectric material (PCB track impedance)



IPC-2141 – Controlled Impedance circuit Boards and High-Speed Logic Design, April 1996.

$$Z_0 \cong \frac{87}{\sqrt{\epsilon_r} + \sqrt{2}} \log_e \left(\frac{5.98D}{0.8d + t} \right) \Big|_{t \ll d, \frac{d}{D} \leq 2} \cong \frac{87}{\sqrt{\epsilon_r} + \sqrt{2}} \log_e \left(7.5 \frac{D}{d} \right) \Big|_{t \rightarrow 0, \frac{d}{D} \leq 2}$$

Wadell, B. C., Transmission Line Design Handbook, Artech House 1991.

$$Z_0 = \frac{60}{\sqrt{2(\epsilon_r + 1)}} \log_e \left(1 + \frac{4D}{d'} \sqrt{A + B} \right)$$

$$A = \frac{14\epsilon_r + 8}{11\epsilon_r} \cdot \frac{4D}{d'} \quad B = \sqrt{A^2 + \sqrt{\pi} \frac{\epsilon_r + 1}{2\epsilon_r}} \quad d' = d + \Delta d' \quad [\text{see Wadell}]$$

Electric permittivity is an electrical property of a dielectric defined in the SI system of units as

$$\epsilon = \epsilon_r \epsilon_0,$$

where ϵ_r is the dielectric constant, sometimes called the relative permittivity, and ϵ_0 is the permittivity of free space,

$$\epsilon_0 \triangleq 1/(c^2 \mu_0) = 8.8542 \times 10^{-12} \text{ Fm}^{-1} = 8.8542 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2},$$

where c is the speed of light, $c \triangleq 2.99792458 \times 10^8 \text{ ms}^{-1}$, μ_0 is the permeability of free space.

Magnetic permeability is the macroscopic quantity given by

$$\mu = \mu_r \mu_0,$$

where μ_r is the relative permeability and μ_0 is the permeability of free space.

$$\mu_0 \triangleq 4\pi \times 10^{-7} \text{ WbA}^{-1} \text{ m}^{-1} = 1.2566 \times 10^{-6} \text{ WbA}^{-1} \text{ m}^{-1}$$

That's all for the present. Mates, April 2004.